7th International Conference on
Combinatorics, Cryptography, Computer Science and Computing
November. 16-17, 2022

Q-soft cosets, characteristic Q-soft and Q-level subsets of Q-soft subgroups

Rasul Rasuli [1]

Department of Mathematics, Payame Noor University(PNU), P. O. Box 19395-4697, Tehran, Iran.

Abstract

In this paper, we introduce the concept Q-soft coset and Q-soft middle coset of group G and investigate some of their properties and structured characteristics. Next we define characteristic Q-soft subgroup and generalized characteristic Q-soft subgroup of groups and obtain some resuls about them. Finally, we introduce Q-level subgroups of Q-soft subgroups and investigate some of their properties.

Keywords: Q-soft subgroups, Q-soft normal subgroups, homomorphism, Q-soft cosets, Q-level subsets.

AMS Mathematical Subject Classification [2010]: 03E72, 08A72, 20N25, 20K27, 20K30.

1 Introduction

Most of our traditional tools for formal modeling, reasoning, and computing are crisp, deterministic, and precise in character. However, there are many complicated problems in economics, engineering, environment, social science, medical science, etc., that involve data which are not always all crisp. We cannot successfully use classical methods because of various types of uncertainties present in these problems. There are theories, viz., theory of probability, theory of fuzzy sets [22], theory of intuitionistic fuzzy sets [2, 3], theory of vague sets [4], theory of interval mathematics [3, 5], and theory of rough sets [10] which can be considered as mathematical tools for dealing with uncertainties. In 1999, Molodtsov [9] introduced the soft set theory as a general mathematical tool for dealing with uncertainty or vagueness. The reason for these difficulties is, possibly, the inadequency of the parametrization tool of the theories. Consequently, Molodtsov [9] initiated the concept of soft theory as a mathematical tool for dealing with uncertainties which is free from the above difficulties. We are aware of the soft sets defined by Pawlak [11], which is a different concept and useful to solve some other type of problems. Soft set theory is still a better approach to deal with problems involving uncertainty. Later, this theory became a very good source of research for many mathematicians and computer scientists of recent years because of its wide range of applicability. The development in the fields of soft set theory and its application has been taking place in a rapid pace. The notion of soft groups is a branch of soft set theory and it was first introduce by Aktas and Cagman [1] in 2007. Later, some authors like Yin and Liao [21] have studied various properties of soft groups. The soft group theory has applications in the theory of computer science. Rosenfeld [18] gave the idea of fuzzy subgroups. A. Solairaju

[1] speaker

and Nagarajan [19, 20] introduce and define a new algebraic structure of Q-fuzzy groups. The author investigated properties of soft algebra [6, 12, 13, 14, 15, 16, 17]. The purpose of this paper is to introduce and investigate of Q-soft coset, Q-soft middle coset, characteristic Q-soft, generalized characteristic Q-soft subgroups of group G and Q-level subgroups of Q-soft subgroups. In section 2, we present basic definitions of soft sets and their operations and we review some elementary aspects that are necessary for this paper. In Section 3, we define Q-soft coset and Q-soft middle coset of group G and we prove that Q-soft middle coset of group G will be Q-soft subgroup of G. Also we show that the cardinal of Q-soft subgroup of G and its Q-soft middle coset will be equal and we characterize Q-soft coset of group G. Finally, by using homomorphisms and anti-homomolrphisms of groups, we obtain some isomorphisms and anti-homomorphisms between Q-soft coset of group G. In Section 4, we define characteristic Q-soft subgroup and generalized characteristic Q-soft subgroup of groups and we prove that any subgroup H of a group G can be realized as a Q-soft subgroup of some Q-soft subgroup of G. In Section 5, we introduce Q-level subgroups of Q-soft subgroups and investigate properties of them. Later, we prove that intersection and union of them is also Q-level subgroups. Next we show that each Q-level subgroup of Q-soft normal subgroup of group G is is a normal subgroup of G. Finally, we obtain some results of Q-level subgroups under product and Q-soft coset of Q-soft subgroup of groups.

2 Preliminaries

Definition 2.1. ([7]) A group is a non-empty set G on which there is a binary operation $(a,b) \to ab$ such that
(1) if a and b belong to G then ab is also in G (closure),
(2) $a(bc) = (ab)c$ for all $a, b, c \in G$ (associativity),
(3) there is an element $e \in G$ such that $ae = ea = a$ for all $a \in G$ (identity),
(4) if $a \in G$, then there is an element $a^{-1} \in G$ such that $aa^{-1} = a^{-1}a = e$ (inverse).

One can easily check that this implies the unicity of the identity and of the inverse. A group G is called abelian if the binary operation is commutative, i.e., $ab = ba$ for all $a, b \in G$.

Remark 2.2. There are two standard notations for the binary group operation: either the additive notation, that is $(a,b) \to a+b$ in which case the identity is denoted by 0, or the multiplicative notation, that is $(a,b) \to ab$ for which the identity is denoted by e.

Example 2.3. (1) $\mathbb{Z}$ with the addition and 0 as identity is an abelian group.
(2) $\mathbb{Z}$ with the multiplication is not a group since there are elements which are not invertible in $\mathbb{Z}$.

Definition 2.4. ([7]) In abstract algebra, a normal subgroup is a subgroup that is invariant under conjugation by members of the group of which it is a part. In other words, a subgroup H of a group G is normal in G if and only if $gH = Hg$ for all g in G. The definition of normal subgroup implies that the sets of left and right cosets coincide. In fact, a seemingly weaker condition that the sets of left and right cosets coincide also implies that the subgroup H of a group G is normal in G. Normal subgroups (and only normal subgroups) can be used to construct quotient groups from a given group. In mathematics, if G is a group, and H is a subgroup of G, and g is an element of G, then $gH = \{gh : h \in H\}$ is the left coset of H in G with respect to g, and $Hg = \{hg : h \in H\}$ is the right coset of H in G with respect to g. Only when H is normal will the set of right cosets and the set of left cosets of H coincide, which is one definition of normality of a subgroup. Although derived from a subgroup, cosets are not usually themselves subgroups of G, only subsets. A coset is a left or right coset of some subgroup in G. Since $Hg = g(g^{-1}Hg)$, the right coset Hg (of H with respect to g) and the left coset $g(g^{-1}Hg)$ (of the conjugate subgroup $g^{-1}Hg$) are the same. Hence it is not meaningful to speak of a coset as being left or right unless one first specifies the underlying subgroup. In other words: a right coset of one subgroup equals a left coset of a different (conjugate) subgroup. If the left cosets and right cosets are the same, then H is a normal subgroup and the cosets form a group called the quotient or factor group.

Proposition 2.5. *([7]) Let G be a group. Let H be a non-empty subset of G. The following are equivalent:*
(1) H is a subgroup of G.
(2) $x, y \in H$ implies $xy^{-1} \in H$ for all x, y.

Definition 2.6. ([7]) Let $(G, .), (H, .)$ be any two groups. The function $f : G \to H$ is called a homomorphism (anti-homomorphism) if $f(xy) = f(x)f(y)(f(xy) = f(y)f(x))$, for all $x, y \in G$.

Throughout this work, Q is a non-empty set, U refers to an initial universe set, E is a set of parameters and $P(U)$ is the power set of U.

Definition 2.7. ([8, 9]) For any subset A of E, a Q-soft subset $f_{A\times Q}$ over U is a set, defined by a function $f_{A\times Q}$, representing a mapping $f_{A\times Q} : E \times Q \to P(U)$, such that $f_{A\times Q}(x, q) = \emptyset$ if $x \notin A$. A soft set over U can also be represented by the set of ordered pairs $f_{A\times Q} = \{((x, q), f_{A\times Q}(x, q)) \mid (x, q) \in E\times Q, f_{A\times Q}(x, q) \in P(U)\}$. Note that the set of all Q-soft subsets over U will be denoted by $QS(U)$. From here on, ”soft set” will be used without over U.

Definition 2.8. ([8, 9]) Let $f_{A\times Q}, f_{B\times Q} \in QS(U)$. Then,
(1) $f_{A\times Q}$ is called an empty Q-soft subset, denoted by $\Phi_{A\times Q}$, if $f_{A\times Q}(x, q) = \emptyset$ for all $(x, q) \in E \times Q$,
(2) $f_{A\times Q}$ is called a $A \times Q$-universal soft set, denoted by $f_{\tilde{A\times Q}}$, if $f_{A\times Q}(x, q) = U$ for all $(x, q) \in A \times Q$,
(3) $f_{A\times Q}$ is called a universal Q-soft subset, denoted by $f_{\tilde{E\times Q}}$, if $f_{A\times Q}(x, q) = U$ for all $(x, q) \in E \times Q$,
(4) the set $Im(f_{A\times Q}) = \{f_{A\times Q}(x, q) : (x, q) \in A \times Q\}$ is called image of $f_{A\times Q}$ and if $A \times Q = E \times Q$, then $Im(f_{E\times Q})$ is called image of $E \times Q$ under $f_{A\times Q}$. (5) $f_{A\times Q}$ is a Q-soft subset of $f_{B\times Q}$, denoted by $f_{A\times Q}\tilde{\subseteq} f_{B\times Q}$, if $f_{A\times Q}(x, q) \subseteq f_{B\times Q}(x, q)$ for all $(x, q) \in E \times Q$,
(6) $f_{A\times Q}$ and $f_{B\times Q}$ are soft equal, denoted by $f_{A\times Q} = f_{B\times Q}$, if and only if $f_{A\times Q}(x, q) = f_{B\times Q}(x, q)$ for all $(x, q) \in E \times Q$,
(7) the set $(f_{A\times Q}\tilde{\cup} f_{B\times Q})(x, q) = f_{A\times Q}(x, q) \cup f_{B\times Q}(x, q)$ for all $(x, q) \in E \times Q$ is called union of $f_{A\times Q}$ and $f_{B\times Q}$,
(8) the set $(f_{A\times Q}\tilde{\cap} f_{B\times Q})(x, q) = f_{A\times Q}(x, q) \cap f_{B\times Q}(x, q)$ for all $(x, q) \in E \times Q$ is called intersection of $f_{A\times Q}$ and $f_{B\times Q}$.

Example 2.9. Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ be an initial universe set and $E = \{x_1, x_2, x_3, x_4, x_5\}$ be a set of parameters. Let $Q = \{q_1, q_2, q_3\}, A = \{x_1, x_2\}, B = \{x_2, x_3\}, C = \{x_4\}, D = \{x_5\}, F = \{x_1, x_2, x_3\}$. Define

$$f_{A\times Q}(x, q) = \begin{cases} \{u_1, u_2, u_3\} & \text{if } x = x_1 \text{ and } q = q_1 \\ \{u_1, u_5\} & \text{if } x = x_2 \text{ and } q = q_2 \end{cases}$$

$$f_{B\times Q}(x, q) = \begin{cases} \{u_1, u_2\} & \text{if } x = x_2 \text{ and } q = q_2 \\ \{u_2, u_4\} & \text{if } x = x_3 \text{ and } q = q_1 \end{cases}$$

$$f_{F\times Q}(x,q)=\begin{cases}\{u_1,u_2,u_3,u_4\} & \text{if } x=x_1 \text{ and } q=q_1\\ \{u_1,u_2,u_5\} & \text{if } x=x_2 \text{ and } q=q_2\\ \{u_2,u_4\} & \text{if } x=x_3 \text{ and } q=q_3\end{cases}$$

$f_{C\times Q}(x_4,q)=U$ and $f_{D\times Q}(x_5,q)=\{\}$. Then we will have

$$(f_{A\times Q}\tilde{\cup} f_{B\times Q})(x,q)=\begin{cases}\{u_1,u_2,u_3\} & \text{if } x=x_1 \text{ and } q=q_1\\ \{u_1,u_2,u_5\} & \text{if } x=x_2 \text{ and } q=q_2\\ \{u_2,u_4\} & \text{if } x=x_3 \text{ and } q=q_1\end{cases}$$

$$(f_{A\times Q}\tilde{\cap} f_{B\times Q})(x,q)=\begin{cases}\{u_1\} & \text{if } x=x_2 \text{ and } q=q_2\\ \{\} & \text{if } x\neq x_2\end{cases}$$

Also, $f_{C\times Q}=f_{C\tilde{\times}Q}$ and $f_{D\times Q}=\Phi_{D\times Q}$. Note that the definition of classical subset is not valid for the soft subset. For example $f_{A\times Q}\tilde{\subseteq} f_{F\times Q}$ does not imply that every element of $f_{A\times Q}$ is an element of $f_{F\times Q}$. Thus $f_{A\times Q}\tilde{\subseteq} f_{F\times Q}$ but $f_{A\times Q}\nsubseteq f_{F\times Q}$ as classical subset.

Definition 2.10. ([6]) Let $(G,.)$ be a group and $f_{G\times Q}\in QS(U)$. Then, $f_{G\times Q}$ is called a Q-soft subgroup over U if $f_{G\times Q}(xy,q)\supseteq f_{G\times Q}(x,q)\cap f_{G\times Q}(y,q)$ and $f_G(x^{-1},q)=f_{G\times Q}(x,q)$ for all $x,y\in G, q\in Q$. Throughout this paper, G denotes an arbitrary group with identity element e_G and the set of all Q-soft subgroup with parameter set G over U will be denoted by $S_{G\times Q}(U)$.

Example 2.11. Let $G=\{1,-1\}$ be a group and $U=\{u_1,u_2,u_3,u_4\}, Q=\{q\}$. Let $f_{G\times Q}=\{((1,q),\{u_1,u_2\}),((-1,q)$ then $f_{G\times Q}\in S_{G\times Q}(U)$.

Proposition 2.12. *([6]) Let $f_{G\times Q}\in S_{G\times Q}(U)$. Then $f_{G\times Q}(e_G,q)\supseteq f_{G\times Q}(x,q)$ for all $x\in G, q\in Q$.*

Proposition 2.13. *([6]) $f_{G\times Q}\in S_{G\times Q}(U)$ if and only if $f_{G\times Q}(xy^{-1},q)\supseteq f_{G\times Q}(x,q)\cap f_{G\times Q}(y,q)$ for all $x,y\in G, q\in Q$.*

Definition 2.14. ([6]) Let $(G,.),(H,.)$ be any two groups and $f_{G\times Q}\in S_{G\times Q}(U), g_{H\times Q}\in S_{H\times Q}(U)$. The product of $f_{G\times Q}$ and $g_{H\times Q}$, denoted by $f_{G\times Q}\tilde{\times} g_{H\times Q}:(G\times H)\times Q\to P(U)$, is defined as $f_{G\times Q}\tilde{\times} g_{H\times Q}((x,y),q)=f_{G\times Q}(x,q)\cap g_{H\times Q}(y,q)$ for all $x\in G, y\in H, q\in Q$. Throughout this paper, H denotes an arbitrary group.

Definition 2.15. ([14]) Let $f_{G\times Q}\in S_{G\times Q}(U)$ and $H=\{x\in G: f_{G\times Q}(x,q)=f_{G\times Q}(e,q)\}$, then $O(f_{G\times Q})$, the order of $f_{G\times Q}$ is defined as $O(f_{G\times Q})=O(H)$.

Definition 2.16. ([14]) Let $f_{G\times Q}\in S_{G\times Q}(U)$ then $f_{G\times Q}$ is said to be a Q-soft normal subgroup of G if $f_{G\times Q}(xy,q)=f_{G\times Q}(yx,q)$, for all $x,y\in G$ and $q\in Q$. Throughout this paper, G denotes an arbitrary group and the set of all Q-soft normal subgroup with parameter set G over U will be denoted by $NS_{G\times Q}(U)$.

Example 2.17. Let $U=\{u_1,u_2,u_3\}$ be an initial universe set and $(\mathbb{R},+)$ be an additive real group. Define $f_{\mathbb{R}\times Q}:\mathbb{R}\times Q\to P(U)$ as

$$f_{\mathbb{R}\times Q}(x,q)=\begin{cases}\{u_1,u_2\} & \text{if } x\in\mathbb{R}^{\geq 0}\\ \{u_3\} & \text{if } x\in\mathbb{R}^{<0}\end{cases}$$

then $f_{\mathbb{R}\times Q}\in NS_{\mathbb{R}\times Q}(U)$.

3 Q-soft cosets of Q-soft subgroups

Recall that $(x) = \{y^{-1}xy : y \in G\}$ is called the conjugate class of x in G.

Proposition 3.1. *Let $f_{G\times Q}$ and $g_{G\times Q}$ be two Q-soft subsets of an abelian group G. Then $f_{G\times Q}$ and $g_{G\times Q}$ are conjugate Q-soft subsets of the abelian group G if and only if $f_{G\times Q} = g_{G\times Q}$.*

Proof. Let $x, y \in G$ and $q \in Q$. If $f_{G\times Q}$ and $g_{G\times Q}$ are conjugate Q-soft subsets of the abelian group G, then $f_{G\times Q}(x, q) = g_{G\times Q}(y^{-1}xy, q) = g_{G\times Q}(yy^{-1}x, q) = g_{G\times Q}(ex, q) = g_{G\times Q}(x, q)$. Therefore $f_{G\times Q} = g_{G\times Q}$. Conversely, if$f_{G\times Q} = g_{G\times Q}$, then for the identity element e of G, we have $f_{G\times Q}(x, q) = f_{G\times Q}(e^{-1}xe, q)$. Hence $f_{G\times Q}$ and $g_{G\times Q}$ are conjugate Q-soft subsets of the abelian group G. □

Definition 3.2. Let $f_{G\times Q} \in S_{G\times Q}(U)$. For any $a \in G$, define $af_{G\times Q}$ defined by $(af_{G\times Q})(x, q) = f_{G\times Q}(a^{-1}x, q)$, for every $x \in G$ and $q \in Q$, is called a Q-soft coset of G.

Definition 3.3. Let $f_{G\times Q} \in S_{G\times Q}(U)$. Then for any $a, b \in G$, a Q-soft middle coset $af_{G\times Q}b$ of G is defined by $(af_{G\times Q}b)(x, q) = f_{G\times Q}(a^{-1}xb^{-1}, q)$, for every $x \in G$ and $q \in Q$.

Proposition 3.4. *If $f_{G\times Q}$ is a Q-soft subgroup of a group G, then for any $a \in G$ the Q-soft middle coset $a(f_{G\times Q})a^{-1}$ of G is also a Q-soft subgroup of G.*

Proof. Let $x, y, a \in G$ and $q \in Q$. Then

$$\begin{aligned}
(a(f_{G\times Q})a^{-1})(xy^{-1}, q) &= f_{G\times Q}(a^{-1}xy^{-1}a, q)\\
&= f_{G\times Q}(a^{-1}xaa^{-1}y^{-1}a, q)\\
&= f_{G\times Q}((a^{-1}xa)(a^{-1}ya)^{-1}, q)\\
&\supseteq f_{G\times Q}(a^{-1}xa, q) \cap f_{G\times Q}((a^{-1}ya)^{-1}, q)\\
&\supseteq f_{G\times Q}(a^{-1}xa, q) \cap f_{G\times Q}(a^{-1}ya, q)\\
&= (a(f_{G\times Q})a^{-1})(x, q) \cap (a(f_{G\times Q})a^{-1})(y, q).
\end{aligned}$$

Thus by Proposition 2.13 we get that $a(f_{G\times Q})a^{-1}$ is a Q-soft subgroup of G. □

Proposition 3.5. *Let $f_{G\times Q}$ be a Q-soft subgroup of a group G and $a(f_{G\times Q})a^{-1}$ be a Q-soft middle coset of G, then $O(a(f_{G\times Q})a^{-1}) = O(f_{G\times Q})$ for any $a \in G$.*

Proof. Let $f_{G\times Q}$ be a Q-soft subgroup of G and $a \in G$. By Proposition 3.4, the Q-soft middle coset $a(f_{G\times Q})a^{-1}$ is a Q-soft subgroup of G. Further by the definition of a Q-soft middle coset of G, we have $(a(f_{G\times Q})a^{-1})(x, q) = f_{G\times Q}(a^{-1}xa, q)$ for every $x \in G$ and $q \in Q$. Hence for any $a \in G$ we obtain $f_{G\times Q}$ and $a(f_{G\times Q})a^{-1}$ are conjugate Q-soft subgroups of a group G as there exists a in G such that $(a(f_{G\times Q})a^{-1})(x, q) = f_{G\times Q}(a^{-1}xa, q)$ for every $x \in G$ and $q \in Q$. Now we get that $O(a(f_{G\times Q})a^{-1}) = O(f_{G\times Q})$ for any $a \in G$. □

Proposition 3.6. *Let $f_{G\times Q}$ be a Q-soft subgroup of a group G and $g_{G\times Q}$ be a Q-soft subset of a group G. If $f_{G\times Q}$ and $g_{G\times Q}$ are conjugate Q-soft subsets of the group G, then $g_{G\times Q}$ is a Q-soft subgroup of a group G.*

Proof. Let $x, y, g \in G$ and $q \in Q$ then $xy^{-1} \in G$. Now

$$\begin{aligned} g_{G\times Q}(xy^{-1}, q) &= f_{G\times Q}(g^{-1}xy^{-1}g, q) \\ &= f_{G\times Q}(g^{-1}xgg^{-1}y^{-1}g, q) \\ &= f_{G\times Q}((g^{-1}xg)(g^{-1}yg)^{-1}, q) \\ &\supseteq f_{G\times Q}(g^{-1}xg, q) \cap f_{G\times Q}((g^{-1}yg), q) \\ &= g_{G\times Q}(x, q) \cap g_{G\times Q}(y, q). \end{aligned}$$

Hence Proposition 2.13 get us that $g_{G\times Q}$ is a Q-soft subgroup of a group G. □

Proposition 3.7. *Let $f_{G\times Q}$ be a Q-soft subgroup of a group G. Then $xf_{G\times Q} = yf_{G\times Q}$ for $x, y \in G$ if and only if $f_{G\times Q}(x^{-1}y, q) = f_{G\times Q}(y^{-1}x, q) = f_{G\times Q}(e, q)$.*

Proof. Let $f_{G\times Q}$ be a Q-soft subgroup of a group G and $x, y \in G, q \in Q$. If $xf_{G\times Q} = yf_{G\times Q}$, then $xf_{G\times Q}(x, q) = yf_{G\times Q}(x, q)$ and $xf_{G\times Q}(y, q) = yf_{G\times Q}(y, q)$ which implies that $f_{G\times Q}(x^{-1}x, q) = f_{G\times Q}(y^{-1}x, q)$ and $f_{G\times Q}(x^{-1}y, q) = f_{G\times Q}(y^{-1}y, q)$. Hence $f_{G\times Q}(e, q) = f_{G\times Q}(y^{-1}x, q)$ and $f_{G\times Q}(x^{-1}y, q) = f_{G\times Q}(e, q)$. Therefore $f_{G\times Q}(x^{-1}y, q) = f_{G\times Q}(y^{-1}x, q) = f_{G\times Q}(e, q)$.
Conversely, let $f_{G\times Q}(x^{-1}y, q) = f_{G\times Q}(y^{-1}x, q) = f_{G\times Q}(e, q)$. Let $g \in G$. Then

$$\begin{aligned} xf_{G\times Q}(g, q) &= f_{G\times Q}(x^{-1}g, q) \\ &= f_{G\times Q}(x^{-1}yy^{-1}g, q) \\ &\supseteq f_{G\times Q}(x^{-1}y, q) \cap f_{G\times Q}(y^{-1}g, q) \\ &= f_{G\times Q}(e, q) \cap f_{G\times Q}(y^{-1}g, q) \\ &= f_{G\times Q}(y^{-1}g, q) \text{ (by Proposition 2.12)} \\ &= yf_{G\times Q}(g, q) = f_{G\times Q}(y^{-1}g, q) \\ &= f_{G\times Q}(y^{-1}xx^{-1}g, q) \\ &\supseteq f_{G\times Q}(y^{-1}x, q) \cap f_{G\times Q}(x^{-1}g, q) \\ &= f_{G\times Q}(e, q) \cap f_{G\times Q}(x^{-1}g, q) \\ &= f_{G\times Q}(x^{-1}g, q) \text{ (by Proposition 2.12)} \\ &= xf_{G\times Q}(g, q). \end{aligned}$$

Thus $xf_{G\times Q} = yf_{G\times Q}$. □

Proposition 3.8. *Let $f_{G\times Q}$ be a Q-soft subgroup of a group G and $xf_{G\times Q} = yf_{G\times Q}$ for all $x, y \in G$. Then $f_{G\times Q}(x, q) = f_{G\times Q}(y, q)$ for all $q \in Q$.*

Proof. Let $x, y \in G$ and $q \in Q$. Then

$$
\begin{aligned}
f_{G\times Q}(x,q) &= f_{G\times Q}(yy^{-1}x,q)\\
&\supseteq f_{G\times Q}(y,q)\cap f_{G\times Q}(y^{-1}x,q)\\
&= f_{G\times Q}(y,q)\cap f_{G\times Q}(e,q)\\
&= f_{G\times Q}(y,q) \text{ (by Proposition 2.12)}\\
&= f_{G\times Q}(xx^{-1}y,q) \supseteq f_{G\times Q}(x,q)\cap f_{G\times Q}(x^{-1}y,q)\\
&= f_{G\times Q}(x,q)\cap f_{G\times Q}(e,q)\\
&= f_{G\times Q}(x,q) \text{ (by Proposition 2.12).}
\end{aligned}
$$

Therefore $f_{G\times Q}(x,q) = f_{G\times Q}(y,q)$. □

Proposition 3.9. *If $f_{G\times Q}$ is a Q-soft normal subgroup of a group G, then the set $\frac{G}{f_{G\times Q}} = \{xf_{G\times Q} : x \in G\}$ is a group with the operation $(xf_{G\times Q})(yf_{G\times Q}) = (xy)f_{G\times Q}$.*

Proof. (1) If $x, y \in G$, then $xy \in G$. Let $xf_{G\times Q}, yf_{G\times Q} \in \frac{G}{f_{G\times Q}}$ then $(xf_{G\times Q})(yf_{G\times Q}) = (xy)f_{G\times Q} \in \frac{G}{f_{G\times Q}}$ (closure).
(2) Let $x, y, z \in G$ then $x(yz) = (xy)z$. Now let $xf_{G\times Q}, yf_{G\times Q}, yf_{G\times Q} \in \frac{G}{f_{G\times Q}}$ so

$$
\begin{aligned}
(xf_{G\times Q})[(yf_{G\times Q})(zf_{G\times Q})] &= [(xf_{G\times Q})](yzf_{G\times Q})\\
&= (x(yz)f_{G\times Q}) = (xy)zf_{G\times Q})\\
&= (xyf_{G\times Q})(zf_{G\times Q})\\
&= [(xyf_{G\times Q})](zf_{G\times Q})\\
&= [(xf_{G\times Q})(yf_{G\times Q})](zf_{G\times Q}). \text{ (associativity)}
\end{aligned}
$$

(3) Let $x \in G$ and e be identity element of G then $xe = ex = x$. Thus $(xf_{G\times Q})(ef_{G\times Q}) = (xef_{G\times Q}) = (exf_{G\times Q}) = xf_{G\times Q}$ (identity).
(4) If $x \in G$, then there is an element $x^{-1} \in G$ such that $xx^{-1} = x^{-1}x = e$. Let $xf_{G\times Q} \in \frac{G}{f_{G\times Q}}$ there is an element $(xf_{G\times Q})^{-1} = x^{-1}f_{G\times Q} \in \frac{G}{f_{G\times Q}}$ such that

$$
\begin{aligned}
(xf_{G\times Q})(xf_{G\times Q})^{-1} &= (xf_{G\times Q})(x^{-1}f_{G\times Q})\\
&= (xx^{-1}f_{G\times Q})\\
&= (x^{-1}xf_{G\times Q})\\
&= ef_{G\times Q} = f_{G\times Q}. \text{ (inverse)}
\end{aligned}
$$

Hence $\frac{G}{f_{G\times Q}}$ is a group. □

Proposition 3.10. *Let $h : G \to H$ be a homomorphism of groups and let $g_{G\times Q}$ be a Q-soft normal subgroup of H and $g_{G\times Q}$ be homomorphic pre-image of Q-soft normal subgroup of H. Then $\varphi : \frac{G}{f_{G\times Q}} \to \frac{H}{g_{G\times Q}}$ such that $\varphi(xf_{G\times Q}) = h(x)g_{G\times Q}$ for every $x \in G$ is an isomorphism of group.*

Proof. Clearly φ is onto. Now we prove that φ is one-one. Let $x, y \in G, q \in Q$ and $xf_{G\times Q}, yf_{G\times Q} \in \frac{G}{f_{G\times Q}}$. If $\varphi(xf_{G\times Q}) = \varphi(yf_{G\times Q})$, then $h(x)g_{G\times Q} = h(y)g_{G\times Q}$. Now Proposition 3.7 implies that $g_{G\times Q}((h(x))^{-1}h(y), q) =$

$g_{G\times Q}((h(y))^{-1}h(x), q) = g_{G\times Q}(h(e), q)$ and so $g_{G\times Q}(h(x^{-1})h(y), q) = g_{G\times Q}(h(y^{-1})h(x), q) = g_{G\times Q}(h(e), q)$ which implies that $g_{G\times Q}(h(yx^{-1}), q) = g_{G\times Q}(h(xy^{-1}), q) = g_{G\times Q}(h(e), q)$. Then Proposition 3.7 implies that $xf_{G\times Q} = yf_{G\times Q}$ and then φ is one-one. Also $\varphi((xf_{G\times Q})(yf_{G\times Q})) = \varphi((xy)f_{G\times Q}) = (h(xy))g_{G\times Q} = (h(x)h(y))g_{G\times Q} = (h(x))g_{G\times Q}(h(y))g_{G\times Q} = \varphi(xf_{G\times Q})\varphi(yf_{G\times Q})$.
Therefore $\varphi((xf_{G\times Q})(yf_{G\times Q})) = \varphi(xf_{G\times Q})\varphi(yf_{G\times Q})$. Hence φ is an isomorphism. □

Proposition 3.11. *Let $h : G \to H$ be an anti-homomorphism of groups and let $g_{G\times Q}$ be a Q-soft normal subgroup of H and $f_{G\times Q}$ be anti-homomorphic pre-image of Q-soft normal subgroup of H. Then $\varphi : \frac{G}{f_{G\times Q}} \to \frac{H}{g_{G\times Q}}$ such that $\varphi(xf_{G\times Q}) = h(x)g_{G\times Q}$ for every $x \in G$ is an anti-isomorphism of group.*

Proof. Let $x, y \in G, q \in Q$ and $xf_{G\times Q}, yf_{G\times Q} \in \frac{G}{f_{G\times Q}}$. Clearly φ is onto and we prove that φ is one-one. Now $\varphi(xf_{G\times Q}) = \varphi(yf_{G\times Q})$ implies that $h(x)g_{G\times Q} = h(y)g_{G\times Q}$. Proposition 3.7 give us that

$$g_{G\times Q}((h(x))^{-1}h(y), q) = g_{G\times Q}((h(y))^{-1}h(x), q) = g_{G\times Q}(h(e), q)$$

and so

$$g_{G\times Q}(h(x^{-1})h(y), q) = g_{G\times Q}(h(y^{-1})h(x), q) = g_{G\times Q}(h(e), q)$$

which implies that $g_{G\times Q}(h(yx^{-1}), q) = g_{G\times Q}(h(xy^{-1}), q) = g_{G\times Q}(h(e), q)$. Then Proposition 3.7 implies that $xf_{G\times Q} = yf_{G\times Q}$ and then φ is one-one. Also $\varphi((xf_{G\times Q})(yf_{G\times Q})) = \varphi((xy)f_{G\times Q}) = (h(xy))g_{G\times Q} = (h(y)h(x))g_{G\times Q} = (h(y))g_{G\times Q}(h(x))g_{G\times Q} = \varphi(yf_{G\times Q})\varphi(xf_{G\times Q})$.
Therefore $\varphi((xf_{G\times Q})(yf_{G\times Q})) = \varphi(yf_{G\times Q})\varphi(xf_{G\times Q})$. Hence φ is an isomorphism. □

4 Characteristic Q-soft subgroups

Definition 4.1. Let $f_{G\times Q} \in S_{G\times Q}(U)$ then $f_{G\times Q}$ is said to be a characteristic Q-soft subgroup of G if $f_{G\times Q}(x, q) = f_{G\times Q}(f(x), q)$, for all $x \in G$ and $f \in AutG$ and $q \in Q$.

Definition 4.2. Let $f_{G\times Q} \in S_{G\times Q}(U)$. we say that $f_{G\times Q}$ is a generalized characteristic Q-soft subgroup if for all $x, y \in G$, condition $O(x) = O(y)$ implies $f_{G\times Q}(x, q) = f_{G\times Q}(y, q)$, for all $q \in Q$.

Remark 4.3. In what follows the symbol $\circ$ stands for the composition operation of functions.

Proposition 4.4. *Let $f_{G\times Q}$ be a Q-soft subgroup of a group H and h is an isomorphism from a group G onto H. Then we have the following:*
(1) If $f_{G\times Q}$ is a generalized characteristic Q-soft subgroup of H , then $f_{G\times Q} \circ h$ is a generalized characteristic Q-soft subgroup of G.
(2) If $f_{G\times Q}$ is a generalized characteristic Q-soft subgroup and h is an automorphism on G, then $f_{G\times Q} \circ h = f_{G\times Q}$.

Proof. (1) Let $f_{G\times Q}$ be a Q-soft subgroup of a group H then from [14, Proposition 3.16] $f_{G\times Q} \circ h$ is a Q-soft subgroup of G. Let $x, y \in G$ and $O(x) = O(y)$. Then we have $(f_{G\times Q} \circ h)(x, q) = f_{G\times Q}(h(x), q) = f_{G\times Q}(h(y), q) = (f_{G\times Q} \circ h)(y, q)$. Therefore $f_{G\times Q} \circ h$ is a generalized characteristic Q-soft subgroup of G.
(2) Clear. □

Proposition 4.5. *Let $f_{G\times Q}$ be a Q-soft subgroup of a group H and h is an anti-isomorphism from a group G onto H. Then we have the following:*

(1) If $f_{G\times Q}$ is a generalized characteristic Q-soft subgroup of H , then $f_{G\times Q}\circ h$ is a generalized characteristic Q-soft subgroup of G.
(2) If $f_{G\times Q}$ is a generalized characteristic Q-soft subgroup and h is an anti-automorphism on G, then $f_{G\times Q}\circ h = f_{G\times Q}$.

Proof. (1) Let $f_{G\times Q}$ be a Q-soft subgroup of a group H then by [14, Proposition 3.16] $f_{G\times Q}\circ h$ is a Q-soft subgroup of G. Let $x, y \in G$ and $O(x) = O(y)$. Then we have $(f_{G\times Q}\circ h)(x, q) = f_{G\times Q}(h(x), q) = f_{G\times Q}(h(y), q) = (f_{G\times Q}\circ h)(y, q)$. Therefore $f_{G\times Q}\circ h$ is a generalized characteristic Q-soft subgroup of G.
(2) Clear. □

Proposition 4.6. *Any subgroup H of a group G can be realized as a Q-soft subgroup of some Q-soft subgroup of G.*

Proof. Let $\alpha \in P(U)$ be fix set and $f_{G\times Q}$ be the Q-soft subset of a group G defined by

$$f_{G\times Q}(x,q) = \begin{cases} \alpha & \text{if } x \in H, q \in Q \\ \emptyset & \text{if } x \notin H, q \in Q \end{cases}$$

We claim that $f_{G\times Q}$ is a Q-soft subgroup of G. Let $x, y \in G$ and $q \in Q$.
(1) If $x, y \in H$, then $xy^{-1} \in H$. Then we get that

$$f_{G\times Q}(xy^{-1}, q) = \alpha \supseteq \alpha = \alpha \cap \alpha = f_{G\times Q}(x, q) \cap f_{G\times Q}(y, q).$$

(2) If $x \in H, y \notin H$, then $xy^{-1} \notin H$. Therefore

$$f_{G\times Q}(xy^{-1}, q) = \emptyset \supseteq \emptyset = \alpha \cap \emptyset = f_{G\times Q}(x, q) \cap f_{G\times Q}(y, q).$$

(3) If $x, y \notin H$, then $xy^{-1} \in H$ or $xy^{-1} \notin H$:
if $xy^{-1} \in H$, then

$$f_{G\times Q}(xy^{-1}, q) = \alpha \supseteq \emptyset = \emptyset \cap \emptyset = f_{G\times Q}(x, q) \cap f_{G\times Q}(y, q)$$

if $xy^{-1} \notin H$, then

$$f_{G\times Q}(xy^{-1}, q) = \emptyset \supseteq \emptyset = \emptyset \cap \emptyset = f_{G\times Q}(x, q) \cap f_{G\times Q}(y, q).$$

Now by (1)-(3) and Proposition 2.13 we obtain that $f_{G\times Q}$ is a Q-soft subgroup of G. □

5 Q-level subset

Definition 5.1. Let $f_{G\times Q}$ be a Q-soft subset of group G. For $\alpha \in P(U)$, a Q-level subset of $f_{G\times Q}$ corresponding to α is the set $f^{\alpha}_{G\times Q} = \{x \in G : f_{G\times Q} \supseteq \alpha\}$

Proposition 5.2. *Let $f_{G\times Q}$ be a Q-soft subgroup of a group G. Then for $\alpha \in P(U)$ such that $\alpha \subseteq f_{G\times Q}(e, q)$, then $f^{\alpha}_{G\times Q}$ a is a subgroup of G.*

Proof. Let $x, y \in f^{\alpha}_{G\times Q}$ and $q \in Q$. Then $f_{G\times Q}(x, q) \supseteq \alpha$ and $f_{G\times Q}(y, q) \supseteq \alpha$. Now $f_{G\times Q}(xy^{-1}, q) \supseteq f_{G\times Q}(x, q) \cap f_{G\times Q}(y, q) \supseteq \alpha \cap \alpha = \alpha$ which implies that $xy^{-1} \in f^{\alpha}_{G\times Q}$ and then by Proposition 2.5 $f^{\alpha}_{G\times Q}$ will be a subgroup of G. □

Proposition 5.3. *Let $f_{G\times Q}$ be a Q-soft subgroup of a group G. Let $\alpha, \beta \in P(U)$ with $\beta \subset \alpha$ and $\alpha \subseteq f_{G\times Q}(e,q)$ and $\beta \subseteq f_{G\times Q}(e,q)$. Then $f^{\alpha}_{G\times Q} = f^{\beta}_{G\times Q}$ if and only if there is no $x \in G$ such that $\alpha \supset f_{G\times Q}(x,q) \supset \beta$.*

Proof. Assume that $f^{\alpha}_{G\times Q} = f^{\beta}_{G\times Q}$ and there exists an $x \in G$ such that $\alpha \supset f_{G\times Q}(x,q) \supset \beta$. Then $f^{\alpha}_{G\times Q} \subset f^{\beta}_{G\times Q}$ which implies that $x \in f^{\beta}_{G\times Q}$ but $x \notin f^{\alpha}_{G\times Q}$. This is a contradiction to $f^{\alpha}_{G\times Q} = f^{\beta}_{G\times Q}$. Thus there is no $x \in G$ such that $\alpha \supset f_{G\times Q}(x,q) \supset \beta$. Conversely, if there is no $x \in G$ such that $\alpha \supset f_{G\times Q}(x,q) \supset \beta$, then $f^{\alpha}_{G\times Q} = f^{\beta}_{G\times Q}$. □

Proposition 5.4. *Let G be a group and $f_{G\times Q}$ be a Q-soft subset of G such that $f^{\alpha}_{G\times Q}$ be a Q-level subgroup of G. If $\alpha \in P(U)$ such that $\alpha \subseteq f_{G\times Q}(e,q)$, then $f_{G\times Q}$ is a Q-soft subgroup of G.*

Proof. Let $x, y \in G$ and $q \in Q$. Let $f_{G\times Q}(x,q) = \alpha$ and $f_{G\times Q}(y,q) = \beta$. Now
(1) If $\alpha \subset \beta$, then $x, y \in f^{\alpha}_{G\times Q}$. Since $f^{\alpha}_{G\times Q}$ is a Q-level subgroup of G so $xy^{-1} \in f^{\alpha}_{G\times Q}$. Then

$$f_{G\times Q}(xy^{-1},q) \supseteq \alpha = \alpha \cap \beta = f_{G\times Q}(x,q) \cap f_{G\times Q}(y,q).$$

(2) If $\alpha \supset \beta$, then $x, y \in f^{\beta}_{G\times Q}$. Since $f^{\beta}_{G\times Q}$ is a Q-level subgroup of G so $xy^{-1} \in f^{\beta}_{G\times Q}$. Then

$$f_{G\times Q}(xy^{-1},q) \supseteq \beta = \alpha \cap \beta = f_{G\times Q}(x,q) \cap f_{G\times Q}(y,q).$$

(3) If $\alpha = \beta$, then it is trivial.
Then from (1)-(3) and Proposition 2.13 we obtain that $f_{G\times Q}$ is a Q-soft subgroup of G. □

Proposition 5.5. *Let G be a group and $f_{G\times Q}$ be a Q-soft subset of G. Let $\alpha, \beta \in P(U)$. If $f^{\alpha}_{G\times Q}$ and $f^{\beta}_{G\times Q}$ be two Q-level subgroups of $f_{G\times Q}$ in G, then $f^{\alpha}_{G\times Q} \cap f^{\beta}_{G\times Q}$ will be Q-level subgroup of $f_{G\times Q}$ in G.*

Proof. Let $\alpha, \beta \in P(U)$ with $\alpha, \beta \subseteq f_{G\times Q}(e,q)$.
(1) If $\alpha \subset f_{G\times Q}(x,q) \subset \beta$, then $f^{\beta}_{G\times Q} \subseteq f^{\alpha}_{G\times Q}$ and so $f^{\alpha}_{G\times Q} \cap f^{\beta}_{G\times Q} = f^{\beta}_{G\times Q}$.
(2) If $\alpha \supset f_{G\times Q}(x,q) \supset \beta$, then $f^{\alpha}_{G\times Q} \subseteq f^{\beta}_{G\times Q}$ and so $f^{\alpha}_{G\times Q} \cap f^{\beta}_{G\times Q} = f^{\alpha}_{G\times Q}$.
(3) If $\alpha = \beta$, then $f^{\alpha}_{G\times Q} = f^{\beta}_{G\times Q}$.
Now (1)-(3) give us that $f^{\alpha}_{G\times Q} \cap f^{\beta}_{G\times Q}$ will be Q-level subgroup of $f_{G\times Q}$ in G. □

Corollary 5.6. *Let $f_{G\times Q}$ be a Q-soft subgroup of a group G and for all $i \in I$, we have $\alpha_i \in P(U)$ with $\alpha_i \subseteq f_{G\times Q}(e,q)$. If $f^{\alpha_i}_{G\times Q}$ be a collection of Q-soft subgroups of $f_{G\times Q}$, then their intersection is also a Q-soft subgroup of $f_{G\times Q}$.*

Proposition 5.7. *Let G be a group and $f_{G\times Q}$ be a Q-soft subset of G. Let $\alpha, \beta \in P(U)$. If $f^{\alpha}_{G\times Q}$ and $f^{\beta}_{G\times Q}$ be two Q-level subgroups of $f_{G\times Q}$ in G, then $f^{\alpha}_{G\times Q} \cup f^{\beta}_{G\times Q}$ will be Q-level subgroup of $f_{G\times Q}$ in G.*

Proof. Let $\alpha, \beta \in P(U)$ with $\alpha, \beta \subseteq f_{G\times Q}(e,q)$.
(1) If $\alpha \subset f_{G\times Q}(x,q) \subset \beta$, then $f^{\beta}_{G\times Q} \subseteq f^{\alpha}_{G\times Q}$ and so $f^{\alpha}_{G\times Q} \cup f^{\beta}_{G\times Q} = f^{\alpha}_{G\times Q}$.
(2) If $\alpha \supset f_{G\times Q}(x,q) \supset \beta$, then $f^{\alpha}_{G\times Q} \subseteq f^{\beta}_{G\times Q}$ and so $f^{\alpha}_{G\times Q} \cup f^{\beta}_{G\times Q} = f^{\beta}_{G\times Q}$.
(3) If $\alpha = \beta$, then $f^{\alpha}_{G\times Q} = f^{\beta}_{G\times Q}$.
Now (1)-(3) give us that $f^{\alpha}_{G\times Q} \cup f^{\beta}_{G\times Q}$ will be Q-level subgroup of $f_{G\times Q}$ in G. □

Corollary 5.8. *Let $f_{G\times Q}$ be a Q-soft subgroup of a group G and for all $i \in I$, we have $\alpha_i \in P(U)$ with $\alpha_i \subseteq f_{G\times Q}(e,q)$. If $f^{\alpha_i}_{G\times Q}$ be a collection of Q-soft subgroups of $f_{G\times Q}$, then their union is also a Q-soft subgroup of $f_{G\times Q}$.*

Proposition 5.9. *Let $f_{G\times Q}$ is a Q-soft normal subgroup of a group G and $\alpha \in P(U)$ with $\alpha \subseteq f_{G\times Q}(e,q)$. Then each Q-level subgroup $f^{\alpha}_{G\times Q}$ is a normal subgroup of G.*

Proof. Let $x \in f^{\alpha}_{G\times Q}$ and $g \in G$ and $q \in Q$ then $f_{G\times Q}(x,q) \supseteq \alpha$. As $f_{G\times Q}$ is a Q-soft normal subgroup of a group G so $f_{G\times Q}(g^{-1}xg,q) = f_{G\times Q}(xgg^{-1},q) = f_{G\times Q}(x,q) \supseteq \alpha$ and then $g^{-1}xg \in f^{\alpha}_{G\times Q}$. This means that $f^{\alpha}_{G\times Q}$ is a normal subgroup of G. □

Proposition 5.10. *Let $f_{G\times Q}$ and $g_{G\times Q}$ be Q-soft subsets of the sets G and H, respectively, and $\alpha \in P(U)$. Then $(f_{G\times Q}\tilde{\times}g_{G\times Q})^{\alpha} = f^{\alpha}_{G\times Q}\tilde{\times}g^{\alpha}_{G\times Q}$.*

Proof. Let $x, y \in G$ and $q \in Q$. Then

$$(x,y) \in (f_{G\times Q}\tilde{\times}g_{G\times Q})^{\alpha} \Longleftrightarrow (f_{G\times Q}\tilde{\times}f_{G\times Q})((x,y),q) \supseteq \alpha$$

$$\Longleftrightarrow f_{G\times Q}(x,q) \cap f_{G\times Q}(y,q) \supseteq \alpha$$

$$\Longleftrightarrow f_{G\times Q}(x,q) \supseteq \alpha \qquad \textit{and} \qquad g_{G\times Q}(y,q) \supseteq \alpha$$

$$\Longleftrightarrow x \in f^{\alpha}_{G\times Q} \qquad \textit{and} \qquad y \in g^{\alpha}_{G\times Q}$$

$$\Longleftrightarrow (x,y) \in f^{\alpha}_{G\times Q}\tilde{\times}g^{\alpha}_{G\times Q}.$$

Therefore $(f_{G\times Q}\tilde{\times}g_{G\times Q})^{\alpha} = f^{\alpha}_{G\times Q}\tilde{\times}g^{\alpha}_{G\times Q}$. □

Proposition 5.11. *Let $f_{G\times Q}$ be a Q-soft subgroup of a group G. Then $af^{\alpha}_{G\times Q} = (af_{G\times Q})^{\alpha}$ for every $a \in G, q \in Q$ and $\alpha \in P(U)$.*

Proof. Let $x \in G$ and $q \in Q$. Then

$$x \in (af_{G\times Q})^{\alpha} \Longleftrightarrow (af_{G\times Q})(x,q) \supseteq \alpha \Longleftrightarrow f_{G\times Q}(a^{-1}x,q) \supseteq \alpha$$

$$\Longleftrightarrow a^{-1}x \in f^{\alpha}_{G\times Q} \Longleftrightarrow x \in af^{\alpha}_{G\times Q}.$$

Thus $af^{\alpha}_{G\times Q} = (af_{G\times Q})^{\alpha}$. □

6 Open problem

One can introduce and define the concepts Q-soft coset, Q-soft middle coset, characteristic Q-soft subring and generalized characteristic Q-soft subring of rings and investigate some of their properties and structured of them as we did of subgroups.

Acknowledgment. It is our pleasant duty to thank referees for their useful suggestions which helped us to improve our manuscript.

References

[1] H. Aktas and N. Cagman, *Soft sets and soft groups*, Inform. Sci., **177**(2007), 2726-2735.

[2] K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, **20** (1986), 87-96.

[3] K. Atanassov, *Operators over interval valued intuitionistic fuzzy sets*, Fuzzy Sets and Systems, **64** (1994), 159-174.

[4] W. L. Gau and D. J. Buehrer, *Vague sets*, IEEE Trans. System Man Cybernet, **23** (1993), 610-614.

[5] M. B. Gorzalzany, *A method of inference in approximate reasoning based on interval-valued fuzzy sets*, Fuzzy Sets and Systems , **21** (1987), 1-17.

[6] F. Hassani and R. Rasuli, *Q-soft Subgroups and Anti-Q-soft Subgroups in Universal Algebra*, The Journal of Fuzzy Mathematics Los Angeles **26** (1) (2018), 139-152.

[7] T. Hungerford, *Algebra*, Graduate Texts in Mathematics. Springer (2003).

[8] P. K. Maji, R. Biswas and A. R. Roy, *Sof tset theory*, Computer Mathematics with Applications, **45** (2003), 555-562.

[9] D. A. Molodtsov, *Soft set theory-First results*, Computers & Mathematics With Applications **37** (4) (1999), 19-31.

[10] Z. Pawlak, *Rough sets*, International Journal of Information and Computer Sciences , **11** (1982), 341-356.

[11] Z. Pawlak, *Hard set and soft sets*, ICS Research Report, Institute of Computer Sczence, Poland, (1994).

[12] R. Rasuli, *Extension of Q-soft ideals in semigroups*, Int. J. Open Problems Compt. Math., **10** (2) (2017), 6-13.

[13] R. Rasuli, *Soft Lie Ideals and Anti Soft Lie Ideals*, The Journal of Fuzzy Mathematics Los Angeles **26** (1) (2018),193-202.

[14] R. Rasuli, *Q-Soft Normal Subgroups*, Journal of New Theory **26** (2019), 13-22.

[15] R. Rasuli, *Anti Q-soft Normal Subgroups*, The Journal of Fuzzy Mathematics Los Angles **28** (2020), 237-248.

[16] R. Rasuli, *Characterization of Q-soft R-submodules over commutative rings*, The Second National Congress on Mathematics and Statistics Conbad Kavous University, 2020.

[17] R. Rasuli and M. M. Moatamedi nezhad, *Anti Q-soft R-submodules*, The First National Conference on Soft Computing and Cognitive Science Conbad Kavous University, July 2020.

[18] A. Rosenfeld, *fuzzy groups*, J. math. Anal. Appl., **35**(1971), 512-517.

[19] A. Solairaju and R. Nagarajan, *Q- fuzzy left R- subgroups of near rings w.r.t T- norms*, Antarctica journal of mathematics, **5**(2008), 51-57.

[20] A. Solairaju and R. Nagarajan, *A New Structure and Construction of Q-Fuzzy Groups*, Advances in Fuzzy mathematics, **4**(2009), 23-29.

[21] X. Yin and Z. Liao, *Study on Soft Groups*, Journal of Computers, **8**(2013), 960-967.

[22] L. A. Zadeh, *Fuzzy sets*, Infor. and Control, **8** (1965), 338-353.

e-mail: rasulirasul@yahoo.com