Iran University of Science &Technology

INTERNATIONAL CONFERENCE ON COMBINATORICS, CRYPTOGRAPHY AND COMPUTATION

Proceedings of the 2nd International Conference on Combinatorics, Cryptography and Computation (I4C2017)

The effect of the skewness of stock distribution on optimal stock selection

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ABSTRACT

Optimal portfolio selection is one of the financial issues which has been considered in several ways in recent decades. Behavior of distribution of returns must be studied carefully for portfolio selection. In this research, it is hypothesized that stock returns have skewed Normal distribution and also it studies optimal portfolio under skewness. Considering skewness, stimulation results showed that the way used in this research is more efficient than Markowitz traditional model.

Keywords: skewed Normal distribution, Monte Carlo stimulation, returns distribution, portfolio optimization.

1. INTRODUCTION

Investors are interested in portfolio optimization, as it is used to increase not only wealth but also maximum investment profit. If portfolio selection was divided into various branches, risk would be decreased under some conditions. So, this is their desire, too. The first modern portfolio theory was introduced by Markowitz in the mid-19th century, and it was used as a basis to solve the portfolio optimization issue. In this model, portfolio optimization was considered according to two criteria: returns increase, and risk reduction. Efficient investment market and risk averse investors were in Markowitz model conditions which may probably not happen in reality. Efficient marketing means a competitive environment provided for all and it makes a normal process statistically. In this research, it is hypothesized that returns distribution have skewed Normal distribution, and also skewness is used as evaluation criteria and model capability increasing. Azaleini introduced skewed Normal distribution in 1985 for the first time to model the behavior of variables with asymmetric structure.

Solving portfolio optimization is done in a variety of ways. Creating a new optimization model or a new definition for risk is considered in all methods. For example, Ziaoohuyang (2008) introduced the semi-variance in risk definition and based on the probabilities of a new model in solving this problem using the hybrid model. A multi-criteria decision model using Pareto-Walter J. Gotthjord et al. (2010) as well as multi-criteria and fuzzy randomization models of returns, and the use of Genetic Algorithm by Jun Lee et al. (2013) is another way of solving this problem. Yang Hyung Shin et al. (2011) optimized portfolios based on retirement age constraints and relative risk aversion. Sylvia Dayo et al. (2015) used a multivariate decision-making model using risk-worth and conditional value at risk and combining them to solve this problem. Among other models are numerical methods and genetic algorithms, as well as fuzzy mathematics, which have been used in recent years to solve the optimization problem. Using skewness coefficient as an important factor in solving the optimization problem in several researches, such as Chiang and colleagues' study on systematic skewness, have shown the utility of using skewness in correcting the traditional model using numerical methods.

This research consists of several sections.In the second part of this research, the model and concepts related to skewed Normal distribution in the portfolio optimization solution will be discussed, and in the third section, the simulation tables and results as well as efficient frontier graphs will be explained. In the end, suggestions will be mentioned in the fourth section.

2. MODEL AND SIMULATION

2.1. THEORETICAL MODEL

Markowitz presented the initial model for solving the portfolio optimization, assuming that investors were introduced as risk averse. By quantifying the concept of risk and using the standard deviation as a amount of risk, Markowitz could provide a framework for solving portfolio optimization. His model was based on maximizing stocks return and minimizing stocks risk. This template can be displayed as follows:

$$\text{Min} \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \tag{1}$$

$$\text{Subject to} \sum_{i=1}^{n} x_i \mu_i = R \tag{2}$$

$$\sum_{i=1}^{n} x_i = 1 \quad , \quad x_i \geq 0 \; , \qquad i = 1, \dots, n \tag{3}$$

Considering that the goal of portfolio optimizing is to maximize stock returns and minimize risks, the above model can be changed to the following one.

$$\text{Max} \sum_{i=1}^{n} x_i \mu_i \tag{4}$$

$$\text{Subject to } \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} = \sigma^2 \tag{5}$$

$$\sum_{i=1}^{n} x_i = 1 \quad , \quad x_i \geq 0 \;, \qquad i = 1, \dots, n \tag{6}$$

Suppose that X is the portfolio vector and μ is the random vector of the mean of returns on probability space with the cumulative distribution $F(.)$. Therefore, the final return of a portfolio is: $\mu^T X$.

Also, we show the matrix of the variance of covariance of returns with Σ. So, the above problem can be generalized in the following Algebraic for:

$$\begin{aligned} &\text{Max } \mu^T X \\ &\text{Subject to : } X^T \textstyle\sum X = \sigma^2 \quad , \quad \bar{1}^T X = 1 \end{aligned} \tag{7}$$

Where $\bar{1}$ is a vector, all of its elements being one. By using the Lagrange optimization method, the answer of the optimal portfolio is obtained from the solution of the following equation:

$$A = \text{Max}\left(\mu^T X + \lambda_0(\bar{1}^T X - 1) + \lambda_1(X^T \textstyle\sum X - \sigma^2)\right) \tag{8}$$

According to the Derivative the result will be:

$$\frac{\partial A}{\partial X} = \mu + \lambda_0 \bar{1} + 2\lambda_1 \textstyle\sum X = 0 \quad \rightarrow \quad X = \sum^{-1} \lambda'_0 \mu + \sum^{-1} \lambda'_1 \bar{1} \tag{9}$$

According to above subjects and the solution of Second-level device, the result must be:

$$\lambda'_0 = \frac{\bar{1}^T \sum^{-1} \bar{1}\, \sigma^2 - 1}{\bar{1}^T \sum^{-1} \bar{1}\, \mu_p - \bar{1}^T \sum^{-1} \mu} \tag{10}$$

$$\lambda'_1 = \frac{\mu_p - \bar{1}^T \sum^{-1} \mu\, \sigma^2}{\bar{1}^T \sum^{-1} \bar{1}\;\, \mu_p - \bar{1}^T \sum^{-1} \mu} \tag{11}$$

The above solution is based on Markowitz optimization and according to the two rates of return and risk that was introduced in the optimization model. The risk aversion of investors is considered in this model.

But since the return on each stock is calculated on the basis of the initial and the end prices of each share over a specified period, and the price is determined by the investor's demand, it should be considered as a random variable. In the following, it is assumed that the return rate of each share has a statistical distribution. So, any statistical distributions are selected for simulation. These distributions are chosen in a way that is skewed. We want to investigate the effect of skewness in solving the optimization problem.

Finally, using the skewed normal distribution, this effect will be investigated. The reason for the final choice of normal distribution is that in reality, the rate of returns is not in a normal distribution situation and is far from it. We assume that if stock returns are distributed asymmetrically, how do they affect the stock market optimality? Therefore, we look at the optimality of the stock portfolio when the distribution of returns is normal, as compared to being a Chi Square.

We assume that the stock return rate in the first stage has a normal distribution and we solve the stock portfolio optimization problem, and then in the second stage, we assume that the distribution of stock returns has a Chi Square distribution with mean and variance similar to the initial normal distribution and we solve the portfolio optimality. In both phases, using Splus software, we simulate and generate random data from normal distribution and Chi Square distributions for a 120-month financial period, and ultimately solve the optimization problem using Lingo software. This simulation is done according to the different degrees of freedom of the Chi Square distribution. The purpose of the study is to investigate the effect of skewness on the optimal solution of portfolios. The simulation results for the efficient frontier of these two distributions are indicated in the following diagrams. As can be seen, a distribution that has skewness is better than the normal distribution in the optimal portfolio response.

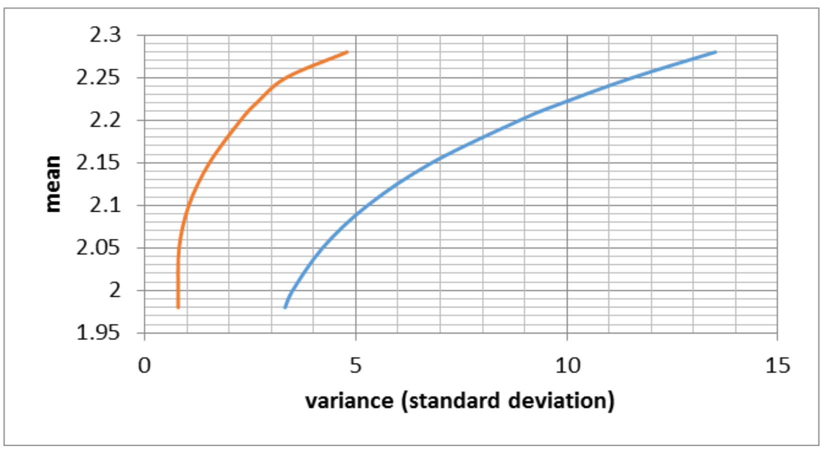

Figure 1: The blue diagram shows the normal distribution and the red diagram of the Chi Square distribution with two degrees of freedom, and both distributions have mean of 2 and variance of 4 **.**

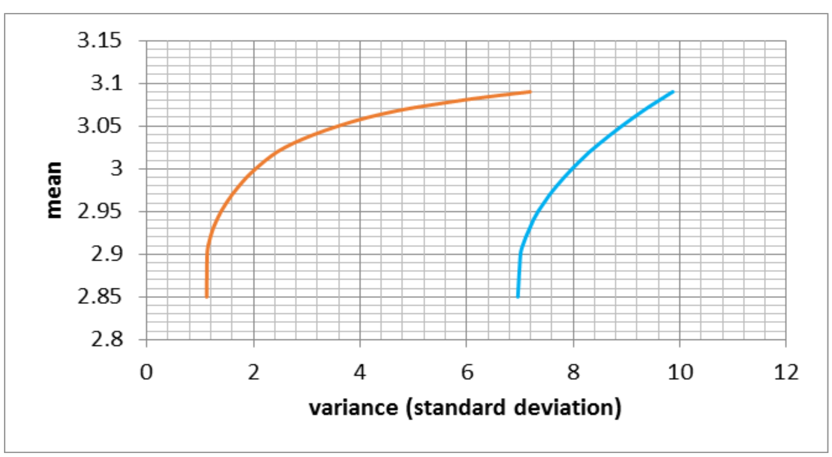

Figure 2: The blue diagram shows the normal distribution and the red diagram of the Chi Square distribution with two degrees of freedom, and both distributions have an average of 3 and a variance of 6.

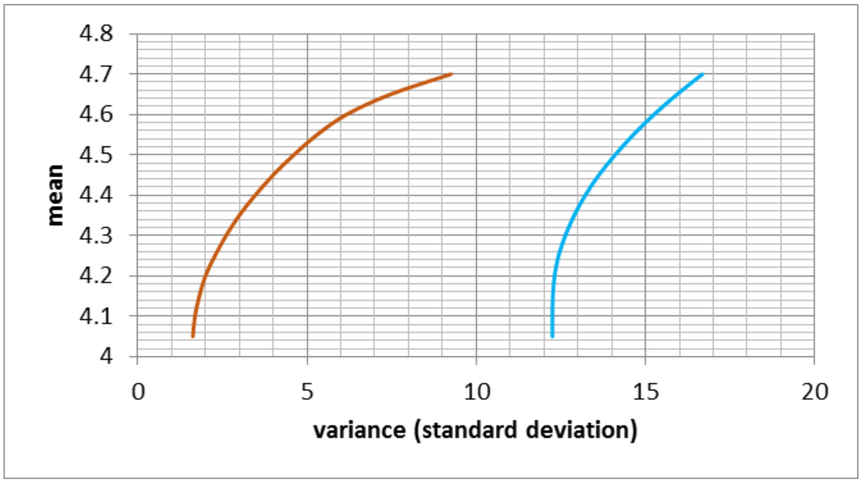

Figure 3: The blue diagram shows a normal distribution and a red diagram of a Chi Square distribution with two degrees of freedom, and both distributions have an average of 4 and a variance of 8.

With respect to the above simulations, it is shown that by increasing the degree of skewness, the optimum distance of the two distributions increases, so paying attention to the skewness of return distribution and entering the value of skewness in the model is effective in achieving the optimal portfolio.

2.2. ENTERING SIMULATION

One of the criteria used in solving the portfolio optimization using the Markowitz method is an efficient market and the normalization of stock returns, and the lower the correlation of the selected stocks, the better the portfolio operates. But it may not happen in a real. For example, returns distribution is normal but skewed. Investigating skewness as a benchmark for evaluating location and scale is useful. We assume that returns distribution is skewed- normal. By definition, the random variable Z has a normal distribution if the form of its density function is as follows:

$$f(x|\lambda) = 2\phi(x)\,\Phi(\lambda x) \qquad -\infty < x < \infty \tag{12}$$

In which φ (.) And Φ (.) is, respectively, the density of the standard normal distribution and the cumulative distribution function of the standard normal distribution. If the random variable Z has a normal distribution with a skew parameter λ, we show it with the symbol X ~ SN (λ). According to the above definitions and based on the main model of the optimization of the Markowitz portfolio optimization, we seek to adopt a model with the assumption of the distribution of stock return distribution, so that the portfolio optimization problem has a more favorable answer. Therefore, we use the following theorem to simulate random samples of skewed- normal distribution.

Theorem: Suppose that Y0 and Y1 are two independent standard normal random variables and δ is a number between (-1 and 1). In this case, $Z = \delta|Y_0| + (1-\delta^2)^{\frac{1}{2}} Y_1$ has a SN (λ) distribution in which $\lambda = \lambda(\delta) = \frac{\delta}{\sqrt{1-\delta^2}}$.

Considering the above assumptions for optimizing stock portfolios, the model is considered as follows:

$$\min \operatorname{var}(X) \tag{13}$$

$$\min \operatorname{skew}(X) \tag{14}$$

$$\text{s.t.} \qquad S(X) > \gamma \tag{15}$$

Or equal:

$$\min(\operatorname{var}(X) + \operatorname{skew}(x)) \tag{16}$$

$$\text{s.t.} \qquad S(X) > \gamma \tag{17}$$

Where γ is a constant value for displaying the minimum expected value. According to the Lagrange optimization method, a portfolio that is consistent with the following conditions is considered to be the optimal portfolio:

$$\frac{\partial\{E\left(x^t\mu - E(x^t\mu)\right)^2 + E\left(x^t\mu - E(x^t\mu)\right)^3 + \alpha_1 E((x^t\mu) - \gamma) + \alpha_2(\sum_{i=1}^n x_i - 1) + \alpha_3 \sum_{i=1}^n x_i\}}{\partial x}$$
$$= 2E\left(x^t\mu - E(x^t\mu)\right) + 3E(x^t\mu - E(x^t\mu))^2 + \alpha_1 E(\mu) + \sum_{i=1}^n (\alpha_2 + \alpha_3)e_i = 0 \tag{18}$$

Where:

$e_i = (0, \dots, 1 \dots, 0)^T \quad i = 1, \dots, n.$

3. MONTE CARLO SIMULATION AND RESULTS

3.1. MONTE CARLO SIMULATION

Suppose we have used the stock of five companies to form stock portfolios. We also assume that these stocks have a return on skewed- normal distribution. It was considered monthly returns for 5 years. As a result, we need up to 120 random samples to simulate. Since simulation is needed to ensure the accuracy of the results, therefore, it is needed to simulate the repetition, and the simulation results are reported, we repeat the simulation process and report three replication simulations. The simulation of this distribution was done using the above mentioned theorem using Splus software. In order to solve the optimization problem to compare the two Markowitz models and the model of this research, the Lingo computational software was used, the results of which were reported in the next section. According to the Monte Carlo simulation in solving the optimization problem, the corresponding model will be calculated as follows:

$$min \frac{1}{n}\left(\sum_{i=1}^{5} x_i^2\sigma_i^2 + x_i^3\gamma^3\right) \tag{19}$$

$$s.t.$$

$$\frac{1}{n}\sum_{j=1}^{N}\sum_{i=1}^{5}(x_i^t\mu_i) > \gamma \tag{20}$$

$$\sum_{i=1}^{5} x_i = 1 \tag{21}$$

$$x_i \geq 0 \quad i = 1, \dots, 5$$

It should also be noted that since the expected values of returns and the returns variance (risk) are calculated on the basis of historical data, for each of these companies, these two values are calculated by the two following formulas and according to the estimators (maximums likelihood) is calculated and is embedded in the formulas:

$$\widehat{\mu_{kt}} = \frac{1}{T}\sum_{i=1}^{T} r_{ki} \tag{22}$$

$$\widehat{\sigma_{kt}} = \frac{1}{T}\sum_{i=1}^{T}(r_{ki} - \widehat{\mu_{kt}})^2 \tag{23}$$

Where $\widehat{\mu_{kt}}$ is the expected return of company k over time t, $\widehat{\sigma_{kt}}$ the value of the variance of company returns (risk) k over the period t, r_{ki} of the return on each of the company's shares k in the period t and eventually T is the financial period or financial year, which is the same sample size that is considered in this simulation for 120 months. As a result, the returns and risks of the portfolio will be calculated from the following formulas:

$$\mu = \sum_{i=1}^{K} x_i \, \widehat{\mu_{it}} \tag{24}$$

$$\sigma = \sum_{i,j} x_i x_j \, \widehat{\sigma_{ijt}} \tag{25}$$

3.2. SIMULATION RESULTS

The results indicate that the model of this research is more desirable considering the existing value of skewness model in the distribution of shares compared to the Markowitz model. In order to generalize these conditions, the efficient frontier for all existing states is also optimized, and the relative efficiency of the model of this research is clearly evident. Using the existing effective boundary, for each expected return and risk, the selection coefficients of each stock (portfolio) are obtained. Also according to Sharp's ratio, a comparison has been made between the two models and the results are reported as follows. It can be seen that the ratio of return to risk (standard deviation) in the model of this research in different portfolios shows a more favorable outcome. This criterion shows that higher returns are obtained in return for less risk, which is the exact investors' utility.
Simulation results for 120 samples and replication simulations:

Table 1: comparative results between this research and Markowitz model.

mean	Variance of portfolio of this re search	Sharp ratio This research	Variance of Markowitz model	Sharp ratio Markowitz model
0.698	0.451	1.039	0.531	0.957
0.695	0.397	1.103	0.461	1.023
0.69	0.332	1.197	0.373	1.129
0.685	0.295	1.261	0.32	1.21
0.68	0.266	1.318	0.286	1.271
0.675	0.24	1.377	0.56	0.902
0.67	0.218	1.434	0.23	1.397
0.665	0.199	1.49	0.208	1.458
0.66	0.184	1.538	0.19	1.514
0.655	0.172	1.579	0.177	1.558
0.65	0.163	1.609	0.166	1.595
0.645	0.155	1.638	0.157	1.627
0.64	0.148	1.663	0.15	1.652
0.635	0.143	1.679	0.144	1.673
0.63	0.139	1.689	0.139	1.689
0.625	0.136	1.694	0.136	1.694

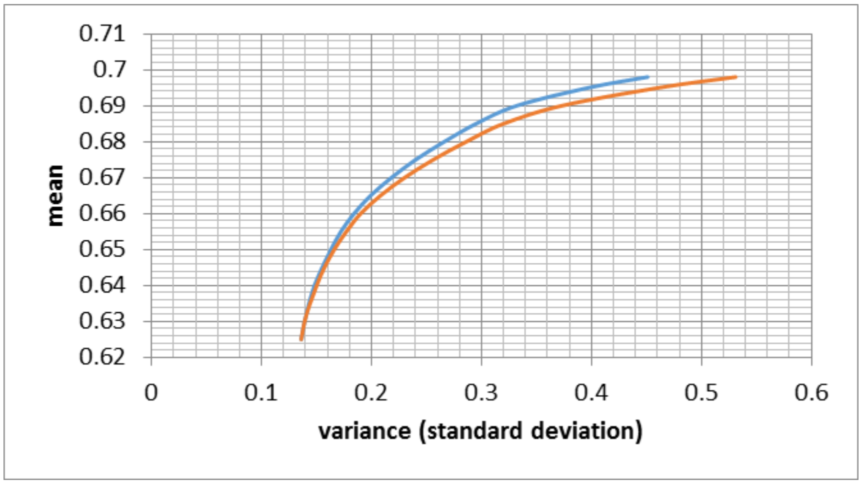

Figure 4: The blue diagram shows a model of this research (using skewness in portfolio optimization model) and a red diagram shows a Markowitz model in simulation 1.

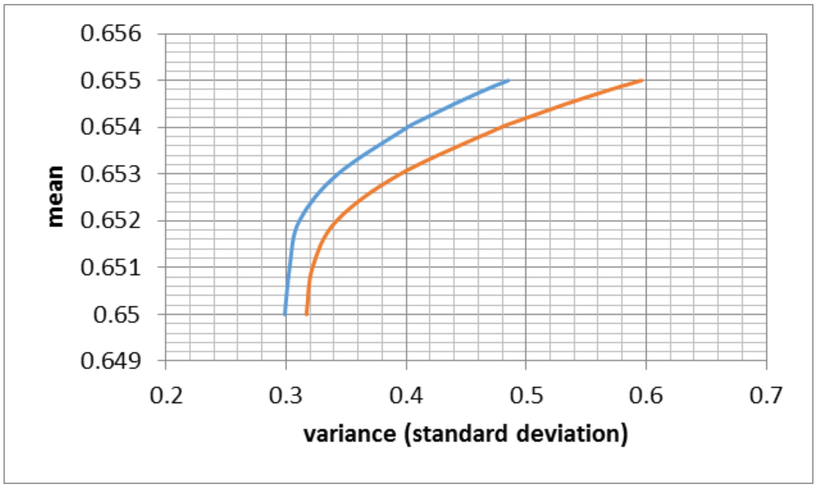

Figure 5: The blue diagram shows a model of this research (using skewness in portfolio optimization model) and a red diagram shows a Markowitz model in simulation 2.

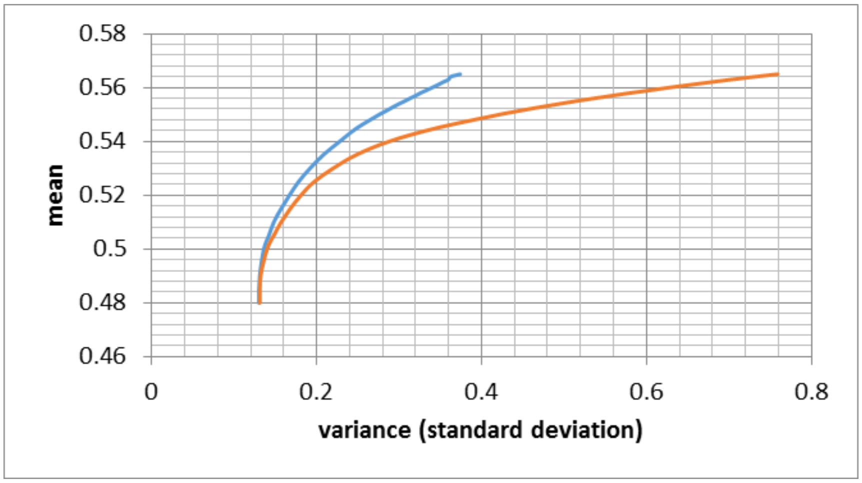

Figure 6: The blue diagram shows a model of this research (using skewness in portfolio optimization model) and a red diagram shows a Markowitz model in simulation 3.

4. FURTHER SUGGESTION

The behavior of stock returns in order to select the optimal portfolio and how to use the effective factors in the final model can be considered as an important criterion in choosing stock portfolios. In this research, based on the simulation, it has been found that the existence of skewness in the distribution of returns as an effective factor in the final model can lead to a better portfolio selection and may have less variance or lower risk than the average variance of the Markowitz model. Also, the use of skewness in the final model of portfolio optimization means simultaneous control of efficiency and risk and also increases the effectiveness of the optimization model. This, of course, can be pointed that higher moment in the optimization model to improve the results.

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