7th International Conference on
Combinatorics, Cryptography, Computer Science and Computing
November. 16-17, 2022

Mixed double Roman domination number in Trees

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Abstract

Let $G = (V, E)$ be a simple graph with vertex set V and edge set E. A*mixed double Roman dominating function* ($MDRDF$) of G is a function $f : V \cup E \to \{0, 1, 2, 3\}$ satisfying the condition every element $x \in V \cup E$ for which $f(x) = 0$, is adjacent or incident to at least two elements $y, y' \in V \cup E$ for which $f(y) = f(y') = 2$ or one element $y'' \in V \cup E$ with $f(y'') = 3$, and if $f(x) = 1$, then element $x \in V \cup E$ must have at least one neighbor $y \in V \cup E$ with $f(y) \geq 2$. The mixed double Roman dominating number of G, denoted by $\gamma^*_{dR}(G)$. The weight of a $MDRDF$ f is $w(f) = \sum_{x \in V \cup E} f(x)$. The mixed double Roman domination number of G is the minimum weight of a mixed double Roman dominating function of G.

Keywords: Double Roman dominating function, Double Roman domination number, Mixed double Roman dominating function, Mixed double Roman domination number
AMS Mathematical Subject Classification [2010]: 05C69

1 Introduction

In this paper, G is a simple graph with vertex set $V = V(G)$ and edge set $E = E(G)$. The *order* $|V|$ of G is denoted by $n = n(G)$. For every vertex $v \in V$, the *open neighborhood* of v is the set $N(v) = \{u \in V(G) : uv \in E(G)\}$ and the *closed neighborhood* of v is the set $N[v] = N(v) \cup \{v\}$. The *degree* of a vertex $v \in V$ is $\deg_G(v) = |N(v)|$. The minimum and maximum degree of a graph G are denote by $\delta = \delta(G)$ and $\Delta = \Delta(G)$, respectively. The *Open neighborhood* of a set $S \subseteq V$ is the set $N(S) = \bigcup_v \in SN(v)$, and the *closed neighborhood*of S is the set $N[S] = N(S) \cup S$. For any $x \in V \cup E$, we denote by $N_m(x) = \{y \in V \cup E$: y is either adjacent or incident with $x\}$, and $N_m[x] = N_m(x) \cup \{x\}$. A *leaf* is a vertex of degree 1, a *support vertex* is a vertex adjacent to a leaf, and a *strong support vertex* is a support vertex adjacent to at least two leaves. An edge incident to a leaf is called a *pendant edge*. A *tree* is an acyclic connected graph. A tree T is a *double star* if it contains exactly two vertices that are not leaves. For $a, b \geq 2$, a double star whose support vertices have degree a and b is denoted by $S(a, b)$. If T is a rooted tree, we for each vertex v, we denote by T_v the sub-rooted tree rooted at v. The height of a rooted is the maximum distance from the root to a leaf. A *fan graph* $F_{1,n}$ is defined as the graph $K_1 + P_n$, where K_1 is the empty graph on one

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vertex and P_n is the path graph on n vertices. A set of vertices S in a graph G is *dominating set*of G if $N[S] = V$, that is, every vertex in $V \setminus S$ is adjacent to a vertex in S. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G. A more general version of domination, where each element $x \in V \cup E$ dominates $N_m[x]$, is *mixed domination*, see for examples [8, 11, 13]. For a mixed dominating set $S \subseteq V \cup E$, every element in denote $\gamma^*(G)$, of G is minimum cardinality of any mixed dominating set of G. A mixed dominating set is also called a total cover in [9, 10].
A *mixed Roman dominating function*$MRDF$ on a graph G is defined by Ahangar, Haynes and Tripodoro in [7] as a function $f : V \cup E \longrightarrow \{0, 1, 2\}$ satisfying the condition every element $x \in V \cup e$ for which $f(x) = 0$ is adjacent or incident to at least one element $y \in V \cup E$ for which $f(y) = 2$. The *weight*, $\omega(f)$, of f is defined as $f(V(G))$. The *mixed Roman domination number* of a graph G, denoted by $\gamma^*_R(G)$, is the minimum weight of any mixed Roman dominating function of G. (for example [2, 3]).
A *double Roman dominating function* on a graph G is defined by Beeler, Haynes and Hedetniemi in [7] as a function $f : V \longrightarrow \{0, 1, 2, 3\}$ having the property that if $f(u) = 0$, then vertex u has at least two neighbors assigned 2 under f or one neighbor w with $f(w) = 3$, and if $f(u) = 1$, then vertex u must have at least one neighbor w with $f(w) \geq 2$. The *weight*, $\omega(f)$, of f is defined as $f(V(G))$. The *double Roman domination number* of a graph G, denoted by $\gamma_{dR}(G)$, is the minimum weight of any double Roman dominating function of G. Further results on the double Roman domination number can be found in [7, 1].
A *Edge double Roman dominating function*($EDRDF$) of graph G is defined by Valinavaz in [4, 5, 6]as function $f : E(G) \longrightarrow \{0, 1, 2, 3\}$ having the property that if $f(e) = 0$, then edge e has at least two neighbors assigned 2 under f or one neighbor e' with $f(e') = 3$, and if $f(e) = 1$, then edge e must have at least one neighbor e' with $f(e') \geq 2$. The *weight* of an edge double Roman dominating number of f, denote by $\omega(f)$, is the value $\sum_{e\in E(G)} f(e)$. The weight of a $EDRDF$, $\sum_{e\in E(G)} f(e)$. The minimum weight of a $EDRDF$ is the edge double roman domination number of G, denoted by $\gamma_{edR}(G)$.

We introduce the mixed version of double Roman domination as follows. Given a graph G, a *mixed double Roman dominating function* ($MDRDF$) of G is a function $f : V \cup E \to \{0, 1, 2, 3\}$ satisfying the condition every element $x \in V \cup E$ for which $f(x) = 0$, is adjacent or incident to at least two elements $y, y' \in V \cup E$ for which $f(y) = f(y') = 2$ or one element $y'' \in V \cup E$ with $f(y'') = 3$, and if $f(x) = 1$, then element $x \in V \cup E$ must have at least one neighbor $y \in V \cup E$ with $f(y) \geq 2$. The mixed double Roman dominating number of G, denoted by $\gamma^*_{dR}(G)$. The weight of a $MDRDF$ f is $w(f) = \sum\limits_{x\in V\cup E} f(x)$. The mixed double Roman domination number of G is the minimum weight of a mixed double Roman dominating function of G. A $MDRDF$ with minimum weight is called a γ^*_{dR}-function on G. Each $MDRDF$ determines a partition of the set $V \cup E = (V_0 \cup E_0) \cup (V_1 \cup E_1) \cup (V_2 \cup E_2) \cup (V_3 \cup E_3)$, where $V_i \cup E_i = \{x \in V \cup E : f(x) = i\}$. For the sake of simplicity, we will denote by $f[x] = f(N_m[x]) = \sum\limits_{v\in N_m[x]} f(v)$, for all $x \in V \cup E$.

Proposition 1.1. *For $n \geq 2$,*

$$\gamma^*_{dR}(P_n) = \begin{cases} \lceil \frac{6n-3}{5} \rceil & \text{if } \; n \equiv 0, 3 \pmod 5 \\ \lceil \frac{6n-3}{5} \rceil + 1 & \text{if } \; n \equiv 1, 2, 4 \pmod 5 \end{cases}$$

2 Trees

In this section, we establish an upper bound for the mixed double Roman domination number of trees. We start the following Observation.

Observation 2.1. Let G be a graph. If G has a vertex v of degree at least two satisfying one of the following conditions:

1. there is a path vu_1 with $deg(u_1) = 1$,
2. there is a path $u_1u_2u_3v$ in G with $deg(u_2) = deg(u_3) = 2$ and $deg(u_1) = 1$,
3. there is a path $u_1u_2u_3u_4v$ in G with $deg(u_2) = deg(u_3) = deg(u_4) = 2$ and $deg(u_1) = 1$, then G has a $\gamma^*_{dR}(G)$-function f such that $f(v) \cup (\cup_{w \in N(v)} f(vw)) \neq 0$.

Proof. Let g be a $\gamma^*_{dR}(G)$-function and $Z = V(G) \cup E(G)$. If $g(v) \cup (\cup_{w \in N(v)} g(vw)) \neq 0$, then we are done. Hence, we assume that $g(v) \cup (\cup_{w \in N(v)} g(vw)) = 0$

(1) Suppose u is a leaf adjacent to v. To dominate vu, we must have $g(u) = 3$. Define $f : Z \to \{0, 1, 2, 3\}$ by $f(vu) = 3$ and $f(z) = g(z)$ for $z \in Z - \{v, u\}$. Obviously, f is a $\gamma^*_{dR}(G)$-function with the desired property.

(2) To dominate vu_3 and u_1, we must have $g(u_3) \cup g(u_3u_2) = 3$ and $g(u_2) + g(u_1u_2) + g(u_1) \geq 3$. Now define $f : Z \to \{0, 1, 2, 3\}$ by $f(u_1) = f(u_2u_3) = f(v) = 2$, $f(u_1u_2) = f(u_2) = f(u_3) = f(u_3v) = 0$ and $f(z) = g(z)$ otherwise. It is easy to see that f is a $\gamma^*_{dR}(G)$-function with the desired property.

(3) Since g is a $MDRDF$ of G, we have $g(u_4) \cup g(u_3u_4) = 3$, $g(u_3) + g(u_3u_2) + g(u_2) + g(u_1u_2) + g(u_1) \geq 3$. Define $f : Z \to \{0, 1, 2, 3\}$ by $f(u_2) = f(u_4v) = 3$, $f(u_1) = f(u_1u_2) = f(u_2u_3) = f(u_3) = f(u_3u_4) = f(u_4) = f(v) = 0$ and $f(z) = g(z)$ otherwise. Obviously f is a $\gamma^*_{dR}(G)$-function with the desired property.

□

A vertex of degree one is a *leaf* and a *support vertex* is a vertex that is adjacent to at least one leaf. A support vertex is said to be *end-support vertex* if all its neighbors except one of them are leaves and an *end-leaf* is a leaf adjacent to an end-support vertex. A *strong support vertex* is a support vertex adjacent to at least two leaves. For $r, s \geq 1$, a *double star* $S(r, s)$ is a tree with exactly two vertices that are not leaves, with one adjacent to r leaves and the other to s leaves.
A *subdivision*of an edge v_1v_2 is obtained by replacing the edge v_1v_2 with a path v_1uv_2, where u is a new vertex. The *subdivision graph* $S(G)$ is the graph obtained from G by subdividing each edge of G. The subdivision star $S(K_{1,t})$ for $t \geq 2$, is called a *healthy spider* S_t. A *wounded spider* S_t is the graph formed by subdividing at most $t - 1$ of the edges of a star $K_{1,t}$ for $t \geq 2$. Note that stars are wounded spiders. A spider is a healthy or a wounded spider.

Observation 2.2. If T is a spider of order $n \geq 3$, then

$$\gamma^*_{dR}(T) \leq \lfloor \tfrac{6n+3s}{5} \rfloor,$$

where s is the number of support vertex that are adjacent to at most two leaves.

Proof. If $T = S(K_{1,t})$ is a healthy spider for some $t \geq 2$, then obviously

$$\gamma^*_{dR}(T) = 3t \leq \lfloor \frac{6(2t+1)+3t}{5} \rfloor = \lfloor \frac{6n+3s}{5} \rfloor.$$

Assume that T is a wounded spider obtained from $K_{1,t}(t \geq 2)$ by subdividing k edges where $0 \leq k \leq t-1$. If $k = 0$, then T is a star and we have $\gamma^*_{dR}(T) = 3 \leq \lfloor \frac{6n}{5} \rfloor$. If $k = t-1$, then $n = 2t, s = t$ and we have

$$\gamma^*_{dR}(T) = 3t < \lfloor \frac{15n}{5} \rfloor + 1 = \lfloor \frac{6n+3s}{5} \rfloor.$$

Suppose $1 \leq k \leq t-2$. Then $n = t+k+1, s \geq k$ and $\gamma^*_{dR}(T) = 3k+3$. It follows that $\gamma^*_{dR}(T) = 3k+3 \leq \lfloor \frac{6n+3s}{5} \rfloor$ and the proof is complete. □

We now ready to present an upper bound for the mixed double Roman domination numbers of trees.

Theorem 2.3. *For any tree T of order $n \geq 3$*

$$\gamma^*_{dR}(G) \leq \lfloor \frac{6n+3s}{5} \rfloor,$$

where $s(T)$ is the number of support vertex that are adjacent to at most two leaves. Furthermore, this bound is sharp.

Proof. Let $s = s(T)$. The proof is by induction on n. The statement holds for all trees of order $n = 2, 3, 4$. For the induction hypothesis, let $n \geq 5$ and suppose that for every tree T of order $3 \leq n' \leq n$ the result is true. Let T be a tree of order n. We may assume that T is not a path for otherwise the result follows by Proposition 1.1. If $diam(T) = 2$, then T is a star and hence $\gamma^*_{dR}(T) = 3 \leq \lfloor \frac{6n+3s}{5} \rfloor$. If $diam(T) = 3$, then T is a double star $S(r,t)$ and it is easy to verify that $\gamma^*_{dR}(T) = 6 \leq \lfloor \frac{6n+3s}{5} \rfloor$. Henceforth, we assume that $diam(T) \geq 4$. We consider the following cases.

Case 1. T has a strong support vertex.
Let T have a strong support vertex v. Suppose u is a vertex in T with maximum distance from v. Root T at u and let w be the parent of v. Assume that $T' = T - T_v$. Clearly, every $\gamma^*_{dR}(T')$-function f can be extended to a $MDRDF$ of T by defining $f(v) = 3$ and $f(t) = 0$ for each element $t \neq w$ adjacent to v. Hence $\gamma^*_{dR}(T) \leq \gamma^*_{dR}(T') + 3$. It follows from the induction hypothesis and the fact $s(T') \leq s(T) + 1$ if $|V(T_v)| \geq 4$ and $s(T') \leq s(T')$ if $|V(T_v)| = 3$ that

$$\gamma^*_{dR}(T) \leq \gamma^*_{dR}(T') + 3 \leq \lfloor \frac{6n+3s}{5} \rfloor,$$

as desired.
Case 2. T has a path u_1u_2v with $deg(v) \geq 3, deg(u_2) = 2$ and $deg(u_1) = 1$.
Suppose that $T' = T - T_{u_2}$. Then every $\gamma^*_{dR}(T')$-function f, can be extended to an $MDRDF$ of T by defining $f(u_2) = 3$ and $f(u_1) = f(u_1u_2) = f(u_2v) = 0$. Hence $\gamma^*_{dR}(T) \leq \gamma^*_{dR}(T') + 3$. By the induction hypothesis and the fact $s(T') = s(T) - 1$, we obtain

$$\gamma^*_{dR}(T) \leq \gamma^*_{dR}(T') + 3 \leq \lfloor \frac{6(n-2)+3(s-1)}{5} \rfloor + 3 = \lfloor \frac{6n+3s}{5} \rfloor,$$

as desired.

By Cases 1 and 2, we may assume that neither T has a strong support vertex nor a path u_1u_2v with $deg(v) \geq 3$, $deg(u_2) = 2$ and $deg(u_1) = 1$.

Let $v_1v_2...v_d$ be a diametral path in T and Root T at v_d. It follows from assumptions that $deg(v_2) = deg(v_3) = deg(v_{d-1}) = deg(v_{d-2}) = 2$. Since T is not a path, we have $diam(T) \geq 6$.

Case 3. $deg(v_4) \geq 3$.

Since T has no a path $v_4u_2u_1$ where $u_2 \neq v_5$, $deg_T(u_2) = 2$ and $deg_T(u_1) = 1$. We consider the following subcases.

Subcase 3.1. There exists a path $v_4u_3u_2u_1$ in T such that $u_3 \notin \{v_3, v_5\}$.

Since $v_1v_2...v_d$ is a diametral path, we deduce that $deg(u_1) = 1$. By assumptions, we have $deg(u_3) = deg(u_2) = 2$. Let $T' = T - T_{v_4}$. Then $s(T') \geq s(T) - 1$ and every $\gamma^*_{dR}(T')$-function f, can be extended to a $MDRDF$ of T by adding 3 to u_2, v_2, v_4 and 0 t0 $v_1, v_1v_2, v_2v_3, v_3v_4, u_1, u_1u_2, u_2u_3, u_3, u_3v_4$. Hence $\gamma^*_{dR}(T) \leq \gamma^*_{dR}(T') + 9$. By the induction hypothesis and the fact $s(T') \leq s(T) - 1$, we obtain

$$\gamma^*_{dR}(T) \leq \gamma^*_{dR}(T') + 9 \leq \lfloor \frac{6(n-7)+3(s-1)}{5} \rfloor + 9 = \lfloor \frac{6n+3s}{5} \rfloor,$$

as desired.

Thus all neighbors of v_4 are leaves with exception of v_3, v_5.

Subcase 3.2. v_4 is adjacent to $\ell \geq 1$ leaves.

Let $T' = T - T_{v_4}$. Then $s(T') \geq s(T) - 1$ and every $\gamma^*_{dR}(T')$-function f, can be extended to a $MDRDF$ of T by defining $f(v_4) = f(v_2) = 3$ and $f(v_1) = f(v_1v_2) = f(v_2v_4) = f(v_3) = f(v_3v_4) = 0$ and $f(u) = f(uv_4) = 0$ for $u \in L_{v_4}$. Hence $\gamma^*_{dR}(T) \leq \gamma^*_{dR}(T') + 6$. If $\ell \leq 2$, then by the induction hypothesis and the fact $s(T') \leq s(T) - 1$, we obtain

$$\gamma^*_{dR}(T) \leq \gamma^*_{dR}(T') + 6 \leq \lfloor \frac{6(n-5)+3(s-1)}{5} \rfloor + 6 \leq \lfloor \frac{6n+3s}{5} \rfloor.$$

Let $\ell \geq 2$. Then $s(T') \leq s(T)$ and the induction hypothesis leads to

$$\gamma^*_{dR}(T) \leq \gamma^*_{dR}(T') + 6 \leq \lfloor \frac{6(n-7)+3(s)}{5} \rfloor + 6 \leq \lfloor \frac{6n+3s}{5} \rfloor.$$

as desired.

Considering Case 3, we may assume that $deg(v_4) = 2$.

Case 4. $deg(v_5) \geq 3$.

By assumption, T has no a path $v_5u_2u_1$ such that $u_2 \neq v_6, deg(u_2) = 3$ and $deg(u_1) = 1$. First let v_5 be a support vertex. Suppose that $T' = T - T_{v_5}$. By Observation 2.1, T' has a $\gamma^*_{dR}(T')$-function f such that

$$f(v_5) \cup (\bigcup_{w \in N(v_5)-\{v_4\}} f(wv_5)) \neq 0$$

We may assume that $3 \subseteq f(v_5) \cup (\bigcup_{w \in N(v_5)-\{v_4\}} f(wv_5))$. Then we can extend f to a $MDRDF$ for T by assigning $f(v_3v_4) = f(v_1) = 3$ and $f(v_2) = f(v_1v_2) = f(v_3) = f(v_4) = f(v_4v_5) = 0$. Hence $\gamma^*_{dR}(T) \leq \gamma^*_{dR}(T') + 6$. By the induction and the fact $s(T') \leq s(T)$, we obtain

$$\gamma^*_{dR}(T) \leq \gamma^*_{dR}(T') + 6 \leq \lfloor \frac{6(n-5)+3s(T)}{5} \rfloor + 6 = \lfloor \frac{6n+3s(T)}{5} \rfloor,$$

as desired.
Now let T has a path $u_1u_2u_3u_4u_5$ where $u_4 \notin \{v_4, v_6\}$. By the choice of diametral path and our assumption, we have $deg(u_2) = deg(u_3) = deg(u_4) = 2$ and $deg(u_1) = 1$. Using an argument similar to that described above, we obtain $\gamma^*_{dR}(T) \leq \lfloor \frac{6n+3s}{5} \rfloor$. Finally, let T have a path $u_1u_2u_3u_5$, where $u_3 \notin \{v_4, v_6\}$ and $deg(u_1) = 1$. By assumption, we have $\deg(u_2) = \deg(u_3) = 2$. Then a similar argument as described above, leads to the desired bound. Hence, we may assume that $\deg(v_5) = 2$. Then, we have that $\deg(v_2) = \deg(v_3) = \deg(v_4) = \deg(v_5) = 2$. First let $\deg(v_6) \geq 3$. Let $T' = T - T_{v_5}$. then every $\gamma^*_{dR}(T')$-function can be extended to a $MDRDF$ for T by assigning 3 to v_2, v_4v_5 and 0 to $v_1, v_1v_2, v_2v_3, v_3, v_3v_4, v_4, v_5, v_5v_6$. Hence $\gamma^*_{dR}(T) \leq \gamma^*_{dR}(T') + 6$. It follows from the induction hypothesis and the fact $s(T') = s(T) - 1$ that

$$\gamma^*_{dR}(T) \leq \gamma^*_{dR}(T') + 6 \leq \lfloor \frac{6(n-5)+3(s(T)-1)}{5} \rfloor + 6 \leq \lfloor \frac{6(n-5)+3s(T)}{5} \rfloor.$$

Now let $\deg(v_6) = 2$. Suppose $T' = T - T_{v_4}$ and let f be a $\gamma^*_{dR}(T')$-function. Then $s(T') = s(T)$. If $f(v_5v_6) \cup f(v_5) \neq 0$, then we may assume that $2 \subseteq f(v_5v_6) \cup f(v_5)$ and f can be extended to a $MDRDF$ of T by assigning 2 to v_2v_3, v_1, v_4 and 0 to $v_1v_2, v_2, v_3v_4, v_3, v_4v_5$. Hence $\gamma^*_{dR}(T) \leq \gamma^*_{dR}(T') + 6$. It follows from the induction hypothesis and the fact $s(T') = s(T)$ that

$$\gamma^*_{dR}(T) \leq \gamma^*_{dR}(T') + 6 \leq \lfloor \frac{6(n-5)+3s(T')}{5} \rfloor + 6 = \lfloor \frac{6(n-5)+3s(T)}{5} \rfloor.$$

Assume that $f(v_5v_6) \cup f(v_5) = 0$. Then to dominate v_5, we must have $f(v_6) = 3$ and f can be extended to a $MDRDF$ of T by assigning 2 to v_2v_3, v_1, v_4, $f(v_1v_2) = f(v_2) = f(v_3v_4) = f(v_3) = f(v_4v_5) = 0$. Hence $\gamma^*_{dR}(T) \leq \gamma^*_{dR}(T') + 6$. Now the result follows as above, and the proof of the bound is complete. The healthy spiders demonstrate that the given bound is sharp. □

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