Design of Finite Impulse Response Filters for Sampling Rate Conversion

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Abstract

As the modern DSP applications are concerned, the increasing requirements of advanced digital systems are to process data at greater than one sampling rate has resulted in the development of a new sub-area in the signal processing known as multirate signal processing. Multirate Signal Processing uses different sampling rates within a system to achieve computational and system efficiencies when compared to system which operates at single sampling rate. Digital filters in Multirate signal processing are used to suppress aliasing and to remove imaging effect in decimator and interpolator respectively. Speed and complexity of multirate filters are dependent on the type, order and design methods of filters. Here, we present the FIR filter design techniques applicable for sampling rate conversion. The design technique decides the order of the filter required, which in turn determines the performance parameters of the Multirate systems. In this paper, single stage decimation and interpolation filter are designed using window techniques and optimal method. The performance metrics like number of multiplications and storage elements are determined and compared. For sampling rate converters, and it is proved as required filter order, number of multiplications and storage elements are less in cases of optimal design methods.

Keywords: Sampling Rate Conversion, Decimation, Interpolation, FIR Filter, Optimal Method.

1. INTRODUCTION

The recent trends such as Software Defined Radio, Internet of Things, Mobile communication have setup a demand of changing the sampling rate of operating signal, either increasing it or decreasing it by integer or non-integer factor. The process of converting a given signal sampling rate to a different sampling rate is called as sampling rate conversion [4, 7]. The systems which operate at different sampling rates throughout the processing of signals are called as multirate signal processing systems. The Multirate systems perform a processing task with improved performance metrics and offering that performance at a significantly lower cost and complexity. Primary operations of sampling rate conversion are Decimation and Interpolation by an integer factor. Decimation is used to reduce the sampling rate. Whereas interpolation is used to increases the sampling rate [1, 4, 5].

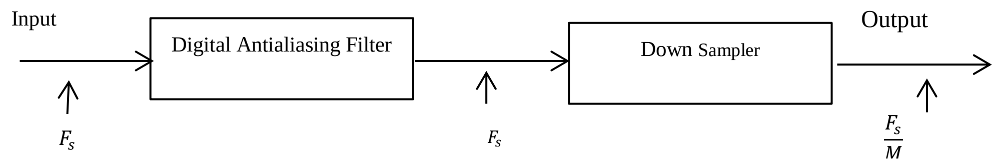

Figure 1. Block Diagram of Decimator

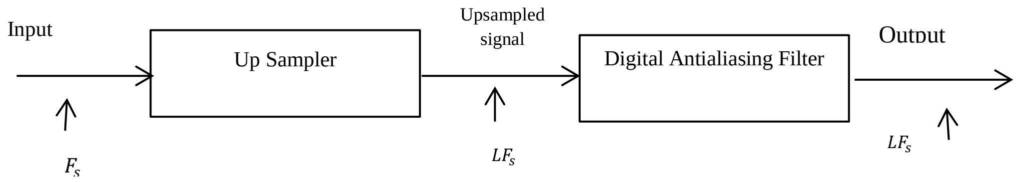

Figure 2. Block Diagram of Interpolator

Decimation by an integer factor M is the process of retaining the every M^{th} sample of input samples and hence results in sampling rate of Fs/M. Decimator consists of an anti-aliasing filter fallowed by an *M*-fold down sampler as shown in Figure 1. Interpolation by a factor L is the process of inserting (L-1) zero-valued samples between each input samples hence resulting in the sampling rate of L*fs. Interpolator consists of an *L*-fold expander, followed by an anti-image filter as shown in Figure 2 [1, 3].

Filtering is the basic signal operation of sampling rate conversion as it executes direct manipulations of digital signals, extracts useful frequency components and removes undesirable components as a noise part. In sampling rate converters, filters are used to suppress aliasing and to remove imaging effect in decimator and interpolator respectively. Performance of the sampling rate converters depend mainly on these filter characteristics hence the major challenge is to design the filters to process and transmit the data within desired frequency band. Contribution of digital filters are significant over the analog filters, hence they preferred for numerous applications [1, 5].

FIR filters or IIR filters, either of these may be used to generate the overall system response in sampling rate conversion. Compared to IIR filters, linear phase FIR filters preserve the waveform of the signal components of interest at the expense of a higher overall complexity. Design techniques for filters are suitably selected by considering the ideal frequency response which is not practically achievable. Hence, the selection of approximate design method by the specifications is the major step in the design of filter. The recent literature reveals that a generalized polyphase FIR filter structure can be used for sampling rate conversion [7].

2. FIR FILTER DESIGN BY WINDOW TECHNIQUES & EQUIRIPPLE METHOD

A FIR filter structure can be used to implement any sort of frequency response for sampling rate conversions. The non-recursive characteristics of FIR filters yield implementation schemes to improve the efficiency of the multirate systems significantly. The advantages of FIR filters which are of great consideration for sampling rate conversion are:

a. Simple and robust filters
b. Easy to obtain linear phase.
c. Implementation of filter is in real time and efficient.
d. More stable and Non-recursive [1, 5].

The important point of consideration of sampling rate converters is to choose the FIR design techniques by considering the ideal frequency domain and time domain criteria. The design of filter is the process of determining the filter co efficient which closely approximates the given specifications. In this section we will discuss the design methods to calculate the FIR filter coefficients applicable to the design of sampling frequency changing systems.

2.1 Window Method

It's a former and reliable approach to design the FIR filters for the Interpolator and Decimator by truncating the ideal low pass filter with a certain bandwidth is generated, and then we use a chosen window to get certain stop band attenuation. Number of windowing techniques are proposed to control the effect of Gibb's phenomenon. Selection of the window to be used and the corresponding order is determined with reference to the Table 1.

Table 1: Windowing Parameters

Window Technique	Main lobe / Order	Peak Error	Side lobe
Rectangular	4π/N	-21 dB	-13 dB
Hamming	8π/N	-44 dB	-32 dB
Hanning	8π/N	-53 dB	-43 dB
Blackman	12π/N	-74 dB	-58 dB

Two most widely used window designs are the 'generalized' Hamming window and Kaiser's window. Window techniques truncate the response to be finite with corresponding window functions. The 'generalized' Hamming window has function as

$$w_H(k) = \gamma + (1-\gamma)\cos\left(\frac{2\pi k}{N}\right), \quad -\frac{N-1}{2} \leq k \leq \frac{N-1}{2}$$

Where γ is in the range of $0 \leq \gamma \leq 1$. If $\gamma = 0.54$, then its Hamming window,
If $\gamma = 0.5$ then its Hanning window.

Conventionally the order is assumed to be odd so as to produce symmetric coefficients. By adjusting the order of the filter it is possible to achieve ideal frequency response. If the order of the filter is increased then width of the main lobe becomes narrower and transition band will be reduced. A simple approximations to these problems have been developed by Kaiser Window in terms of the Bessel function $I_o(x)$. In Kaiser Window technique, length of filter N can be determined with respect to the stop band attenuation and is adjusted to meet a specified roll-off rate in the transition band. The Kaiser window has the finite function in the form

$$W_K(k) = \frac{I_0\left(\beta\sqrt{1-\left(\frac{(2k)^2}{(N-1)^2}\right)}\right)}{I_0(x)}$$

Where, β is the constant that specifies frequency trade off, with A being the attenuation in db, order of the filter is by Kaiser Window is approximated by

$$N = \frac{A - 7.95}{14.6\,\Delta f}$$

2.2 Optimal (Equiripple) Method

FIR filters may also be designed by means of more sophisticated design techniques. Optimal filter design which is derived from chebyshev approximation method designs the FIR filters at the rate of less peak error. Antialiasing and anti-imaging filters for the Multirate signal processing are designed with respect to the tolerance scheme of the ideal low pass filter. For sampling rate converter, pass band frequency and stop band edge frequency are considered according to the specifications to determine the order of the filter. The important part of the equiripple method is to first estimate the order and then to find the filter co efficient by the chebyshev approximation. With the given tolerances, Order of the filter for optimal design may be approximated by

$$N = -\frac{20\log\left(\sqrt{R_p\,R_s} - 13\right)}{14.6\,\Delta f}$$

Where Δf is frequency deviation, Rp and Rs are the ripples in the bands. The Parks-McLellan method solves for filter coefficients.
As examples of a Sampling rate converters, filters for the single stage decimator and interpolator are designed for the following specifications and responses are as shown.

Table 2: Sampling Rate Converter's Specifications

Parameters	**Decimator Specifications**	**Interpolator Specifications**
Sampling frequency	60 kHz	9 kHz
Input frequency	60 kHz	600 Hz
Ripples	Rp=0.02, Rs=0.01	Rp=0.002, Rs=0.004
Band edge frequencies	fp=1250 Hz, fs=1500 Hz	fp=200 Hz, fs=300 Hz
Sampling rate factors	20	15

Linear-phase FIR filters are used in most sampling rate converters as these do not have feedback loops and are nearly easy to implement. The design constraints and steps for decimator and interpolator are as below.

Step 1: Obtain the sampling rate converters specifications.
Step 2: Select the proper class of filters for single stage conversion.
Step 3: Estimate the filter order from the different methods.
Step 4: Obtain the filter co efficient h(n) and analyse the filter response.
Step 5: Increase filter order if frequency response doesn't meet ideal response & repeat step 4.
Step 5: Choose the best method of filter design which requires lesser order with better frequency response.
Step 6: Design the decimator and interpolator as shown in the figures 1 and 2 using the designed FIR filter.

3. RESULTS AND DISCUSSIONS

In order to design the FIR filters for above sampling rate converters windowing techniques and equiripple method are used. For windowing techniques, the length of the anti-aliasing filter is estimated with reference to the Table 1. As another method of designing techniques, optimal equiripple design is used with length of the FIR filter required is estimated by the order approximation formula. Linear phase FIR filters are used to design (to obtain the co efficient theoretically) the decimator. Practically, the desired filter co efficient are obtained using MATLAB and FDA tool. Filter response is analyzed in both cases. The frequency responses of the filter are as shown in Figure 1, Figure 2 and Figure 3.

Table 3: Sampling rate converters

Parameter	**Decimator**			**Interpolator**		
Design Method	Hamming Window Method	Kaiser's Window Method	Optimal Method	Hanning Window Method	Kaiser's Window Method	Optimal Method
Order of the filter	481	427	395	360	247	235
Number of multiplications (in terms of 10^6)	28.8	25.62	23.7	3.24	2.2	2.05
Storage elements	481	427	395	360	247	235

We note that order of the filter is inversely proportional to the frequency deviation Δf. It is also observed, the required orders of the filter for decimators and interpolators are low in case of equiripple design method. Hence, it is proved as optimal methods of FIR design results lesser order filter for sampling rate converters as compared to other FIR designs. This in turn minimizes the delay elements and the storage elements during the implementation, hence linear phase FIR filters designed by optimal methods are preferred for the decimators and interpolators as Coefficient symmetry may be exploited and the number of multiplications per second, storage elements are also minimized during the implementation.

However, the filters for interpolation and decimation always need not to be low-pass filters, which then invoke the filter design problem to make use of other filters for some applications. In practise, filters like half band filters and multirate complementary filters are preferred for such applications.

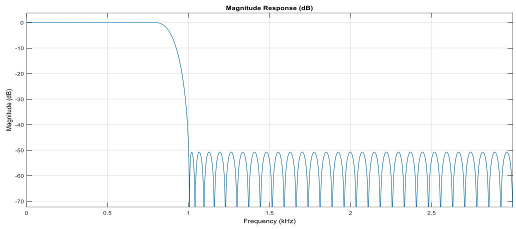

Figure 1**.** Interpolation Filter Response from Optimal Method

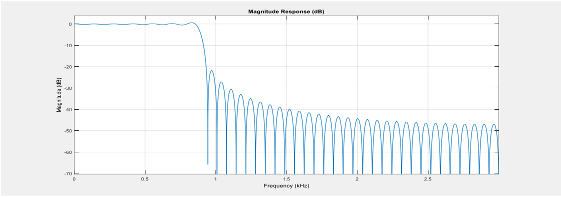

Figure 2. Interpolation Filter Response from Kasier Window Method

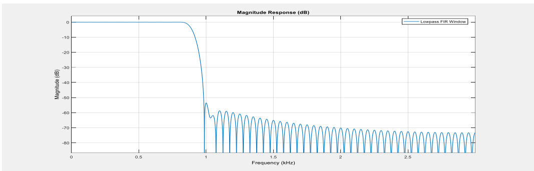

Figure 3. Interpolation Filter Response from Hamming Window Method

In practical FIR filters, the magnitude will not be constant in the pass band. The antialiasing filter of this particular examples have negligible amount of ripples in the pass band. Similarly, there will be ripples in stop band. From the responses, it is clear that the filter designed by the optimal method closely approximates the requirements and can be used to design the sampling rate converters. Due to the small transition band, signal filtering is accurate in the case of FIR filter designed by the optimal method. Whereas window method results in wider transition band with equal frequencies on both the sides of ideal cut off frequency.

The advantages of window technique are its simplicity and relatively easy to use in determining the filter coefficients. The set of predefined equations to determine coefficient have made window method to pay attention for the beginners. But equiripple methods of filter design provides control over the pass band and stop band thereby controls the frequency response. Among the different window techniques, Kaiser Window has parameter β that allows adjustment of the compromising in the ripples. By the optimal method, it is possible to obtain the controlled filter response which is of great interest in audio processing applications.

4. CONCLUSION

In multirate systems, Anti-aliasing and anti-imaging filters are employed to suppress aliasing effect and remove imaging effect of the converters. Here, the design aspects of multirate filters, their merits and demerits are discussed. Appropriate filter and design method can be adopted for multirate signal processing depending upon the application and the required response, Symmetric Linear FIR filters designed by optimal method are used for sampling rate converters because of their advantages. For multirate systems, optimal method results less order FIR filters and hence minimizes the storage elements and number of multiplications. Referring to the Table 2, we believe that the reduction in the order of filter reduces the complexity involved in the implementation of sampling rate converters and hence many of the multirate systems employ optimal FIR filter design methods.

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