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Neighbourhood Degree - Based Topological Indices of $TuC_4C_8[\alpha, \beta]$ and its corresponding Paraline Graph

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ABSTRACT

Topological descriptors defined on chemical structures are effective in understanding those properties and activities of the chemical molecules. This research area has drawn the attention of many researchers, primarily due to its biological and empirical chemistry applications. In this paper, we estimate a few neighbourhood degrees sum based topological indices such as SK index, SK1 index, SK2 index, Modified Randić index, and Inverse Sum Index for the $TuC4C8[\alpha, \beta]$ and its corresponding paraline graph.

KEYWORDS: $TuC_4C_8[\alpha, \beta]$, Paraline Graphs, Topological Indices.

1 INTRODUCTION

Chemical reaction network theory is a field of applied mathematics that aims to mimic real world chemical structure activity. It has gained an increasing scientific community following since its start in the 19th century, predominantly because of the developments in organic chemistry and theoretical chemistry. Cheminformatics is an active research area where quantitative structure behaviour and structure property relations predict nanomaterial biological activities and properties [1 - 2]. A few physicochemical characteristics and topological indices have been used in research findings to predict the bioactivity of organic molecules [3 - 5]. In a chemical graph, nodes represent atoms or molecules and the links denote the chemical bonding between the atoms or molecules. The degree of a vertex represents the number of edges that are incident on that vertex [6]. The arbitrary degree of any chemical network is at most 4.

Topological indices are numerical parameters associated with a graph that characterize its topology and they are usually invariant graph properties. The neighbourhood degree of a node $u \in V$, indicated as $\boldsymbol{\delta}$ **(u) /** $\boldsymbol{\delta_u}$, is the sum of degrees of all adjacent nodes of the node u. In this paper we estimate a few topological descriptors like *SK, SK1, SK2* indices, Modified Randić index, and Inverse Sum Index based on the neighbourhood degree.

We have defined a few new neighbourhood degree-based topological indices [7] denoted as $SK_N(\mathrm{G})$, $SK1_N(\mathrm{G})$, $SK2_N(\mathrm{G})$, $mR_N(\mathrm{G})$, $ISI_N(\mathrm{G})$ as follows:

$$\boldsymbol{SK_N(G) = \sum_{uv \in E(G)} \left[\frac{\delta(u)+\delta(v)}{2}\right]} \quad (1)$$

$$\boldsymbol{SK1_N(G) = \sum_{uv \in E(G)} \left[\frac{\delta(u) * \delta(v)}{2}\right]} \quad (2)$$

$$\boldsymbol{SK2_N}(\mathbf{G}) = \sum_{uv \in E(G)} \left[\frac{\delta(u)+\delta(v)}{2}\right]^2 \quad (3)$$

$$\boldsymbol{mR_N}(\mathbf{G}) = \sum_{uv \in E(G)} \left[\frac{1}{max\{\delta(u),\delta(v)\}}\right] \quad (4)$$

$$\boldsymbol{ISI_N}(\mathbf{G}) = \sum_{uv \in E(G)} \left[\frac{\delta(u) * \delta(v)}{\delta(u) + \delta(v)}\right] \quad (5)$$

where $\delta(u) = \sum_{v \in N(u)} deg(v)$, $N(u)$ is the Neighbourhood set of the vertex u. Many researchers [8 - 9] have defined and estimated topological indices based on the sum of the neighbourhood degrees. For the 2D representation of the *TuC4C8[α, β]* and *L(*S(TuC4C8[α, β])*)* structure one can refer in [10]. The edges of *TuC4C8[α, β]* [10] can be partitioned into two types based on the neighbourhood degrees of the vertices of each edge denoted by $\mathbf{E}_{(\delta(u),\delta(v))}$ / $E_{(\delta_u,\delta_v)}$. The edge partitions are given in Table 1 and Table 2.

Table 1 Edge Partition of *TuC4C8[α, β]* when α > 1, β > 1

$E((\delta_u, \delta_v))$	$\lvert E((\delta_u, \delta_v))\rvert$
(6, 6)	4
(6, 7)	8
(7, 7)	2(α+β-4)
(7, 11)	4(α+β-2)
(11, 12)	8(α+β-2)
(12, 12)	2(9αβ+10)-19(α+β)

Table 2 Edge Partition of *TuC4C8[α, β]* when α > 1, β = 1

$E((\delta_u, \delta_v))$	$\lvert E((\delta_u, \delta_v))\rvert$
(6, 6)	6
(6, 7)	4
(7, 7)	2(α-2)
(7, 11)	4(α-1)
(11, 11)	2(α-1)
(11, 12)	4(α-1)
(12, 12)	(α-1)

2. Results and Discussion on *TuC_4C_8[α, β]*

In this section, we compute neighbourhood degree based topological indices of *TuC4C8[α, β]* namely SK_N(G), $SK1_N$(G), $SK2_N$(G), mR_N(G), ISI_N(G).

Theorem 2.1: The SK_N *index of TuC_4C_8[α, β] is*

$$\boldsymbol{SK_N} = \begin{cases} \mathbf{216\alpha\beta - 314\alpha + 118\beta + 4} & \boldsymbol{if\ \alpha > 1, \beta > 1} \\ \mathbf{130\alpha - 82} & \boldsymbol{if\ \alpha > 1, \beta = 1} \end{cases}$$

Proof: We establish the proof for the following two cases:

Case 1: We use the edge partition for ***α > 1, β > 1*** given Table **1** in equation **1** and obtain,

$SK_N = |E_{(6,6)}| \left[\frac{12}{2}\right] + |E_{(6,7)}| \left[\frac{13}{2}\right] + |E_{(7,7)}| \left[\frac{14}{2}\right] + |E_{(7,11)}| \left[\frac{18}{2}\right] + |E_{(11,12)}| \left[\frac{23}{2}\right] + |E_{(12,12)}| \left[\frac{24}{2}\right]$

$= 4 \left[\frac{12}{2}\right] + 8 \left[\frac{13}{2}\right] + 2(\alpha + \beta - 4) \left[\frac{14}{2}\right] + 4(\alpha + \beta - 2) \left[\frac{18}{2}\right] + 8(\alpha + \beta - 2) \left[\frac{23}{2}\right] + (2(9\alpha\beta + 10) - 19(\alpha+\beta)) \left[\frac{24}{2}\right]$

$= \mathbf{216\alpha\beta - 314\alpha + 118\beta + 4}$

Case 2: Using the edge partition for ***α*** > ***1, β = 1*** given in Table **2** in equation **1** we obtain,

$SK_N = |E_{(6,6)}| \left[\frac{12}{2}\right] + |E_{(6,7)}| \left[\frac{13}{2}\right] + |E_{(7,7)}| \left[\frac{14}{2}\right] + |E_{(7,11)}| \left[\frac{18}{2}\right] + |E_{(11,11)}| \left[\frac{22}{2}\right] + |E_{(11,12)}| \left[\frac{23}{2}\right] + |E_{(12,12)}| \left[\frac{24}{2}\right]$

$= 6 \left[\frac{12}{2}\right] + 4 \left[\frac{13}{2}\right] + 2(\alpha - 2) \left[\frac{14}{2}\right] + 4(\alpha - 1) \left[\frac{18}{2}\right] + 2(\alpha - 1) \left[\frac{22}{2}\right] + 4(\alpha - 1) \left[\frac{23}{2}\right] + (\alpha - 1) \left[\frac{24}{2}\right]$

$= \mathbf{130\alpha - 82}$

Theorem 2.2: The $SK1_N$ *index of TuC4C8[α, β] is*

$$SK1_N = \begin{cases} 1926\alpha\beta - 2005\alpha + 587\beta + 120 & if\ \alpha > 1, \beta > 1 \\ 660\alpha - 517 & if\ \alpha > 1, \beta = 1 \end{cases}$$

Proof: We establish the proof for the following two cases:

Case 1: We use the edge partition for ***α > 1, β > 1*** given Table **1** in equation **2** and obtain,

$SK1_N = |E_{(6,6)}| \left[\frac{36}{2}\right] + |E_{(6,7)}| \left[\frac{42}{2}\right] + |E_{(7,7)}| \left[\frac{49}{2}\right] + |E_{(7,11)}| \left[\frac{77}{2}\right] + |E_{(11,12)}| \left[\frac{132}{2}\right] + |E_{(12,12)}| \left[\frac{1444}{2}\right]$

$= 4 \left[\frac{36}{2}\right] + 8 \left[\frac{42}{2}\right] + 2(\alpha + \beta - 4) \left[\frac{49}{2}\right] + 4(\alpha + \beta - 2) \left[\frac{77}{2}\right] + 8(\alpha + \beta - 2) \left[\frac{132}{2}\right] + (2(9\alpha\beta + 10) - 19(\alpha+\beta)) \left[\frac{144}{2}\right]$

$= \mathbf{1926\alpha\beta - 2005\alpha + 587\beta + 120}$

Case 2: Using the edge partition for ***α*** > ***1, β = 1*** given in Table **2** in equation **2** we obtain,

$SK1_N = |E_{(6,6)}| \left[\frac{36}{2}\right] + |E_{(6,7)}| \left[\frac{42}{2}\right] + |E_{(7,7)}| \left[\frac{49}{2}\right] + |E_{(7,11)}| \left[\frac{77}{2}\right] + |E_{(11,11)}| \left[\frac{121}{2}\right] + |E_{(11,12)}| \left[\frac{132}{2}\right] + |E_{(12,12)}| \left[\frac{144}{2}\right]$

$= 6 \left[\frac{36}{2}\right] + 4 \left[\frac{42}{2}\right] + 2(\alpha - 2) \left[\frac{49}{2}\right] + 4(\alpha - 1) \left[\frac{77}{2}\right] + 2(\alpha - 1) \left[\frac{121}{2}\right] + 4(\alpha - 1) \left[\frac{132}{2}\right] + (\alpha - 1) \left[\frac{144}{2}\right]$

$= \mathbf{660\alpha - 517}$

Theorem 2.3: The $SK2_N$ *index of TuC4C8[α, β] is*

$$SK2_N = \begin{cases} 2592\alpha\beta - 3992\alpha + 1193\beta + 206 & if\ \alpha > 1, \beta > 1 \\ 1337\alpha - 1050 & if\ \alpha > 1, \beta = 1 \end{cases}$$

Proof: We establish the proof for the following two cases:

Case 1: We use the edge partition for ***α > 1, β > 1*** given Table **1** in equation **3** and obtain,

$$SK2_N = |E_{(6,6)}| \left[\frac{12}{2}\right]^2 + |E_{(6,7)}| \left[\frac{13}{2}\right]^2 + |E_{(7,7)}| \left[\frac{14}{2}\right]^2 + |E_{(7,11)}| \left[\frac{18}{2}\right]^2 + |E_{(11,12)}| \left[\frac{23}{2}\right]^2 + |E_{(12,12)}| \left[\frac{24}{2}\right]^2$$

$$= 4\left[\frac{12}{2}\right]^2 + 8\left[\frac{13}{2}\right]^2 + 2(\alpha+\beta-4)\left[\frac{14}{2}\right]^2 + 4(\alpha+\beta-2)\left[\frac{18}{2}\right]^2 + 8(\alpha+\beta-2)\left[\frac{23}{2}\right]^2 + (2(9\alpha\beta+10)-19(\alpha+\beta))\left[\frac{24}{2}\right]^2$$

$$= \mathbf{2592\alpha\beta - 3992\alpha + 1193\beta + 206}$$

Case 2: Using the edge partition for ***α > 1, β = 1*** given in Table **2** in equation **3** we obtain,

$$SK2_N = |E_{(6,6)}| \left[\frac{12}{2}\right]^2 + |E_{(6,7)}| \left[\frac{12}{2}\right]^2 + |E_{(7,7)}| \left[\frac{14}{2}\right]^2 + |E_{(7,11)}| \left[\frac{18}{2}\right]^2 + |E_{(11,11)}| \left[\frac{22}{2}\right]^2 + |E_{(11,12)}| \left[\frac{23}{2}\right]^2 + |E_{(12,12)}| \left[\frac{24}{2}\right]^2$$

$$= 6\left[\frac{12}{2}\right]^2 + 4\left[\frac{13}{2}\right]^2 + 2(\alpha-2)\left[\frac{14}{2}\right]^2 + 4(\alpha-1)\left[\frac{18}{2}\right]^2 + 2(\alpha-1)\left[\frac{22}{2}\right]^2 + 4(\alpha-1)\left[\frac{23}{2}\right]^2 + (\alpha-1)\left[\frac{24}{2}\right]^2$$

$$= \mathbf{1337\alpha - 1050}$$

Theorem 2.4: The mR_N *index of TuC4C8[α, β] is*

$$mR_N = \begin{cases} \dfrac{3[77\alpha\beta - 97\alpha + 59\beta + 14]}{154} & if\ \alpha > 1, \beta > 1 \\ \dfrac{1153\alpha + 35}{924} & if\ \alpha > 1, \beta = 1 \end{cases}$$

Proof: We establish the proof for the following two cases:

Case 1: We use the edge partition for ***α > 1, β > 1*** given Table **1** in equation **4** and obtain,

$$mR_N = |E_{(6,6)}| \left[\frac{1}{max\{6,6\}}\right] + |E_{(6,7)}| \left[\frac{1}{max\{6,7\}}\right] + |E_{(7,7)}| \left[\frac{1}{max\{7,7\}}\right] + |E_{(7,11)}| \left[\frac{1}{max\{7,11\}}\right] + |E_{(11,12)}| \left[\frac{1}{max\{11,12\}}\right] + |E_{(12,12)}| \left[\frac{1}{max\{12,12\}}\right]$$

$$= 4\left[\frac{1}{6}\right] + 8\left[\frac{1}{7}\right] + 2(\alpha+\beta-4)\left[\frac{1}{7}\right] + 4(\alpha+\beta-2)\left[\frac{1}{11}\right] + 8(\alpha+\beta-2)\left[\frac{1}{12}\right] + (2(9\alpha\beta+10)-19(\alpha+\beta))\left[\frac{1}{12}\right]$$

$$= \frac{3[77\alpha\beta - 97\alpha + 59\beta + 14]}{154}$$

Case 2: Using the edge partition for ***α > 1, β = 1*** given in Table **2** in equation **4** we obtain,

$$mR_N = |E_{(6,6)}| \left[\frac{1}{max\{6,6\}}\right] + |E_{(6,7)}| \left[\frac{1}{max\{6,7\}}\right] + |E_{(7,7)}| \left[\frac{1}{max\{7,7\}}\right] + |E_{(7,11)}| \left[\frac{1}{max\{7,11\}}\right] + |E_{(11,11)}| \left[\frac{1}{max\{11,11\}}\right] + |E_{(11,12)}| \left[\frac{1}{max\{11,12\}}\right] + |E_{(12,12)}| \left[\frac{1}{max\{12,12\}}\right]$$

$$= 6\left[\frac{1}{6}\right] + 4\left[\frac{1}{7}\right] + 2(\alpha-2)\left[\frac{1}{7}\right] + 4(\alpha-1)\left[\frac{1}{11}\right] + 2(\alpha-1)\left[\frac{1}{11}\right] + 4(\alpha-1)\left[\frac{1}{12}\right] + (\alpha-1)\left[\frac{1}{12}\right]$$

$$= \frac{1153\alpha+35}{924}$$

Theorem 2.5: The ISI_N *index of TuC4C8[α, β] is*

$$ISI_N = \begin{cases} \dfrac{290628\alpha\beta - 425113\alpha + 156143\beta + 10220}{2691} & if\ \alpha > 1, \beta > 1 \\ \dfrac{172406\alpha - 108029}{2691} & if\ \alpha > 1, \beta = 1 \end{cases}$$

Proof: We establish the proof for the following two cases:

Case 1: We use the edge partition for ***α > 1, β > 1*** given Table **1** in equation **5** and obtain,

$$ISI_N = |E_{(6,6)}| \left[\frac{6*6}{6+6}\right] + |E_{(6,7)}| \left[\frac{6*7}{6+7}\right] + |E_{(7,7)}| \left[\frac{7*7}{7+7}\right] + |E_{(7,11)}| \left[\frac{7*11}{7+11}\right] + |E_{(11,12)}| \left[\frac{11*12}{11+12}\right] + |E_{(12,12)}| \left[\frac{12*12}{12+12}\right]$$

$$= 4\left[\frac{36}{12}\right] + 8\left[\frac{42}{13}\right] + 2(\alpha+\beta-4)\left[\frac{49}{14}\right] + 4(\alpha+\beta-2)\left[\frac{77}{18}\right] + 8(\alpha+\beta-2)\left[\frac{132}{23}\right] + (2(9\alpha\beta + 10) - 19(\alpha+\beta))\left[\frac{144}{24}\right]$$

$$= \frac{290628\alpha\beta - 425113\alpha + 156143\beta + 10220}{2691}$$

Case 2: Using the edge partition for ***α > 1, β = 1*** given in Table **2** in equation **5** we obtain,

$$ISI_N = |E_{(6,6)}| \left[\frac{6*6}{6+6}\right] + |E_{(6,7)}| \left[\frac{6*7}{6+7}\right] + |E_{(7,7)}| \left[\frac{7*7}{7+7}\right] + |E_{(7,11)}| \left[\frac{7*11}{7+11}\right] + |E_{(11,11)}| \left[\frac{11*11}{11+11}\right] + |E_{(11,12)}| \left[\frac{11*12}{11+12}\right] + |E_{(12,12)}| \left[\frac{12*12}{12+12}\right]$$

$$= 6\left[\frac{36}{12}\right] + 4\left[\frac{42}{13}\right] + 2(\alpha-2)\left[\frac{49}{14}\right] + 4(\alpha-1)\left[\frac{77}{18}\right] + 2(\alpha-1)\left[\frac{121}{22}\right] + 4(\alpha-1)\left[\frac{132}{23}\right] + (\alpha-1)\left[\frac{144}{24}\right]$$

$$= \frac{172406\alpha - 108029}{2691}$$

3. Results on Paraline graph of *TuC4C8[α, β]*

In this section, we compute the neighbourhood degree based topological indices of the line graph of subdivision graph of *TuC4C8[α, β]*. We denote such a graph as **H**. The edges of **H** are partitioned as given in Table 3 and Table 4.

Table 3 Edge Partition of H when α > 1, β > 1

$E((\delta_u, \delta_v))$	$\lvert E((\delta_u, \delta_v))\rvert$
(7, 7)	2α
(7, 11)	4α
(11, 12)	8α
(12, 12)	18αβ-19α

Table 4 Edge Partition of H when α > 1, β = 1

E((δu, δv))	**| E((δu, δv))|**
(7, 7)	2α
(7, 11)	4α
(11, 11)	2α
(11, 12)	4α
(12, 12)	α

Theorem 3.1: The SK_N *index of H is*

$$SK_N = \begin{cases} 216\alpha\beta - 86\alpha & if\ \alpha > 1, \beta > 1 \\ 130\alpha & if\ \alpha > 1, \beta = 1 \end{cases}$$

Proof: We establish the proof for the following two cases:

Case 1: We use the edge partition for ***α > 1, β > 1*** given Table **3** in equation **1** and obtain,

$SK_N = |E_{(7,7)}| \left[\frac{14}{2}\right] + |E_{(7,11)}| \left[\frac{18}{2}\right] + |E_{(11,12)}| \left[\frac{23}{2}\right] + |E_{(12,12)}| \left[\frac{24}{2}\right]$

$= (2\alpha) \left[\frac{14}{2}\right] + (4\alpha) \left[\frac{18}{2}\right] + (8\alpha) \left[\frac{23}{2}\right] + (18\alpha\beta - 19\alpha) \left[\frac{24}{2}\right]$

$= 216\alpha\beta - 86\alpha$

Case 2: Using the edge partition for ***α > 1, β = 1*** given in Table **4** in equation **1** we obtain,

$SK_N = |E_{(7,7)}| \left[\frac{14}{2}\right] + |E_{(7,11)}| \left[\frac{18}{2}\right] + |E_{(11,11)}| \left[\frac{22}{2}\right] + |E_{(11,12)}| \left[\frac{23}{2}\right] + |E_{(12,12)}| \left[\frac{24}{2}\right]$

$= (2\alpha) \left[\frac{14}{2}\right] + (4\alpha) \left[\frac{18}{2}\right] + (2\alpha) \left[\frac{22}{2}\right] + (4\alpha) \left[\frac{23}{2}\right] + (\alpha) \left[\frac{24}{2}\right]$

$= 130\alpha.$

Theorem 3.2: The $SK1_N$ *index of H is*

$$SK1_N = \begin{cases} 1296\alpha\beta - 637\alpha & if\ \alpha > 1, \beta > 1 \\ 660\alpha & if\ \alpha > 1, \beta = 1 \end{cases}$$

Proof: We establish the proof for the following two cases:

Case 1: We use the edge partition for ***α > 1, β > 1*** given Table **3** in equation **2** and obtain,

$SK1_N = |E_{(7,7)}| \left[\frac{49}{2}\right] + |E_{(7,11)}| \left[\frac{77}{2}\right] + |E_{(11,12)}| \left[\frac{132}{2}\right] + |E_{(12,12)}| \left[\frac{144}{2}\right]$

$= (2\alpha) \left[\frac{49}{2}\right] + (4\alpha) \left[\frac{77}{2}\right] + (8\alpha) \left[\frac{132}{2}\right] + (18\alpha\beta - 19\alpha) \left[\frac{144}{2}\right]$

$= 1296\alpha\beta - 637\alpha$

Case 2: Using the edge partition for ***α > 1, β = 1*** given in Table **4** in equation **2** we obtain,

$SK1_N = |E_{(7,7)}| \left[\frac{49}{2}\right] + |E_{(7,11)}| \left[\frac{77}{2}\right] + |E_{(11,11)}| \left[\frac{121}{2}\right] + |E_{(11,12)}| \left[\frac{132}{2}\right] + |E_{(12,12)}| \left[\frac{144}{2}\right]$

$= (2\alpha) \left[\frac{49}{2}\right] + (4\alpha) \left[\frac{77}{2}\right] + (2\alpha) \left[\frac{121}{2}\right] + (4\alpha) \left[\frac{132}{2}\right] + (\alpha) \left[\frac{144}{2}\right]$

$= \boldsymbol{660\alpha}.$

Theorem 3.3: The $SK2_N$ *index of H is*

$$SK2_N = \begin{cases} 2592\alpha\beta - 1256\alpha & if\ \alpha > 1, \beta > 1 \\ 1337\alpha & if\ \alpha > 1, \beta = 1 \end{cases}$$

Proof: We establish the proof for the following two cases:

Case 1: We use the edge partition for ***α > 1, β > 1*** given Table **3** in equation **3** and obtain,

$$SK2_N = |{}'E_{(7,7)}| \left[\frac{14}{2}\right]^2 + |{}'E_{(7,11)}| \left[\frac{18}{2}\right]^2 + |{}'E_{(11,12)}| \left[\frac{23}{2}\right]^2 + |{}'E_{(12,12)}| \left[\frac{24}{2}\right]^2$$

$$= (2\alpha) \left[\frac{14}{2}\right]^2 + (4\alpha) \left[\frac{18}{2}\right]^2 + (8\alpha) \left[\frac{23}{2}\right]^2 + (18\alpha\beta - 19\alpha) \left[\frac{24}{2}\right]^2$$

$$= \boldsymbol{2592\alpha\beta} - 1256\boldsymbol{\alpha}$$

Case 2: Using the edge partition for ***α > 1, β = 1*** given in Table **4** in equation **3** we obtain,

$$SK2_N = |{}'E_{(7,7)}| \left[\frac{14}{2}\right]^2 + |{}'E_{(7,11)}| \left[\frac{18}{2}\right]^2 + |{}'E_{(11,11)}| \left[\frac{22}{2}\right]^2 + |{}'E_{(11,12)}| \left[\frac{23}{2}\right]^2 + |{}'E_{(12,12)}| \left[\frac{24}{2}\right]^2$$

$$= (2\alpha) \left[\frac{14}{2}\right]^2 + (4\alpha) \left[\frac{18}{2}\right]^2 + (2\alpha) \left[\frac{22}{2}\right]^2 + (4\alpha) \left[\frac{23}{2}\right]^2 + (\alpha) \left[\frac{24}{2}\right]^2$$

$$= \boldsymbol{1337\alpha}.$$

Theorem 2.4: The mR_N *index of H is*

$$mR_N = \begin{cases} \dfrac{\alpha[1386\beta - 247]}{924} & if\ \alpha > 1, \beta > 1 \\ \dfrac{1153\alpha}{924} & if\ \alpha > 1, \beta = 1 \end{cases}$$

Proof: We establish the proof for the following two cases:

Case 1: We use the edge partition for ***α > 1, β > 1*** given Table **3** in equation **4** and obtain,

$$mR_N = |{}'E_{(7,7)}| \left[\frac{1}{max\{7,7\}}\right] + |{}'E_{(7,11)}| \left[\frac{1}{max\{7,11\}}\right] + |{}'E_{(11,12)}| \left[\frac{1}{max\{11,12\}}\right] + |{}'E_{(12,12)}| \left[\frac{1}{max\{12,12\}}\right]$$

$$= (2\alpha) \left[\frac{1}{7}\right] + (4\alpha) \left[\frac{1}{11}\right] + (8\alpha) \left[\frac{1}{12}\right] + (18\alpha\beta - 19\alpha) \left[\frac{1}{12}\right]$$

$$= \frac{\alpha[1386\beta - 247]}{924}$$

Case 2: Using the edge partition for ***α > 1, β = 1*** given in Table **4** in equation **4** we obtain,

$$mR_N = |{}'E_{(7,7)}| \left[\frac{1}{max\{7,7\}}\right] + |{}'E_{(7,11)}| \left[\frac{1}{max\{7,11\}}\right] + |{}'E_{(11,11)}| \left[\frac{1}{max\{11,11\}}\right] + |{}'E_{(11,12)}| \left[\frac{1}{max\{11,12\}}\right] + |{}'E_{(12,12)}| \left[\frac{1}{max\{12,12\}}\right]$$

$$= (2\alpha) \left[\frac{1}{7}\right] + (4\alpha) \left[\frac{1}{11}\right] + (2\alpha) \left[\frac{1}{11}\right] + (4\alpha) \left[\frac{1}{12}\right] + (\alpha) \left[\frac{1}{12}\right]$$

$$= \frac{\boldsymbol{1153\alpha}}{\boldsymbol{924}}.$$

Theorem 2.5: The ISI_N *index of H is*

$$ISI_N = \begin{cases} \dfrac{\alpha[22356\beta - 9103]}{207} & if\ \alpha > 1, \beta > 1 \\ \dfrac{13768\alpha}{207} & if\ \alpha > 1, \beta = 1 \end{cases}$$

Proof: We establish the proof for the following two cases:

Case 1: We use the edge partition for ***α > 1, β > 1*** given Table **3** in equation **5** and obtain,

$$ISI_N = |E_{(7,7)}| \left[\frac{7*7}{7+7}\right] + |E_{(7,11)}| \left[\frac{7*11}{7+11}\right] + |E_{(11,12)}| \left[\frac{11*12}{11+12}\right] + |E_{(12,12)}| \left[\frac{12*12}{12+12}\right]$$

$$= (2\alpha)\left[\frac{49}{14}\right] + (4\alpha)\left[\frac{77}{18}\right] + (8\alpha)\left[\frac{132}{23}\right] + (18\alpha\beta - 19\alpha)\left[\frac{144}{24}\right]$$

$$= \frac{\alpha[22356\beta - 9103]}{207}$$

Case 2: Using the edge partition for ***α > 1, β = 1*** given in Table **4** in equation **5** we obtain,

$$ISI_N = |E_{(7,7)}| \left[\frac{7*7}{7+7}\right] + |E_{(7,11)}| \left[\frac{7*11}{7+11}\right] + |E_{(11,11)}| \left[\frac{11*11}{11+11}\right] + |E_{(11,12)}| \left[\frac{11*12}{11+12}\right] + |E_{(12,12)}| \left[\frac{12*12}{12+12}\right]$$

$$= (2\alpha)\left[\frac{49}{14}\right] + (4\alpha)\left[\frac{77}{18}\right] + (2\alpha)\left[\frac{121}{22}\right] + (4\alpha)\left[\frac{132}{23}\right] + (\alpha)\left[\frac{144}{24}\right]$$

$$= \frac{13768\alpha}{207}.$$

4 CONCLUSION

The computation of various topological indices of graphs associated with chemical graphs enables the analysis of molecules in molecular chemistry and study of how the indices relate to their molecular properties. In this paper we estimated a few topological indices based on the neighbourhood degree and obtained results based on the sum of the cardinality of the edge partitions of TuC_4C_8 structure and its corresponding paraline graph.

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