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Computing the Geometric Arithmetic Index of Vertex Gluing Graphs

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ABSTRACT

The first geometric–arithmetic index GA and the atom-bond connectivity index ABC are two well-known molecular descriptors, which are found to be useful tools in QSPR/QSAR investigations. The first geometric–arithmetic index GA was proposed by Vukičević and Furtula. This index is defined as $GA(G) = \sum_{uv\in E(G)} \frac{2\sqrt{d_u d_v}}{d_u+d_v}$, we denotes d_u the degree of vertex u in G. In this paper, we compute the general formula for geometric arithmetic GA index of certain graphs such as $k - Bridge$. In addition to, we found general formula to the geometric arithmetic GA index for the vertex gluing of graphs such as n, m −Bridge graph $Q_n^m(v_1)$.

KEYWORDS: Geometric–arithmetic GA index, $k - Bridge$, Vertex Gluing.

1 INTRODUCTION

Topological indices are useful tools for modeling physical and chemical properties of molecules, for design of pharmacologically active compounds, for recognizing environmentally hazardous materials, etc. [1]. There are many publications on the topological indices; see [2]–[5]. The geometric arithmetic GA index gives better correlation coefficients than Randic index for these properties, but the differences between them are not significant. However, the predicting ability of the GA index compared with Randic index is reasonably better [6]. The first geometric–arithmetic index GA was proposed by Vukičević and Furtula [7]. This index is defined as $GA(G) = \sum_{uv\in E(G)} \frac{2\sqrt{d_u d_v}}{d_u+d_v}$. It was demonstrated [7], on the example of octane isomers, that the GA index is well correlated with a variety of physico-chemical properties such as entropy, enthalpy of vaporization, standard enthalpy of vaporization, enthalpy of formation and acentric factor. Moreover, the quality of these correlations was found to be better than for other often employed molecular descriptors [8]. Sigarreta obtained new inequalities involving the geometric-arithmetic index and other well known topological indices such as relate the geometric-arithmetic index of a graph G with its first Zagreb index , its second Zagreb index, its modified Zagreb index and its Randic index [9]. Abdelgader et al. computed the geometric arithmetic and atom bond connectivity indices of special graphs such as Cayley tree, Square lattice and Complete bipartite [10]. Sardar et al. computed of the line graphs such as Banana tree and Firecracker graph [11]. Farahani and Rajesh Kanna computed the geometric-arithmetic and another index to V-phenylenic Nanotubes and Nanotori [12]. The mathematical properties of the GA and other indices were reported in [13]–[19]. Such a graph will be denoted by $G = (V, E)$, where $V = \{v_1, \dots, v_{n-1}, v_n\}$ and $E = E(G)$ are the vertex set and edge set of G, respectively. If $v_i v_j \in E$, then $G -$

v_iv_j will denote the graph obtained from G by deleting the edge v_iv_j. If $v_iv_j \notin E$, then $G - v_iv_j$ will denote the graph obtained from G by adding the edge v_iv_j. When examining a topological index, one of the fundamental questions that needs to be answered is for which graphs this index assumes minimal and maximal values and what are these extremal values. Mohanad et al. studied the general formula for ABC index of certain graphs and vertex gluing of graphs such as (K_4-homeomorphism, complete bipartite, k −bridge graph and vertex gluing) [20]. Das and Trinajstić [21] compared the ABC index and the GA indices for molecular graphs and general graphs. Bin Yang et al. obtained extremal acyclic, unicyclic and bicyclic graphs with minimum and maximum EA index by a unified method, respectively [22]. In this paper, we investigate in the first section the GA index of a special graph as well as r −bridge graphs. Also, we compute geometric arithmetic of a new graph by joining two special graphs such as (r −bridge graphs) as vertex gluing graphs finding general formulas.

2 BASIC DEFINITION AND KNOWN RESULTS:

A graph consisting of r paths joining two vertices is called an $r - bridge$ graph, which is denoted by $Q(a_1, a_2, \dots, a_r)$**,** where $a_1, a_2, \dots, a_r$**,** are the lengths of r paths ($E = \sum_{i=1}^{r} a_i$, E the number of edges). Clearly an $r - bridge$ graph is a generalized polygon tree as shown in Figure 1.

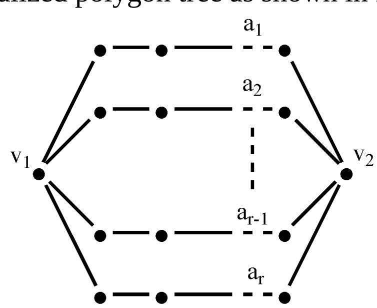

Figure 1. $r - bridge$ graph.

We first give results of the geometric arithmetic GA index for some simple graphs. Let K_n, C_n, S_n *and* G be the complete, cycle, star and regular graphs respectively with n vertices[3].

Lemma2.1.

Consider the complete graph K_n of order n. The Geometric-arithmetic (GA) index of this graph is computed as follows: $GA(K_n) = \frac{n(n-1)}{2}$

Lemma 2.2.

Suppose C_n is a cycle of length n labeled by $1, 2, \dots, n$. Then the Geometric-arithmetic (GA) index of this cycle is $GA(C_n) = n$.

Lemma 2.3.

Suppose S_n is the star on n vertices, then the Geometric-arithmetic (GA) Index of this graph is computed as follows: $GA(S_n) = \frac{2(n-1)^{\frac{3}{2}}}{n}$

Lemma 2.4.

If G is a regular graph of degree $r > 0$, then $GA(G) = \frac{nr}{2}$.

3 THE GEOMETRIC ARITHMETIC GA INDEX OF SPECIAL GRAPH:

In this section, we obtain the general formulas for a special graphs such as $k - Bridge$ graphs.

Theorem 3.1.

Let r be a positive integer, the geometric arithmetic GA index of a $r-Bridge$ graph denoted by $(a_1, a_2, \dots, a_r)$, $a_r \geq 2$ is

$GA\big(Q(a_1, a_2, \dots, a_r)\big) = \frac{4r\sqrt{2r}-4r-2r^2}{2+r} + E$, ($E = \sum_{i=1}^{r} a_i$, E the number of edges)

Proof:

In $r-Bridge$ graph there are $E = \sum_{i=1}^{r} a_i$ edges, $2r$ edges of them are incident on two vertices of degree r and degree 2. The remaining $E-2r$ edges are incident on two vertices of degree 2.

Now, we have,

$E_1 = \{uv \in E(G) | d_u = r \,\&\, d_v = 2\} = 2r.$

$E_2 = \{uv \in E(G) \,\big|\, d_u = d_v = 2\} = E - 2r.$

By using the definition of geometric arithmetic (GA) index of G, we have following computation for the geometric-arithmetic GA index of $r-Bridge$ graph as follow:

$$GA\big(Q(a_1, a_2, \dots, a_r)\big) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}$$

$$= \sum_{e=uv \in E_1} \frac{2\sqrt{d_u d_v}}{d_u + d_v} + \sum_{e=uv \in E_2} \frac{2\sqrt{d_u d_v}}{d_u + d_v}$$

$$= E_1 \frac{2\sqrt{2r}}{2+r} + E_2 \frac{2\sqrt{2\times 2}}{2+2}$$

$$= 2r \frac{2\sqrt{2r}}{2+r} + (E - 2r)$$

$$= \frac{4r\sqrt{2r}-4r-2r^2}{2+r} + E$$

■

4 THE GEOMETRIC ARITHMETIC GA INDEX OF CERTAIN VERTEX GLUING GRAPH:

In this section, we obtain the general formulas for geometric arithmetic GA index to the vertex gluing of graphs.

Let v_1-gluing of $n, m-$Bridge graph be a graph obtained from two different $K-$Bridge graphs Q_1 and Q_2 with one common vertex v_1 denoted by $Q_n^m(v_1)$ (vertex gluing of graph), as shown in Figure 7.

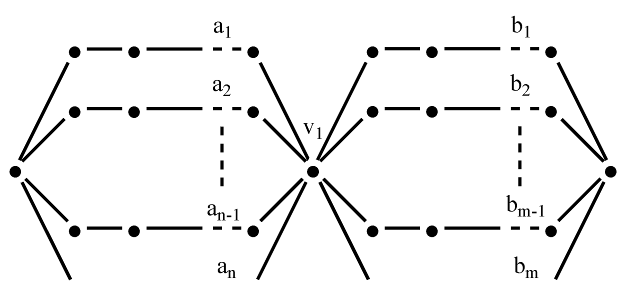

Figure 7. v_1-gluing of $n, m-$Bridge graph $Q_n^m(v_1)$

Theorem 4.1.

Let n, m and s be positive integers. Then, the geometric arithmetic GA index of the v_1-gluing of $n, m-$Bridge graph $G = Q_n^m(v_1)$, $a_i, b_j \geq 2$ is

$$GA\left(Q_n^m(v_1)\right) = \frac{2n\sqrt{2n}}{n+2} + \frac{2m\sqrt{2m}}{m+2} + \frac{2(n+m)\sqrt{2(n+m)}}{n+m+2} - 2(n+m) + \sum_{i=1}^{n} a_i + \sum_{j=1}^{m} b_j$$

Proof:

In the v_1-gluing of n, m −Bridge graph $Q_n^m(v_1)$ there are $\left(\sum_{i=1}^{n} a_i + \sum_{j=1}^{m} b_j\right)$ edges, n of them are incident on two vertices of degree n and 2 also m of them are incident on two vertices of degree m and 2. Therefore, $(n+m)$ edges are incident on two vertices of degree $(n+m)$ and 2. The remaining $\left(\sum_{i=1}^{n}(a_i - 2) + \sum_{j=1}^{m}(b_j - 2)\right)$ edges are incident on two vertices of the same degree 2.

Now, we have,

$E_1 = \{uv \in E(G) | d_u = n \;\&\; d_v = 2\} = n.$

$E_2 = \{uv \in E(G) | d_u = m \;\&\; d_v = 2\} = m.$

$E_3 = \{uv \in E(G) \,|\, d_u = n + m \;\&\; d_v = 2\} = n + m.$

$E_4 = \{uv \in E(G) | d_u = d_v = 2\} = \sum_{i=1}^{n}(a_i - 2) + \sum_{j=1}^{m}(b_j - 2).$

By using the definition of geometric arithmetic (GA) index of G, we have following computation for the geometric-arithmetic GA index of the v_1-gluing of n, m −Bridge graph as follow:

$$GA\left(Q_n^m(v_1)\right) = \sum_{uv\in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}$$

$$= \sum_{e=uv\in E_1} \frac{2\sqrt{d_u d_v}}{d_u + d_v} + \sum_{e=uv\in E_2} \frac{2\sqrt{d_u d_v}}{d_u + d_v} + \sum_{e=uv\in E_3} \frac{2\sqrt{d_u d_v}}{d_u + d_v} + \sum_{e=uv\in E_4} \frac{2\sqrt{d_u d_v}}{d_u + d_v}$$

$$= E_1 \frac{2\sqrt{2n}}{n+2} + E_2 \frac{2\sqrt{2m}}{m+2} + E_3 \frac{2\sqrt{2(n+m)}}{n+m+2} + E_4 \left(\frac{2\sqrt{2\times 2}}{2+2}\right)$$

$$= \frac{2n\sqrt{2n}}{n+2} + \frac{2m\sqrt{2m}}{m+2} + \frac{2(n+m)\sqrt{2(n+m)}}{n+m+2} - 2(n+m) + \sum_{i=1}^{n} a_i + \sum_{j=1}^{m} b_j \qquad \blacksquare$$

5 CONCLUSION:

The study of topological descriptors are very useful to acquire the basic topologies of graphs. In this paper, we obtained some relations for degrees between a derived graph and its parent graph. Using these new structures, we computed geometric arithmetic indices of some derived graphs of some the parent graphs such as k −Bridge graphs and vertex gluing of graphs.

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