On ev-degree Based Zagreb and Randic Indices of Sierpinski Graphs

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ABSTRACT

In this paper, we computed ev-degree based Zagreb and Randic topological indices of Sierpinski graphs.

KEYWORDS: ev-degree, ev-degre based topological indices, Sierpinski graphs

1 INTRODUCTION

Computing topological indices for Sierpinski graphs has been intensively rising recently. Until now, many classical degree based topological indices such as Randic, Zagreb, atom-bond connectivity, geometric-arithmetic, sum-connectivity indices [1-7]. As a continuation of these studies, we computed some ev-degree based topological indices of Sierpinski graphs.

2 PRELIMINARIES

In this section we give some basic and preliminary concepts which we shall use later. A graph $G=(V,E)$ consists of two nonempty sets V and 2-element subsets of V namely E. The elements of V are called vertices and the elements of E are called edges. For a vertex v, $deg(v)$ show the number of edges that incident to v. The set of all vertices which adjacent to v is called the open neighborhood of v and denoted by $N(v)$. If we add the vertex v to $N(v)$, then we get the closed neighborhood of v, $N[v]$.

Definition 1 **(ev-degree)** Let G be a connected graph and $e=uv\in E(G)$. The ev-degree of the edge e, $deg_{ev}(e)$, equals the number of vertices of the union of the closed neighborhoods of u and v. Or the ev-degree of the edge e, $deg_{ev}(e)=degu+degv-n_e$, where n_e means the number of triangles in which the edge e lies in.

Definition 2 (**the ev-degree Zagreb index**) Let G be a connected graph and $v\in V(G)$. The ev-degree Zagreb index of the graph G defined as;

$$M^{ev}(G) = \sum_{e\in E(G)} (deg_{ev}e)^2 \quad (1)$$

Definition 3 (**the ev-degree Randic index**) Let G be a connected graph and $uv\in E(G)$. The first ve-degree Zagreb beta index of the graph G defined as;

$$R^{\beta}(G) = \sum_{e\in E(G)} (deg_{ev}e)^{-1/2} \quad (2)$$

Definition 4 (**Sierpinski graphs**) Sierpinski graphs, $S(n,k)$, has vertex set $\{1, 2, \cdots, k\}^n$, and there is an edge between two vertices $u=(u_1, u_2 \cdots, u_n)$ and $v=(v_1, v_2, \cdots, v_n)$ iff there is an $h \in \{1, 2, \cdots, n\}$ such that:

- $u_j = v_j$ for $j=1,2,\ldots,h-1$
- $u_h \neq v_h$
- $u_i = v_j$ and $u_j = v_i$ for $j=h+1,h+2,\ldots,n$.

See Figures 1,2 and 3.

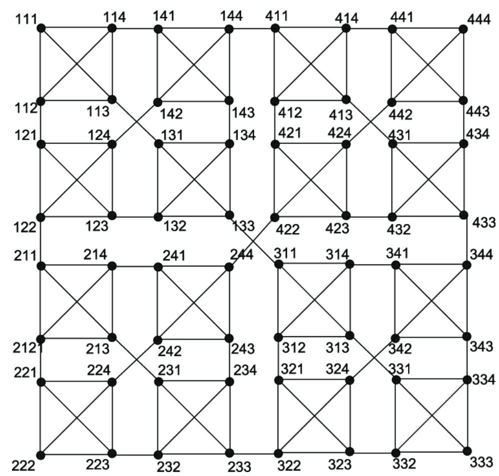

Figure 1: S(3,4)

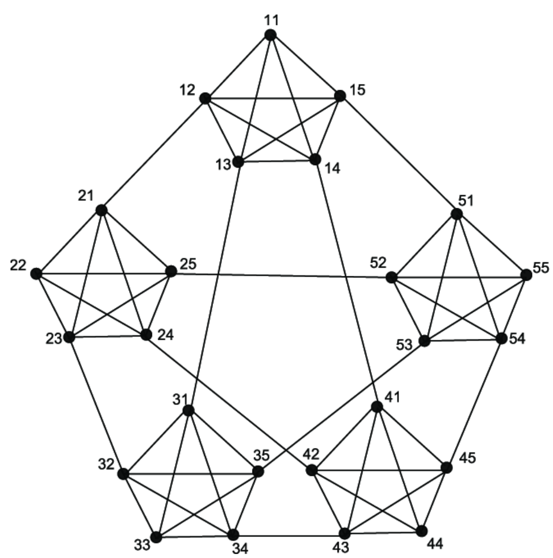

Figure 2: S(2,5)

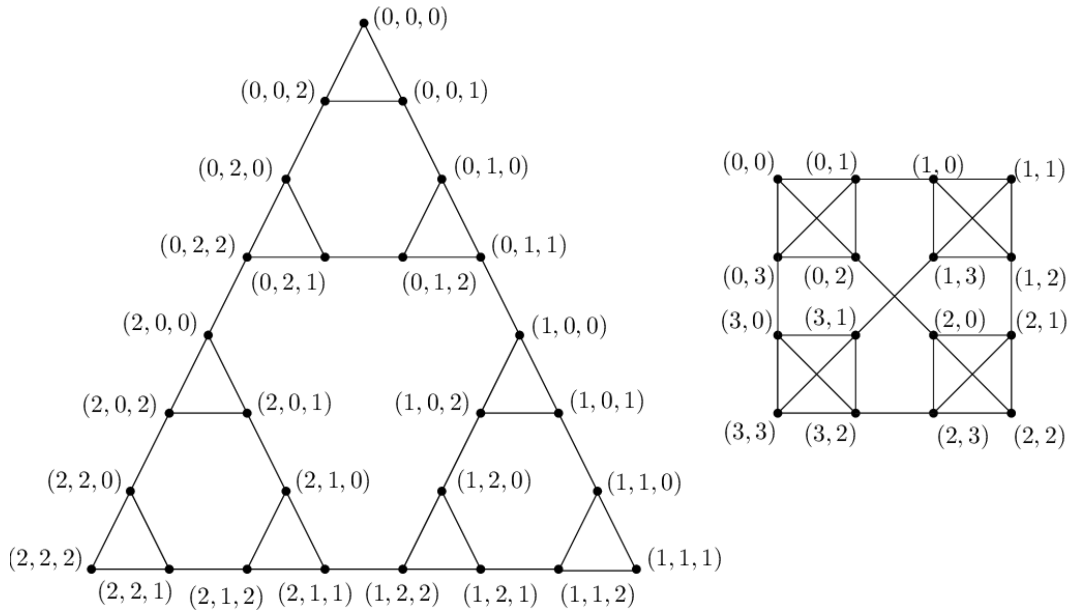

Figure 3: S(3,3) and S(2,4)

A vertex of $S(n,k)$ of the form $i,i,\ldots,i \in \{1,2,\ldots,k\}$ will be called an extreme vertex and the other vertices are called inner vertices. The degree of extreme vertices is $k-1$, while the degree of the inner vertices is k. The edges of $S(n,k)$ that lie in no induced K_k are called bridge edges. Note that bridge edges consist of the vertices $(i,j,\ldots,j)$ and $(j,i,\ldots,i)$ for $i \neq j$.

3 RESULTS

Observation 1. [9] $S(n,k)$ has k extreme vertices with degree $k-1$ and k^n-k inner vertices with degree k.

Observation 2. [9] $S(n,k)$ has $\frac{k}{2}(k^n-1)$ edges.

Observation 3. $S(n,k)$ has $\frac{k}{2}(k^n-2k-3)$ edges with end points are only inner vertices.

Observation 4. $S(n,k)$ has k^2+2k edges with end points are between inner and extreme vertices.

Observation 5. Let $e \in E(S(n,k)), e$ is a bridge edge and $k \geq 4$,then e lies on no any triangle.

Observation 6. Let $e \in E(S(n,k))$, e is not a bridge edge and $k \geq 4$,then e lies on exactly $k-2$ different triangles.

Corollary 7. Let $e \in E(S(n,k))$, e is a bridge edge and $k \geq 4$, then $deg_{ev}e = 2k$

Corollary 8. Let $e \in E(S(n,k))$ with endpoints inner-extreme vertices. Then $deg_{ev}e = k+1$.

Corollary 9. Let $e \in E(S(n,k))$ with endpoints extreme-extreme vertices. Then $deg_{ev}e = k+2$.

Proposition 10. The ev-degree Zagreb index of the graph $S(n,k)$ is $4k^3 + k(k-1)(k+1)^2 + \left(\frac{k(k^n-1)}{2} - k^2\right)(k+2)^2$

Proposition 11. The ev-degree Randic index of the graph $S(n,k)$ is $k\sqrt{2k} + k(k-1)\sqrt{k+1} + \left(\frac{k(k^n-1)}{2} - k^2\right)\sqrt{k+2}$

CONCLUSION

In this paper, we computed ev-degree based Randic and Zagreb topological indices of Sierpinski graphs.

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