# ON THE TOPOLOGY INDUCED BY A INTUITIONISTIC GENERALIZED FUZZY METRIC

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ABSTRACT. In this paper, we study the topology induced by a intuitionistic generalized fuzzy metric and show some results follow directly from well–known theorems in generalized fuzzy metric spaces.

#### 1. INTRODUCTION

Intuitionistic fuzzy metric spaces were investigated by Park [1]. He introduced and studied intuitionistic fuzzy metric spaces based both on the idea of intuitionistic fuzzy sets due to Atanassov [2] and the concept of fuzzy metric spaces given by George and Veeramani in [3]. The topology of intuitionistic fuzzy metric spaces modified by Grigori et. al. [4] also see [5]. Sun and Yang [6] defined generalized fuzzy metric spaces using the ideas of generalized metric spaces due to Mustafa and Sims [7] and fuzzy sets; see [10, 11] for some applications. Recently, Mohiuddine and Alotaibi [12] introduced and studied intuitionistic generalized fuzzy metric spaces. In this note we prove that the topology  $\tau_{(G,H)}$  generated by an intuitionistic generalized fuzzy metric space  $(X, G, H, *, \diamond)$  coincides with the topology  $\tau_G$  generated by the generalized fuzzy metric space (X, G, \*), and thus, the results obtained in [12] are immediate consequences of the corresponding results for generalized fuzzy metric spaces. Our paper is motivated from ideas in [5].

# 2. Preliminaries

A binary operation  $*:[0,1]\times[0,1]\longrightarrow[0,1]$  is a continuous t-norm if it satisfies the following conditions

- (a) \* is associative and commutative,
- (b) \* is continuous,
- (c) a \* 1 = a for all  $a \in [0, 1]$ ,

(d)  $a * b \le c * d$  whenever  $a \le c$  and  $b \le d$ , for each  $a, b, c, d \in [0, 1]$ .

Two typical examples of continuous *t*-norm are a \* b = a.b and  $a * b = \min(a, b)$ . A binary operation  $\diamond : [0, 1] \times [0, 1] \longrightarrow [0, 1]$  is a continuous *t*-conorm if it satisfies the following conditions:

- (a)  $\diamond$  is associative and commutative,
- (b)  $\diamond$  is continuous,
- (c)  $a \diamond 0 = a$  for all  $a \in [0, 1]$ ,
- (d)  $a \diamond b \leq c \diamond d$  whenever  $a \leq c$  and  $b \leq d$ , for each  $a, b, c, d \in [0, 1]$ .

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Two typical examples of continuous t-conorm are  $a \diamond b = \min(a + b, 1)$  and  $a \diamond b = \max(a, b)$ .

Mustafa and Sims [7] introduced and studied the concept of  $\mathcal{G}$ -metric spaces. Let X be a non-empty set. A function  $\mathcal{G} : X \times X \times X \longrightarrow [0, +\infty)$  is called a  $\mathcal{G}$ -metric if the following conditions are satisfied :

- (i)  $\mathcal{G}(x, y, z) = 0$  if x = y = z(coincidence),
- (ii)  $\mathcal{G}(x, x, y) > 0$  for all  $x, y \in X$ , where  $x \neq y$ ,
- (iii)  $\mathcal{G}(x, x, z) \leq \mathcal{G}(x, y, z)$  for all  $x, y, z \in X$ , with  $z \neq y$ ,
- (iv)  $\mathcal{G}(x, x, z) = \mathcal{G}(p\{x, y, z\})$ , where p is a permutation of x, y, z (symmetry),
- (v)  $\mathcal{G}(x, y, z) \leq \mathcal{G}(x, a, a) + \mathcal{G}(a, y, z)$  for all  $x, y, z, a \in X$ (rectangle inequality).

A  $\mathcal{G}$ -metric is said to be symmetric if  $\mathcal{G}(x, y, y) = \mathcal{G}(y, x, x)$  for all  $x, y \in X$ . Now, we give two examples of  $\mathcal{G}$ -metrics.

Let (X, d) be a metric space. The function  $\mathcal{G}_1: X^3 \longrightarrow [0, +\infty)$  defined by

$$\mathcal{G}_1(x, y, z) = \max\{d(x, y), d(y, z), d(x, z)\}$$

for all  $x, y, z \in X$  is a  $\mathcal{G}$ -metric.

Let  $X = \mathbb{R}$ . The function  $\mathcal{G}_2$  defined by

$$\mathcal{G}_2(x, y, z) = \frac{1}{3}(|x - y| + |y - z| + |x - z|)$$

for all  $x, y, z \in \mathbb{R}$  is a  $\mathcal{G}$ -metric.

In 2010, Sun and Yang [6] introduced the concept of generalized fuzzy metric spaces with the help of generalized metric spaces due to Mustafa and Sims [7] and fuzzy metric spaces due to George and Veeramani [3].

**Definition 2.1.** A 3-tuple (X, G, \*) is said to be a *G*-fuzzy metric space (denoted by GF-space) if X is an arbitrary nonempty set, \* is a continuous t-norm and *G* is a fuzzy set on  $X^3 \times (0, +\infty)$  satisfying the following conditions for each t, s > 0:

(GF-1) G(x, x, y, t) > 0 for all  $x, y \in X$  with  $x \neq y$ ;

(GF-2) 
$$G(x, x, y, t) = G(x, y, z, t)$$
 for all  $x, y, z \in X$  with  $z \neq y$ ;

(GF-3) G(x, y, z, t) = 1 if and only if x = y = z;

(GF-4) G(x, y, z, t) = G(p(x, y, z), t), where p is a permutation function;

(GF-5)  $G(x, a, a, t) * G(a, y, z, s) \le G(x, y, z, t+s)$  (the triangle inequality);

(GF-6)  $G(x, y, z, \cdot) : (0, +\infty) \longrightarrow (0, 1]$  is continuous (the function G is continuous in the fourth place).

For example, if a \* b = a.b for  $a, b \in [0, 1]$  and

$$G(x, y, z, t) = \frac{t}{t + \mathcal{G}_1(x, y, z)}$$

for all  $x, y, z \in X$  and t > 0. Then G is a (standard) G-fuzzy metric and (X, G, \*) is a G-fuzzy metric space.

**Example 2.2.** Let  $X = \mathbb{R}$  and a \* b = a.b for  $a, b \in [0, 1]$  and

$$M(x, y, t) = \begin{cases} \frac{x+t}{y+t}, & \text{if } x \le y, \\ \frac{y+t}{x+t}, & \text{if } y \le x, \end{cases}$$

for all  $x, y \in X$  and t > 0. Then M is a fuzzy metric and (X, M, \*) is a fuzzy metric space [8]. Now, by example 2.10 of [9]

$$G(x, y, z, t) = M(x, y, t) * M(y, z, t) * M(z, x, t)$$

is a G-fuzzy metric which is not induced by any G-metrics.

Sun and Yang showed in [6] that every generalized fuzzy metric (G, \*) on X generates a first countable topology  $\tau_G$  on X which has as a base the family of open sets of the form  $\{B_G(x, r, t) : x \in X, r \in (0, 1), t > 0\}$  where  $B_G(x, r, t) = \{y \in X : G(x, y, y, t) > 1 - r\}$  for all  $x \in X, r \in (0, 1)$  and t > 0.

## 3. INTUITIONISTIC GENERALIZED FUZZY METRIC SPACE

Recently, Mohiuddine and Alotaibi [12] introduced intuitionistic generalized fuzzy metric spaces using the concepts of continuous t-norm and t-conorm.

**Definition 3.1.** [12] The 5-tuple  $(X, G, H, *, \diamond)$  is said to be an intuitionistic generalized fuzzy metric space (for short, IGFM-space) if X is an arbitrary nonempty set, \* is a continuous t-norm,  $\diamond$  is a continuous t-conorm, and G, H are fuzzy sets on  $X^3 \times (0, +\infty)$  satisfying the following conditions. For every  $x, y, z, a \in X$  and s, t > 0,

(i)  $G(x, y, z, t) + H(x, y, z, t) \le 1$ , (ii) G(x, x, y, t) > 0 for  $x \neq y$ , (iii) G(x, x, y, t) = G(x, y, z, t) for  $y \neq z$ , (iv) G(x, y, z, t) = 1 if and only if x = y = z, (v) G(x, y, z, t) = G(p(x, y, z), t), where p is a permutation function, (vi)  $G(x, a, a, t) * G(a, y, z, s) \le G(x, y, z, t+s),$ (vii)  $G(x, y, z, .): (0, +\infty) \rightarrow [0, 1]$  is continuous, (viii) G is a non-decreasing function on  $\mathbb{R}^+$  $\lim_{t \to +\infty} G(x, y, z, t) = 1, \ \lim_{t \to 0} G(x, y, z, t) = 0, \ \text{for all } x, y, z \in X, \ t > 0,$ (xi) H(x, x, y, t) < 1 for  $x \neq y$ , (x) H(x, x, y, t) = H(x, y, z, t) for  $y \neq z$ , (xi) H(x, y, z, t) = 0 if and only if x = y = z, (xii) H(x, y, z, t) = H(p(x, y, z), t), where p is a permutation function, (xiii)  $H(x, a, a, t) \diamond H(a, y, z, s) \ge H(x, y, z, t+s),$ (xiv)  $H(x, y, z, .): (0, +\infty) \to [0, 1]$  is continuous, (xv) H is a non-increasing function on  $\mathbb{R}^+$  $\lim_{t \to +\infty} H(x, y, z, t) = 0, \ \lim_{t \to 0} H(x, y, z, t) = 1, \ \text{for all } x, y, z \in X, \ t > 0.$ 

In this case, the pair (G, H) is called an intuitionistic generalized fuzzy metric on X.

Note that, the conditions (i) and (ii) imply the condition (xi), therefore we can remove it. Also the conditions (i) and part two of (viii) imply the part two of condition (xv), therefore we can remove it too.

**Example 3.2.** [12] Let  $(X, \mathcal{G})$  be a  $\mathcal{G}$ -metric space. For all  $x, y, z \in X$  and every t > 0, consider G and H, to be fuzzy sets on  $X^3 \times (0, +\infty)$  defined by

$$G(x, y, z, t) = \frac{t}{t + \mathcal{G}(x, y, z)},$$
$$H(x, y, z, t) = \frac{\mathcal{G}(x, y, z)}{t + \mathcal{G}(x, y, z)},$$

and denote a \* b = ab and  $a \diamond b = \min\{a + b, 1\}$  for  $a, b \in [0, 1]$ . Then  $(X, G, H, *, \diamond)$  is a IGFM-space. Notice that the above example holds even with the t-norm  $a * b = \min(a, b)$  and the t-conorm  $a \diamond b = \max(a, b)$ . This kind of intuitionistic generalized fuzzy metric is said to be, the standard intuitionistic generalized fuzzy metric.

Mohiuddine and Alotaibi proved in [12] that every intuitionistic generalized fuzzy metric (G, H) on X generates a first countable topology  $\tau_{(G,H)}$  on X which has as a base the family of open sets of the form  $\{B_{(G,H)}(x,r,t): x \in X, r \in (0,1), t > 0\}$  where  $B_{(G,H)}(x,r,t) = \{y \in X : G(x,y,y,t) > 1 - r, H(x,y,y,t) < r\}$  for all  $x \in X, r \in (0,1)$  and t > 0.

**Lemma 3.3.** Let  $(X, G, H, *, \diamond)$  be an IGFM-space. Then, for each  $x \in X$ ,  $r \in (0,1)$  and t > 0 we have  $B_{(G,H)}(x, r, t) = B_G(x, r, t)$ .

*Proof.* It is clear that  $B_{(G,H)}(x,r,t) \subseteq B_G(x,r,t)$ .

Now, suppose that  $y \in B_G(x, r, t)$ . Then G(x, y, y, t) > 1 - r, so, by condition (i) of Definition 3.1, we have

$$1 \ge G(x, y, y, t) + H(x, y, y, t) > 1 - r + H(x, y, y, t).$$

Hence H(x, y, y, t) < r, and consequently  $y \in B_{(G,H)}(x, r, t)$ . The proof is finished.

From Lemma 3.3, we deduce the following.

**Theorem 3.4.** Let  $(X, G, H, *, \diamond)$  be an IGFM-space. Then the topologies  $\tau_{(G,H)}$  and  $\tau_G$  coincide on X.

# 4. Coupled coincidence point theorems for contractions in generalized fuzzy metric spaces and in intuitionistic generalized fuzzy metric spaces

In Section 3 we showed that the topology induced by the generalized fuzzy metric G and the intuitionistic generalized fuzzy metric (G, H) coincide on X. Now, we show that, the results obtained in [12] are immediate consequences of the corresponding results for generalized fuzzy metric spaces due to Hu and Luo [11]. For deep study and more results we refer to [13]–[35]

**Remark 4.1.** Example 2.3 in [12] is the same as Example 2.1 of [11].

*Proof.* Since in [12, Example 2.3], the authors supposed

$$H(x, y, z, t) = 1 - G(x, y, z, t)$$

for every  $x, y, z \in X$  and t > 0. Then, we have

$$H(x, y, z, t) = \frac{|x - y| + |y - z| + |z - x|}{t + |x - y| + |y - z| + |z - x|}.$$

By [11, Example 2.1] we have  $G(F(x, y), F(x, y), F(u, v), \phi(t)) = 1$  which implies that  $H(F(x, y), F(x, y), F(u, v), \phi(t)) = 0$  and vice versa. Thus, it is verified that the functions  $F, g, \phi$  satisfy all the conditions of [11, Theorem 3.1]. Here (0,0) is the coupled coincidence point of F and g in X, which is also their common coupled fixed point which imply that the same result for [12, Example 2.3].

**Remark 4.2.** Definition 2.5, Remark 2.6, Definition 2.7 and Theorem 2.10 from [12] are the same as Remark 2.3 and Definition 2.7 of [11].

*Proof.* By the same method used in Remark 4.1 the proof is straightforward. Note that, in Theorem 2.8 of [12] if  $G(x_j, x_j, x, t)$  tends to 1 then by Definition 3.1 (i),  $H(x_j, x_j, x, t)$  tends to 0.

Remark 4.3. Definition 2.9 of [12] is the same as Definition 2.7 (2) of [11].

*Proof.* Let  $\{x_n\}$  be a Cauchy sequence at (X, G, \*). Then by Definition 2.7 (2) of [11] we have, for any  $\varepsilon > 0$  and for each t > 0, there exists  $n_0 \in \mathbb{N}$  such that

$$G(x_n, x_n, x_m, t) > 1 - \varepsilon, \tag{4.1}$$

for  $m, n \ge n_0$ . Now, by Definition 3.1 (i) and (4.1) we have

$$H(x_n, x_n, x_m, t) < \varepsilon. \tag{4.2}$$

Then,  $\{x_n\}$  is a Cauchy sequence at  $(X, G, H, *, \diamond)$ . The converse is easy.

Remark 4.4. Definition 3.1 of [12] has additional conditions.

Proof. If

$$\lim_{n \to +\infty} G(gF(x_n, y_n), gF(x_n, y_n), F(g(x_n), g(y_n)), t) = 1$$

then by Definition 3.1 (i) we have

$$\lim_{n \to +\infty} H(gF(x_n, y_n), gF(x_n, y_n), F(g(x_n), g(y_n)), t) = 0.$$
(4.3)

Also, if

$$\lim_{n \to +\infty} G(gF(y_n, x_n), gF(y_n, xy_n), F(g(y_n), g(x_n)), t) = 1$$

then by Definition 3.1 (i) we have

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$$\lim_{n \to +\infty} H(gF(y_n, x_n), gF(y_n, x_n), F(g(y_n), g(x_n)), t) = 0.$$
(4.4)

Thus conditions (4.3) and (4.4) are additional and if we remove them we get the original Definition 3.5 of [11].  $\Box$ 

**Remark 4.5.** From Remark 4.3 and Theorem 3.4, Lemma 3.3 of [12] is the same as Lemma 2.5 of [11].

*Proof.* By the same method used in Remark 4.1 the proof is straightforward.  $\Box$ 

Now we state the main theorem of [12].

**Theorem 4.6.** Let  $(X, \geq)$  be a partially ordered set and  $(X, G, H, *, \diamond)$  be a complete IGFM-space. Suppose that  $F : X \times X \to X$  and  $g : X \to X$  are mappings such that F has the mixed g-monotone property, and also assume that there exists  $\phi \in \Phi$  such that

$$G(F(x,y), F(x,y), F(u,v), \phi(t))$$

$$\geq G(gx, gx, gu, t) * G(gx, gx, F(x,y), t) * G(gu, gu, F(u,v), t)$$
(4.5)

and

$$H(F(x,y),F(x,y),F(u,v),\phi(t))$$

$$\leq H(gx,gx,gu,t) \Diamond H(gx,gx,F(x,y),t) \Diamond H(gu,gu,F(u,v),t)$$

$$(4.6)$$

for all  $x, y, u, v \in X$  and t > 0 with  $g(x) \leq g(u)$  and  $g(y) \geq g(v)$ , or  $g(x) \geq g(u)$ and  $g(y) \leq g(v)$ . Suppose that  $F(X \times X) \subset g(X)$ , g is continuous and F and g are compatible with respect to (G, H), and also suppose that either

- (a) F is continuous, or
- (b) X has the following property:
  - (i): if a non-decreasing sequence  $x_n \to x$  with respect to (G, H) then  $x_n \leq x$  for all n,
  - (ii): if a non-decreasing sequence  $y_n \to x$  with respect to (G, H) then  $y_n \ge y$  for all n.

If there exists  $x_0, y_0 \in X$  such that  $g(x_0) \leq F(x_0, y_0)$  and  $g(y_0) \geq F(y_0, x_0)$ , then there exists  $x, y \in X$  such that g(x) = F(x, y) and g(y) = F(y, x); that is, F and ghave a coupled coincidence point.

*Proof.* Let  $x_0, y_0 \in X$  such that  $g(x_0) \leq F(x_0, y_0)$  and  $g(y_0) \geq F(y_0, x_0)$ . Then by Theorem 3.1 of [11] there are sequences  $\{g(x_n)\}$  and  $\{g(y_n)\}$  which are Cauchy with respect to the topology induced by G, so from Remark 4.3 and Theorem 3.4 they are Cauchy with respect to the topology induced by (G, H). Since X is complete then there exists  $x, y \in X$  such that

$$\lim_{n \to +\infty} F(x_n, y_n) = \lim_{n \to \infty} g(x_n) = x,$$

$$\lim_{n \to +\infty} F(y_n, x_n) = \lim_{n \to \infty} g(y_n) = y,$$

and g(x) = F(x, y) and g(x) = F(x, y). Hence F and g have a coupled coincidence point in X.

**Remark 4.7.** Example 3.6 of [12] is the same as Example 3.2 of [11], since

$$H(x, y, z, t) = 1 - G(x, y, z, t).$$

*Proof.* By the same method used in Remark 4.1 the proof is straightforward.  $\Box$ 

# 5. Conclusion

In this paper, we showed although the topology induced by fuzzy G-metrics is different of topology induced by G-metrics, but the topology induced by a intuitionistic generalized fuzzy metric which is different by topology induced by G-metric and showed some results followed directly from well-known theorems in generalized fuzzy metric spaces. There are some new problems which are very interesting to prove in fuzzy G-metric spaces, therefore we recommend them which ones can be find in [36, 37, 38, 39, 40, 41, 42].

#### AUTHOR CONTRIBUTIONS

All authors conceived the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

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#### CONFLICTS OF INTEREST

The authors declare that they have no competing interests.

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