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A Left (Right) Normal Near Idempotent Semigroup

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ABSTRACT

In this work, we define near-idempotency in a new form called Normal Near Idempotent Semigroup. A near idempotent semigroup is said to be left (right) normal near – idempotent semigroup if xuvwy = xuwvy(xuvwy = xvuwy), for all x, y, z, w in S. An element a of a semigroup is called an idempotent element if a^2 = a. We generalize this concept to near idempotency by defining a near idempotent element of a semigroup S as an element a of S such that $xa^2y = xay$ for all x, y in S. A near - idempotent element a of a semigroup is perhaps not the same as a^2 but produces the same effect as a^2 on multiplication by arbitrary elements of S on either side.

KEYWORDS: Idempotent semigroup, near idempotent, normality, Regularity, Band, Inflation.

1 INTRODUCTION

Idempotent semigroup was first introduced by Klein – Barmen [5] who called it schief. An idempotent element in a semigroup is an element a such that a^2 = a. An idempotent semigroup or a band is a semigroup in which every element is an idempotent. Naoki Kimura [4] has classified an idempotent semigroup B through the identities satisfied by three arbitrary elements of B. We classify Near idempotent semigroups by means of identities satisfied by them on four arbitrary elements.

We define below a normal Near Idempotent Semigroup.

We now define a left normal near – idempotent semigroup.

2. Left(Right) normal near idempotent semigroup

2.1 Definition

A near idempotent semigroup S is called a left normal near – idempotent semigroup if xuvwy = xuwvy for all x, y, u, v, w in S.

2.2 Example

Left normal near idempotent semigroup

Let S = { 1, 2, 3, 4, 5, 6 } and define a*b = a (mod 3)

*	1	2	3	4	5	6
1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	1	1	1	1	1	1
5	2	2	2	2	2	2
6	3	3	3	3	3	3

1, 2, 3 are idempotents and hence near idempotent elements.

From the table, it is clear that x.4 = x.1 for all x in S and $4^2 = 1$. Thus $x.4.1 = x.4^2.1$ for all x in S. By similar arguments, we can show that

$x.5.y = x.5^2.y$ and $x.6.y = x.6^2.y$

Thus S is a near idempotent semigroup.

When u = 1 or u = 4, the products uvw and uwv are both equal to 1, so that uvw = uwv for all v,w in S

When u = 2 or u = 5, the products uvw and uwv are both equal to 2, so that uvw = uwv for all v,w in S.

When u = 3 or u = 6, the products uvw and uwv are both equal to 3, so that uvw = uwv for all v,w in S. Thus uvw = uwv for all u,v,w in S. But S is not a band since the elements 4,5,6 are not idempotents.

Thus in S, for all x, y, u, v, w $\in$ S, xuvwy = xuwvy is true

Therefore S is a left normal near – idempotent semigroup.

2.3 Example

Right normal near idempotent semigroup

Consider the elements $1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $3 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $4 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

of $M_2(Z_2)$ under matrix multiplication modulo 2. These elements form a semigroup with the following multiplication table

X	1	2	3	4
1	1	1	1	1
2	1	2	3	4
3	1	1	1	1
4	1	2	3	4

To prove S is a near idempotent semigroup

Here 1, 2, 4 are idempotent elements

Now we have to prove the non idempotent element 3 is a near idempotent element.

$3^2 = 1 \implies 3.y = 1.y$, so that $x.1.y = x.3.y$

$$\implies x.\,3^2.y = x.3.y$$

Since 3 is also a near idempotent element

Therefore S is a near idempotent semigroup.

To check the above semigroup S is right normal near idempotent semigroup. We have to verify the identity $xuvwy = xvuwy$, for all $x, y, u, v, w \in S$.

If $uv = vu$ there is nothing to prove this is the case for the pairs of elements (1, 2), (1, 3), (1, 4) because $1.2 = 1 = 2.1$, $1.3 = 1 = 3.1$, $1.4 = 1 = 4.1$

When $u = 2, v = 3$, $2.3 = 3$, $3.2 = 1$

From the table it is clear that $3.w = 1.w$, for all $w \in S$

There fore $uvw = vuw$ in this case

When $u = 2$, $v = 4$, $2.4 = 4$, $4.2 = 2$ from the table above it is clear that $4.w = 2.w$ for all $w \in S$

Therefore $uvw = vuw$ in this case also.

Suppose $u = 3$, $v = 4$, $3.4 = 1$ and $4.3 = 3$

From the table above it is clear that $1.w = 3.w$, for all $w \in S$

Therefore $uvw = vuw$ in this case also.

Therefore for all $u, v, w \in S$ the identity $uvw = vuw$.

So that for all $x, u, v, w, y \in S$, $xuvwy = xvuwy$

S is a right normal near idempotent semigroup.

The following lemma shows that left (right) normality implies left (right) regularity in a near idempotent semigroup.

2.4 Lemma

In a near idempotent semigroup, left normality implies left regularity.

Proof:

Let S be a left normal near idempotent semigroup.

Then xuvwy = xuwvy for all x,y,u,v,w in S. Let x,y,z,w be in S. Then xyzyw = xyyzw by left normality. Hence xyzyw = xy^2zw = xyzw by near idempotency. Thus S is left regular.

Dually,

We can prove that the right normal near idempotent semigroup is a right regular near – idempotent semigroup.

CONCLUSION

An attempt to generalize a band is also not new. One such attempt is a semigroup whose factorizable elements form a band. Ananth K. Atre [1] has studied a semigroup whose factorizable elements form a rectangular band, with the additional condition that the semigroup is an inflation of that rectangular band. A. Jayalakshmi [3] has studied a semigroup in which the factorizable elements form a band.

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