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Combinatorics, Fibonacci numbers and Mathematical finance

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ABSTRACT

In this paper, we try to identify a specific type of Fibonacci numbers using combinatorial methods.

KEYWORDS: Combinatorics, Fibonacci number, Mathematical finance, Timezone.

1 INTRODUCTION

In finance, Fibonacci retracement is a method of technical analysis for determining support and resistance levels. It is named after the Fibonacci sequence of numbers, whose ratios provide price levels to which markets tend to retrace a portion of a move, before a trend continues in the original direction.

In mathematics, the Fibonacci numbers, denoted by F_n , form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. That is,

$$F_{0,} = 0,\ F_{1,} = 1,\ F_n = F_{n-1} + F_{n-2} \text{ for } n > 1.$$

$$\{0,1,1,2,3,5,8,13,21,34,55,89,144,\dots\}.$$

$$\frac{1+\sqrt{5}}{2} = \varphi \simeq 1.618\dots \quad , \frac{1-\sqrt{5}}{2} \simeq -0.618\dots$$

Johannes Kepler proved that $\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \varphi$.

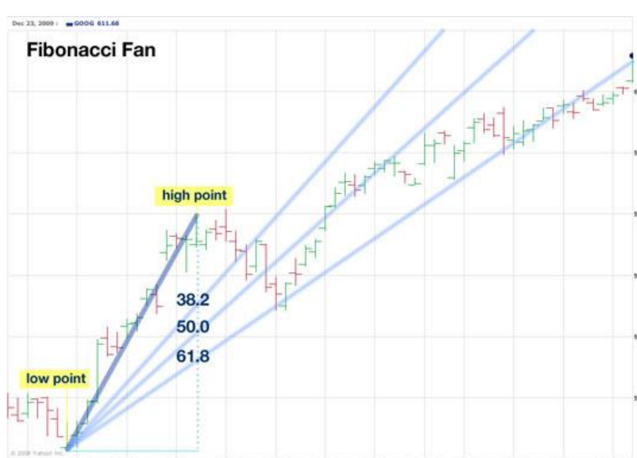

Figure 1. Fibo-Retracement

According to Fibonacci, any nature-driven market, such as the financial market, is prone to make retracements that are either 0.618 (61.8%) or 0.382 (38.2%) of the distance a stock, currency, or index has moved. There are four Fibonacci tools have been able to utilize the golden ratio. They are calculated by locating the high and low of the market chart, then drawing five horizontal lines to indicate support and resistance areas. The first line is drawn at the highest point of the chart (100%), then the second to the fifth are drawn at 61.8%, 50%, 38.2% and 0% (lowest point on the chart) in that order. When a significant price movement happens, new support and resistance levels are established near these horizontal lines.

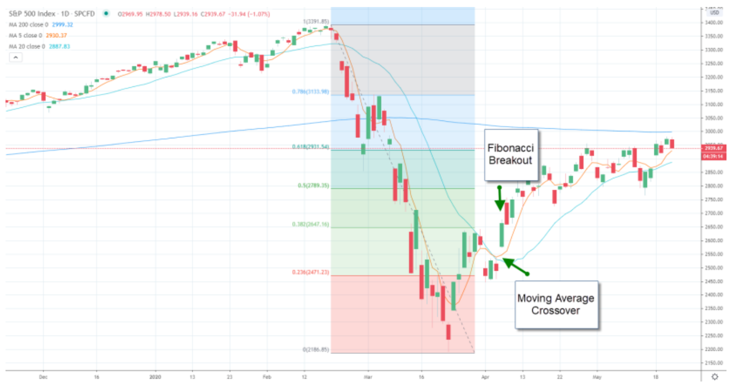

Figure 2. Fibo-Retracement

Fibonacci time zones are vertical lines that represent potential areas where a swing high, low, or reversal could occur. Fibonacci time zones may not indicate exact reversal points. They are time-based areas to be aware of. Fibonacci time zones only indicate potential areas of importance related to time.

Fibonacci Time Zones and Fibo Trend-Based Time are tools that do not count on price actions and fundamental analysis. In simple words, they do not care what happens in the market, neither technically nor fundamentally.

'The retracement level forecast' is a technique that can identify up to which level retracement can happen. These retracement levels provide a good opportunity for the traders to enter new positions in the trend direction. The Fibonacci ratios, i.e. 61.8%, 38.2%, and 23.6%, help the trader identify the retracement's possible extent. The trader can use these levels to position himself for trade.

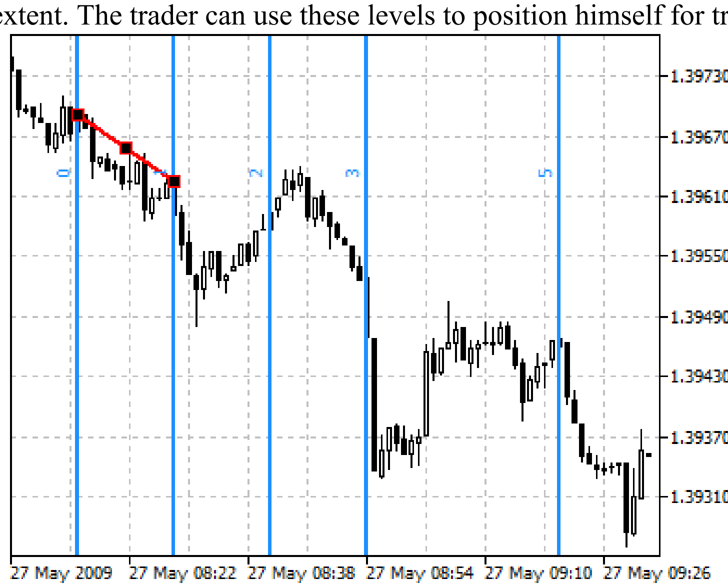

Figure 3. Fibo-Timezones

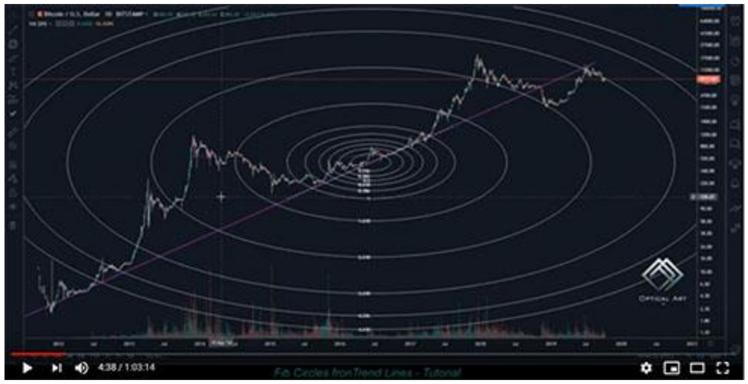

Figure 4. Fibo-Circles

According to the introduction about Fibonacci numbers and their wide applications in financial mathematics, There is this interest so that we can study more about these numbers as much as possible to make more use of the applications of Fibonacci numbers. One of the useful tools used in this case is the combination tools. Many studies have been done on this matter. Some of the studies that we have done in the past conferences on this issue are briefly mentioned here.

Theorem A. we define a special type of Fibonacci polynomials as following:

$$F_{n+2}(x) = F_{n+1}(x)(2-4x) - F_n(x),\ F_0(x) = 1, F_1(x) = 3-4x.$$

With previous notation, we have :

$$F_n(x) = \sum_{k=0}^{n} (-1)^k \binom{2n+1}{2k+1} (1-x)^{n-k}\, x^k$$

Also, when

$$F_n(x) = xF_{n-1}(x) + F_{n-2}(x),\ F_1(x) = 1, F_2(x) = x.$$

and

$$J_n(x) = J_{n-1}(x) + xJ_{n-2}(x),\ J_1(x) = 1, J_2(x) = 1.$$

We have:

$$J_n(x) = (\sqrt{x})^{n-1} F_n\left(\frac{1}{\sqrt{x}}\right)$$

Cassini's identity states that:

$$(\mathrm{F_n})^2 - \mathrm{F_{n+1}F_{n-1}} = (-1)^{\mathrm{n-1}}.$$

Catalan's identity is a generalization:

$$(\mathrm{F_n})^2 - \mathrm{F_{n+r}F_{n-r}} = (-1)^{\mathrm{n-1}}(\mathrm{F_r})^2.$$

d'Ocagne's identity

$$F_{n+1}F_m - F_{m+1}F_n = (-1)^n F_{m-\mathrm{n}}.$$

Theorem B. Assume that $\mathrm{F_n}$, for $\mathrm{n} \in \mathbb{Z}$, be the general sentence of the Fibonacci sequence.

Then:

$F_n = (-1)^n F_{-n} = \frac{1}{2}\left[-\binom{n}{1}F_0 + \binom{n}{2}F_1 - \cdots + (-1)^n\binom{n}{n-1}F_{n-1}\right]$, when n is an even number.

And also,

$\left[-\binom{n}{1}F_0 + \binom{n}{2}F_1 - \cdots + (-1)^n\binom{n}{n-1}F_{n-1}\right] = 0$, when n is an odd number.

For more result, see [1], [2], [3], [4] , [5] and [6].

2 MAIN RESULTS

Main Theorem. Assume that $\mathrm{F_m}$, for $\mathrm{m} \in \mathbb{Z}$, be the general sentence of the Fibonacci sequence.

Then:

$$F_{2m+2} = \sum_{k=0}^{m}\sum_{s=0}^{m}\binom{m-k}{s}\binom{m-s}{k}$$

Proof.

$$I = \sum_{m=0}^{\infty}\sum_{k=0}^{m}\sum_{s=0}^{m}\binom{m-k}{s}\binom{m-s}{k}x^m$$
$$= \sum_{m=0}^{\infty}(\sum_{k=0}^{m}\sum_{s=0}^{k}\binom{k}{s}\binom{m-s}{m-k})x^m$$
$$= \sum_{m=0}^{\infty}\sum_{k=0}^{m}\sum_{s=0}^{k}\binom{k}{s}\binom{m-s}{m-k}x^m$$
$$= \sum_{m=0}^{\infty}\sum_{k=0}^{\infty}\sum_{s=0}^{k}\binom{k}{s}\binom{m+k-s}{m}x^{m+s}$$
$$= \sum_{k=0}^{\infty}\sum_{s=0}^{k}\binom{k}{s}x^s(\sum_{m=0}^{\infty}\binom{m+k-s}{m}x^m)$$

We have:

$$\sum_{m=0}^{\infty}\binom{m+k-s}{m}x^m = (1-x)^{-k+s-1}$$

Therefore,

$$I = \sum_{k=0}^{\infty}\sum_{s=0}^{k}\binom{k}{s}x^s((1-x)^{-k+s-1})$$

$$= \frac{1}{1-\text{x}}(\sum_{k=0}^{\infty}\sum_{s=0}^{k}\binom{k}{s}x^s((1-x)^{-k+s}))$$

$$= \frac{1}{1-\text{x}}(\sum_{k=0}^{\infty}\sum_{s=0}^{k}\binom{k}{s}(\frac{\text{x}}{1-\text{x}})^k((1-x)^s))$$

$$= \frac{1}{1-\text{x}}(\sum_{k=0}^{\infty}(\frac{\text{x}}{1-\text{x}})^k(\sum_{s=0}^{k}\binom{k}{s}(1-x)^s))$$

$$= \frac{1}{1-\text{x}}(\sum_{k=0}^{\infty}(\frac{\text{x}}{1-\text{x}})^k(2-x)^k)$$

$$= \frac{1}{1-\text{x}}(\sum_{k=0}^{\infty}(\frac{2\text{x}-x^2}{1-\text{x}})^k)$$

$$= \frac{1}{1-\text{x}}(\frac{1}{1-\frac{2\text{x}-x^2}{1-\text{x}}})$$

□

$$= \frac{1}{1-\text{x}}(\frac{1}{1-\frac{2\text{x}-x^2}{1-\text{x}}})$$

$$= \frac{1}{1-\text{x}}(\frac{1-\text{x}}{1-3\text{x}+x^2})$$

$$= \frac{1}{1-3\text{x}+x^2}$$

$$= \left(\frac{1-\text{x}}{1-3\text{x}+x^2}\right) + \left(\frac{\text{x}}{1-3\text{x}+x^2}\right)$$

Set:

$$l(x) = \sum_{m=0}^{\infty}F_{2m+1}\text{x}^m$$

And

$$s(x) = \sum_{m=0}^{\infty}F_{2m}\text{x}^m$$

Then, we have:

$$s(x) = 0 + x + \sum_{m=2}^{\infty} F_{2m}\mathrm{x}^{m}$$

$$s(x) = x + \sum_{m=2}^{\infty} F_{2m-2}\mathrm{x}^{m} + \sum_{m=2}^{\infty} F_{2m-1}\mathrm{x}^{m}$$

$$s(x) = x + x\sum_{m=0}^{\infty} F_{2m}\mathrm{x}^{m} + x[\left(\sum_{m=0}^{\infty} F_{2m+1}\mathrm{x}^{m}\right) - x]$$

Therefore,

$$s(x) = x + xs(x) + x[l(x) - x]$$

Using similar calculation on

$$l(x) = \sum_{m=0}^{\infty} F_{2m+1}\mathrm{x}^{m}$$

We obtain that:

$$l(x) = \left(\frac{1 - \mathrm{x}}{1 - 3\mathrm{x} + x^{2}}\right)$$

$$s(x) = \left(\frac{\mathrm{x}}{1 - 3\mathrm{x} + x^{2}}\right)$$

Consequently,

$$I = \sum_{m=0}^{\infty} F_{2m+1}\mathrm{x}^{m} + \sum_{m=0}^{\infty} F_{2m}\mathrm{x}^{m}$$

$$= \sum_{m=0}^{\infty} F_{2m+2}\mathrm{x}^{m}$$

Thus,

$$F_{2m+2} = \sum_{k=0}^{m}\sum_{s=0}^{m}\binom{m-k}{s}\binom{m-s}{k}.$$

As we claimed.

3 ACKNOWLEDGEMENTS

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