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A dynamic model for weapon-target assignment problem

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ABSTRACT

In the present research, we are going to obtain the solution of the Weapon-Target Assignment (WTA) problem. According to our search in the scientific reported papers, this is the first scientific attempt for resolving of WTA problem by projection recurrent neural network models. Here, by reformulating the original problem to an unconstrained problem a novel projection recurrent neural network model as high performance tool to provide the solution of the problem is proposed. In continuous, the global exponential stability of the system was proved in this research. In the final step, simulation results are presented to depict the performance and the feasibility of the method. Reported results were compared with some other published papers.

KEYWORDS: Weapon-target assignment problem, nonlinear optimization problem, projection recurrent neural network, global exponential stability.

1 INTRODUCTION

The weapon-target assignment (WTA) problem is to find a proper assignment of weapons to targets with the objective of minimizing the expected damage of own-force assets. This is an NP-complete problem [١]. Some methods such as combinatorial optimization [٢], pseudo-boolean programming [٣] (and the references therein) were given to solve these types of problems. These methods are based on graph search approaches and usually result in exponential computational complexities. As a consequence, it is difficult to solve these types of problems directly while the number of targets or weapons are large. Moreover, Genetic Algorithms (GAs) have been applied to solve this problem [4,5,6]. Even though those approaches could find the best solution in those simulated cases, the search efficiency did not seem good enough. The dynamic system method is one of the efficient approaches for solving programming problems. In fact, recurrent neural network models are well tool to transfer the optimization problems into a dynamic system. The main idea of such method for solving the mathematical programming problem is to apply a non-negative function which is called energy function and a dynamic system. An important requirement is that the energy function decreases monotonically as the dynamic system approaches an equilibrium point. Note that, the nature of the dynamic system method is parallel and distributed which is the main advantage of this scheme.

The pioneering works on recurrent neural network (RNN) models to optimization are for Hopfield and Tank [٧]. Neurodynamic optimization has received great success in recent years . For instance, Eshaghnezhad et. al [٨] developed an RNN for solving non-linear pseudo-monotone projection equation, Effati et. al [٩] gave a projection type NN model to solve bilinear programs, He et. al [١٠]

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proposed an inertial projection NN to solve VIs, Mansoori et. al [۱۱,۱۲] presented an efficient NN model to solve the absolute value equations.

Motivated by the former discussion, in spite of the fact that the several success in RNNs, it has some limits in solving zero-one programs with general convex objective functions. The WTA problem is a zero-one optimization problem with linear constraints. In this research, we try to develop a novel projection RNN for this problem. For this purpose, we assume the unconstrained form of the problem. Then, the novel proposed projection RNN model used to solve the WTA problem. On the final step, the globally exponentially stability of the proposed projection RNN is stated.

2 WEAPON-TARGET ASSIGNMENT PROBLEM FORMULATION

In this section a mathematical formulation of the WTA problem is presented. On modern battlefields, it is an important task for battle managers to make a proper WTA to defend own-force assets. As an example in considering anti-aircraft weapons of naval battle force platforms, threat targets may be launched from surface ships, aircrafts, or submarines. These targets have different probabilities of killing to platforms which are dependent on the target types. Thus, a WTA decision-aided system is strongly desired in helping and training planners to make proper decisions on the battlefield. Throughout the paper, in the WTA problem, we assume that there are W weapons and T targets and all weapons must be assigned to targets. In fact, this assumption is further restricted to W=T and that all targets must also be assigned. Such a restricted assumption, in fact, has largely reduced the search space, but also may restrict the diversity of solutions. Also, the individual probability of killing ($p_{\{ij\}}$) by assigning the i-th weapon to the j-th target is known for all i and j. This probability defines the effectiveness of the j-th weapon to destroy the i-th target. Hence, the WTA problem is to minimize the following cost function [۱۳]:

$$\min_{x} \quad J=\sum_{i=1}^{T} u_i \prod_{j=1}^{W}\left(1-p_{ij}\right)^{x_{ij}} \tag{1}$$

where $\prod_{j=1}^{W}\left(1-p_{ij}\right)^{x_{ij}}$ verifies the overall probability of the i-th target being destroyed and u_i is the expected damage value of the i-th target to the asset. Since, all weapons must be assigned to targets therefore,

$$\sum_{i=1}^{T} x_{ij} = 1, \quad for \quad j=1,2,\ldots,W, \tag{2}$$

where x_{ij} is a boolean value indicating whether the j-th weapon is assigned to the i-th target. $x\_ij=1$ indicates that the j-th weapon is assigned to the i-th target. Thus, equation (۱) summarizes the overall damage for all targets. Moreover, equation (2) can be rewritten as follow when W=T=N:

$$\sum_{i=1}^{N} x_{ij} = 1, \qquad \sum_{j=1}^{N} x_{ij} = 1, \quad for \quad j=1,2,\ldots,N.$$

Summarizing the former discussion, the WTA problem can be modelled as zero-one nonlinear ptimization problem:

$$\begin{aligned} &\min_{x} \quad J=\sum_{i=1}^{T} u_i \prod_{j=1}^{W}\left(1-p_{ij}\right)^{x_{ij}} \\ &s.t. \quad \sum_{i=1}^{N} x_{ij} = 1, \qquad j=1,2,\ldots,N, \end{aligned} \tag{3}$$

$$\sum_{j=1}^{N} x_{ij} = 1, \qquad j = 1,2, \dots, N,$$
$$x_{ij} \in \{0,1\}^{N^2}.$$

Because of the total unimodality property of the constraint coefficient matrix defined in (3) [١٤], the integrality constraint in the WTA problem formulation can be equivalently replaced with the non-negativity constraint . By the above discussion the following equivalent nonlinear programming problem can be formulated:

$$\begin{aligned} \min_{x} \quad & J = \sum_{i=1}^{T} u_i \prod_{j=1}^{W} \left(1 - p_{ij}\right)^{x_{ij}} \\ s.t. \quad & \sum_{i=1}^{N} x_{ij} = 1, \qquad j = 1,2, \dots, N, \\ & \sum_{j=1}^{N} x_{ij} = 1, \qquad j = 1,2, \dots, N, \\ & x_{ij} \geq 0. \end{aligned} \tag{4}$$

Next result shows that the cost function J in (4) is convex.

Theorem 2.1. The function $J: [0,1]^{N^2} \to R$ is convex (note that $x_{ij} \geq 0$ can not exceed from 1).

From the above analysis, it can be seen that the WTA problem formulation is a convex nonlinear programming problem. Thus, in next section we consider the general form of the convex nonlinear programming problem with linear constraints and proposing a neurodynamic model to solve the original problem.

3 NEURODYNAMIC MODEL

Consider the following convex nonlinear programming problem with linear constraints:

$$\begin{aligned} \min \quad & f(x) \\ s.t. \quad & Ax = b, \\ & x \geq 0. \end{aligned} \tag{5}$$

where f(.) is twice continuously differentiable and convex in R^n. The Lagrangian of (٥) is constructed by:

$$L(x,u,v) = f(x) - \frac{1}{2}u^2 x + v(Ax - b), \tag{6}$$

where u and v are the Lagrange multipliers. Thereby, the KKT optimality conditions are given as:

$$KKT\ conditions: \begin{cases} u^*, x^* \geq 0, u^* x^* = 0, \\ \nabla f(x^*) - u^* + A^T v^* = 0\,, \\ Ax^* = b. \end{cases} \tag{7}$$

Theorem ٢,١ shows that the objective function of the problem is convex and therefore, (5) is a convex problem. Thus, the KKT optimality conditions are both the necessary and the sufficient conditions for optimality. Now, our goal is to describe a neurodynami model satisfying the KKT optimality conditions. In accordance with the convexity of ((٥ the gradient of L (.,.,.) must be zero at optimal point.

Therefore, we describe the neurodynamic model for solving the original problem (٥) as the following nonlinear dynamical system:

$$\frac{dx}{dt} = -\left(\nabla f(x) - \frac{1}{2}u^2 + A^T v\right) \tag{8}$$

$$\frac{du}{dt} = -diag(u)x \tag{9}$$

$$\frac{dv}{dt} = Ax - b. \tag{10}$$

For simplifying the proposed neurodynamic model we define:

$$\Theta(y) = \Theta(x,u,v) = \begin{pmatrix} -\left(\nabla f(x) - \frac{1}{2}u^2 + A^T v\right) \\ -diag(u)x \\ Ax - b \end{pmatrix},$$

then the proposed model can be summarized as follow:

$$\frac{dy}{dt} = \zeta\Theta(y), \tag{11}$$

where $\zeta > 0$ is the convergence rate.

Theorem 3.1. $(x^*, u^*, v^*)^T$ is the optimal solution of (7), if and only if, y^* is the equilibrium point of the neurodynamic model (11).

4 STABILITY ANALYSIS

Here, we study the stability analysis of the proposed model (11).

Lemma 4.1. There exists a unique solution y(t) for the model (11).

Theorem٤.٢ . The proposed model (11) is stable in the sense of Lyapunov.

5 SIMULATION RESULTS

This section, gives some simulation results to show the performance of the proposed model. The codes are developed using symbolic computation software MATLAB (ode45) and the calculations are implemented on a machine with Intel core 7 Duo processor 2 GHz and 8 GB RAM. Consider the WTA problem with N=5. We set the random parameters as follow:

$$u = (1, 3, 5, 2, 4), \quad p_{i,j} = \begin{bmatrix} 0.6 & 0.5 & 0.8 & 0.7 & 0.4 \\ 0.9 & 0.5 & 0.7 & 0.8 & 0.4 \\ 0.8 & 0.9 & 0.4 & 0.5 & 0.8 \\ 0.8 & 0.9 & 0.8 & 0.6 & 0.4 \\ 0.7 & 0.4 & 0.5 & 0.6 & 0.7 \end{bmatrix}$$

By using the proposed method and applying the above parameters the optimal solution of the problem is:

$$x_{\{1,4\}} = x_{\{2,1\}} = x_{\{3,2\}} = x_{\{4,3\}} = x_{\{5,5\}} = 1, \quad J = 2.7.$$

Fig. 1. displays the transient behaviour of the continuous-time neurodynamic model (11).

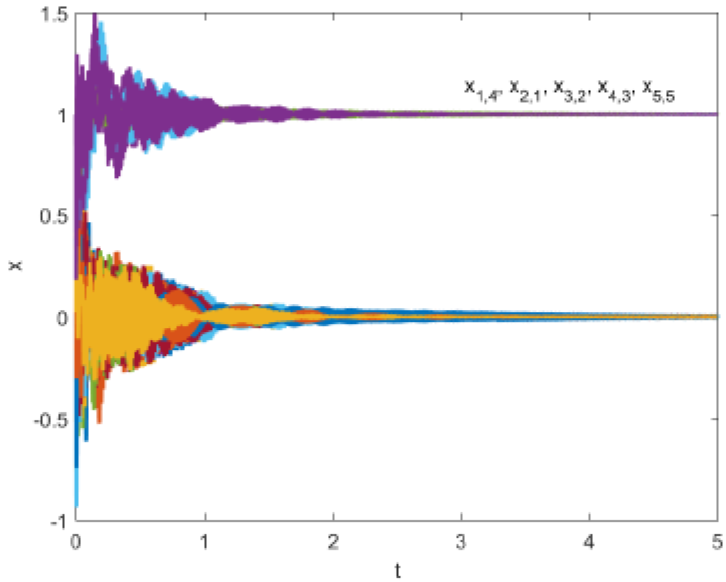

Figure 1: Transient behaviour of the continuous-time model (11) for N = 5.

6 CONCLUSIONS

Through the paper, a novel dynamic model for solving the WTA problem was proposed. The given model was solved by an ODE. The proposed model is a one-layer design. Reported results were compared with some other methods. Moreover, the dynamic system approach does not depend on the initial point; the reason is that our model is globally convergent to the optimal solution of the problem. At last, the work is in progress to extend the other recurrent models for solving other optimization problems.

REFERENCES

[1] S. P. Lloyd and H. S. Witsenhausen, “Weapon allocation is NP-complete," in IEEE Summer Simulation Conf., Reno, NV, 1986.

[2] A. M. H. Bjorndal et al., “Some thoughts on combinatorial optimization", *European Journal of Operational Research*, pp. 253-270, 1995.

[3] P. L. Hammer, Some network ow problems solved with pseudo-boolean programming", *Operations Research*, vol. 13, pp. 388-399, 1965.

[4] P. Merz and B. Freisleben, Fitness landscape analysis and memetic algorithms for quadratic assignment problem"' *IEEE Transactions on Evolutionary Computation*, vol. 4, pp. 337-352, 2000.

[5] Z. -J. Lee, C. -Y. Lee, and S. -F. Su, \A fuzzy-genetic based decision aided system for the naval weapon-target assignment problems", in Proc. 2000 R.O.C. Automatic Control Conf., 2000, pp. 163-168.

[6] Z. -J. Lee, S. -F. Su, and C. -Y. Lee, \Efficiently Solving General Weapon-Target Assignment Problem by Genetic Algorithms With Greedy Eugenics", *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 33, no. 1, pp. 113-121, 2003.

[7] D. W. Tank and J. J. Hop_eld, \Simple 'neural' optimization networks: an A/D converter, signal decision circuit, and a linear programming circuit", *IEEE Transactions on Circuits and Systems*, vol. 33, pp. 533-541, 1986.

[٨] M. Eshaghnezhad, S. Effati, A. Mansoori, \A Neurodynamic Model to Solve Nonlinear Pseudo-Monotone Projection Equation and Its Applications", *IEEE Transactions on Cybernetics*, vol. 47, no. 10, pp. 3050-3062, 2017.

[٩] S. Effati, A. Mansoori, M. Eshaghnezhad, \An efficient projection neural network for solving bilinear programming problems", *Neurocomputing*, vol. 168, pp. 1188-1197, 2015.

[١٠] X. He, T. Huang, J. Yu, C. Li, and C. Li, An Inertial Projection Neural Network for Solving Variational Inequalities", *IEEE Transactions on Cybernetics*, vol. 47, no. 3, pp. 809 - 814, 2017.
[١١] A. Mansoori and M. Erfanian, A dynamic model to solve the absolute value equations, *Journal of Computational and Applied Mathematics*, **333** (2018), 28-35.
[١٢] A. Mansoori, M. Eshaghnezhad, and S. Effati, An efficient neural network model for solving the absolute value equations", *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 65, no. 3, pp. 391-395, 2018.
[١٣] Hosein, P. A Class of Dynamic Non Linear Resource Allocation Problems", PhD Thesis, Massachusetts Institute of Technology, MA, 1989.
[١٤] M. S. Bazaraa, C. Shetty, and H. D. Sherali, *Linear Programming and Network Flows*, New York: John Wiley and Sons, 2010.