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Medical Applications of the New Fuzzy H-Transformation

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ABSTRACT

Integral transformations are a very general technique for solving differential equations. These powerful methods can be used to make even the most complex problem easier to solve. This research takes a novel integral transformation, called the "h- transformation," and applies it to the problem of solving ordinary differential equations as a specific case. Concepts, characteristics, and defining terms of significance were also discussed. H-fuzzy transformation, a concept drawn from the study of differential equations where the notions of fuzzy groups are extensively applied, is used to solve the drug concentration in plasma equation due to the precision of the integrated transformations. Any time after a drug dose has been administered, a blood sample can be collected to measure the drug's concentration by measuring the amount of plasma containing the medication

KEYWORDS: h-transform, Fuzzy h-transform, find the medication concentration equation

1 INTRODUCTION

"Mathematicians use a method known as an integral transform to convert the values of one set of functions to another. If you have a function that has been transformed, you can put it back to its original form by using the inverse transform. We construct a novel integral transform, which we call the "H-transform," and establish certain theorems concerning it in this study. Fuzzy approximation models frequently employ fuzzy transformations as a technique. A fuzzy derivative was first proposed by Chang and Zadeh [6,11] in 1972, while fuzzy differential equations [7] were first proposed by Kandel and Byatt [8,9]. Abbasbandy and Allahviranloo (2002) [5] were the first to suggest the idea of using a numerical solution to solve fuzzy differential equations. This study builds on prior work by introducing an unique fuzzy transform, the H-transform, that can be utilized to more precisely solve fuzzy differential equations. It is shown that the measured drug concentration may be interpreted using this fuzzy method. The plasma concentration is commonly used as an indicator of a drug's effectiveness. The H-transform kernel may be improved by including in this article.

2 MAIN RESULT

Definition1:: Let $\Im(\delta)$ which function definitions are made for $\delta \geq 0$, let $\mu(\varepsilon)=\varepsilon, \varepsilon \neq 0$ be non-negative real functions, and $\psi(\varepsilon)=\frac{i}{\sqrt[2]{\varepsilon}}$ be a positive complex function, we have the following formula to describe its general integral transform $\Im(\delta)$:

$$H\left\{\Im(\delta),\varepsilon\right\}=\mu(\varepsilon)\int_0^{\infty}\Im(\delta)\, e^{-\psi(\varepsilon)\delta}d\,\delta=\varepsilon\int_0^{\infty}\Im(\delta)\, e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}d\,\delta\ ,\qquad .$$

Definition 2(H-Transform): Inverse H-transform of is represented by $H^{-1}\left[S\left(\varepsilon\right)\right]$ is a function that breaks up into pieces $\Im(\delta)$ on $[0,\infty)$ which satisfies: $H\left|\Im(\delta)\right| = S\left(\varepsilon\right)$.

3. H-Transform for Some Basic Functions:

Let us assume that $\Im(\delta)$ the integral in equation (1) holds true for any function. Therefore, the H-transform's case is:

Function $\Im(\delta) = T^{-1}\left\{H(\varepsilon)\right\}$	$H(\varepsilon) = H\left\{\Im(\delta), \varepsilon\right\}$
$\Im(\delta) = 1$	$H\left\{\Im(\delta), \varepsilon\right\} = \dfrac{\varepsilon}{\frac{i}{\sqrt[2]{\varepsilon}}}$,
$\Im(\delta) = \delta$	$H\left\{\Im(\delta), \varepsilon\right\} = \dfrac{\varepsilon}{\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^2}$
$\Im(\delta) = \delta^{\alpha}$	$H\left\{\Im(\delta), \varepsilon\right\} = \dfrac{\Gamma\left(\alpha+1\right)\varepsilon}{\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^{\alpha+1}}$, $\alpha \geq 0$.
$\Im(\delta) = \sin\delta$	$H\left\{\Im(\delta), \varepsilon\right\} = \dfrac{\varepsilon}{(\frac{i}{\sqrt[2]{\varepsilon}})^2 + 1}$
$\Im(\delta) = \cos\delta$	$H\left\{\Im(\delta), \varepsilon\right\} = \dfrac{\varepsilon(\frac{i}{\sqrt[2]{\varepsilon}})}{(\frac{i}{\sqrt[2]{\varepsilon}})^2 - 1}$
$\Im(\delta) = \sinh\delta$	$H\left\{\Im(\delta), \varepsilon\right\} = \dfrac{\varepsilon}{(\frac{i}{\sqrt[2]{\varepsilon}})^2 - 1}$
$\Im(\delta) = \cosh\delta$	$H\left\{\Im(\delta), \varepsilon\right\} = \dfrac{\varepsilon(\frac{i}{\sqrt[2]{\varepsilon}})^2}{(\frac{i}{\sqrt[2]{\varepsilon}})^2 - 1}$
$\Im(\delta) = e^{\delta}$	$H\left\{\Im(\delta), \varepsilon\right\} = \dfrac{\varepsilon}{\frac{i}{\sqrt[2]{\varepsilon}} - 1}$

Some functions' H-transforms have been proven as follows:

1- If $\Im(\delta) = 1$, then $H\left\{\Im(\delta), \varepsilon\right\} = \dfrac{\varepsilon}{\frac{i}{\sqrt[2]{\varepsilon}}}$

$$H\left\{\Im(\delta),\varepsilon\right\}=\varepsilon\int_0^\infty 1\, e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}\, d\delta=\frac{\varepsilon}{-\frac{i}{\sqrt[2]{\varepsilon}}}\int_0^\infty -e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}(\frac{i}{\sqrt[2]{\varepsilon}})\, d\delta=\frac{\varepsilon}{\frac{i}{\sqrt[2]{\varepsilon}}}\left[e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}\right]_0^\infty=\frac{\varepsilon}{\frac{i}{\sqrt[2]{\varepsilon}}}\ .$$

2-If $\Im(\delta)=\sin\delta$ then

$$H\left\{\Im(\delta),\varepsilon\right\}=\varepsilon\int_0^\infty \sin\delta\ e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}\, d\delta \quad integrating\ \ by\ \ parts:$$

$$(3)\qquad \int_0^\infty \sin\delta\ e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}\, d\delta=\left[\sin\delta\,\frac{-e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}}{\frac{i}{\sqrt[2]{\varepsilon}}}\right]_0^\infty-\int_0^\infty \cos\delta\ \frac{-e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}}{\frac{i}{\sqrt[2]{\varepsilon}}}\ d\delta$$

$Integrating\ \ by\ \ parts:$

$$(4)\qquad \int_0^\infty \cos\delta\ \frac{-e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}}{\frac{i}{\sqrt[2]{\varepsilon}}}\ d\delta=\left[\mathrm{con}\delta\,\frac{-e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}}{\frac{i}{\sqrt[2]{\varepsilon}}}\right]_0^\infty-\int_0^\infty \text{-}\sin\delta\ \frac{-e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}}{\frac{i}{\sqrt[2]{\varepsilon}}}\ d\delta$$

$We\ \ compensate\ (4)\ in\ (3)$ $\varepsilon\int_0^\infty \sin\delta\ e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}\, d\delta=\dfrac{\varepsilon}{(\frac{i}{\sqrt[2]{\varepsilon}})^2+1}$

$$\text{then}\ \ H\left\{\Im(\delta),\varepsilon\right\}=\varepsilon\int_0^\infty \sin\delta\ e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}\, d\delta=\frac{\varepsilon}{(\frac{i}{\sqrt[2]{\varepsilon}})^2+1}$$

3-If $\Im(\delta)=e^{\delta}$ then $SHA\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\varepsilon}{\frac{i}{\sqrt[2n]{\varepsilon}}+\varepsilon-1}$

$$H\left\{\Im(\delta),\varepsilon\right\}=\varepsilon\int_0^\infty \mathrm{e}^{\delta}e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}\, d\delta=\frac{\varepsilon}{e^{\frac{i}{\sqrt[2]{\varepsilon}}}-1}.$$

Theorem1: Let $\Im(\delta)$ is a continuous function with respect to $\delta\geq 0$ and act as a genuinely useful function, $\psi(\varepsilon)=\dfrac{i}{\sqrt[2]{\varepsilon}},\varepsilon\neq 0$ be positive complex function then :

(I) $H\left\{\Im(\delta)',\varepsilon\right\}=(\frac{i}{\sqrt[2]{\varepsilon}})H(\varepsilon)-\varepsilon\ \Im(0)$,

(Π) $H\left\{\Im(\delta)'',\varepsilon\right\}=(\frac{i}{\sqrt[2]{\varepsilon}})^2 H(\varepsilon)-\varepsilon\ (\frac{i}{\sqrt[2]{\varepsilon}})\Im(0)-\varepsilon\ \Im(0)'$,

(IΠ) $H\left\{\Im(\delta)^{(n)},\varepsilon\right\}=(\frac{i}{\sqrt[2]{\varepsilon}})^n H(\varepsilon)-\varepsilon\ \sum_{k=0}^{n-1}(\frac{i}{\sqrt[2]{\varepsilon}})^{n-1-k}\ \Im(0)^k$.

Proof :

(I) since $H\{\Im(\delta)',\varepsilon\} = \varepsilon\int_0^\infty \Im(\delta)' e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta} d\delta$, *integrating by parts* :

$$H\{\Im(\delta)',\varepsilon\} = \varepsilon\left[\Im(\delta)e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}\right]_0^\infty + \int_0^\infty \Im(\delta)e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta} d\delta = (\frac{i}{\sqrt[2]{\varepsilon}})H(\varepsilon) - \varepsilon\,\Im(0) .$$

(Π) since $H\{\Im(\delta)'',\varepsilon\} = \varepsilon\int_0^\infty \Im(\delta)'' e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta} d\delta$, *integrating by parts* :

$$= (\frac{i}{\sqrt[2n]{\varepsilon}} + \varepsilon)H\{\Im(\delta)',\varepsilon\} - \varepsilon\Im(0)' = (\frac{i}{\sqrt[2]{\varepsilon}})^2 H(\varepsilon) - \varepsilon\,(\frac{i}{\sqrt[2]{\varepsilon}})\Im(0) - \varepsilon\,\Im(0)^{\backslash} ,$$

(ΠI) For this n-th derivative can be prove using mathematical induction.

Theorem2: Suppose that $\Im_1(\delta)$ and $\Im_2(\delta)$ are integrable functions with H-transforms on [0, +∞),and there are H-transforms defined when $\varepsilon \neq 0$, $H\{\Im_1(\varepsilon)\} = \wp$,$H\{\Im_2(\varepsilon)\} = \mathrm{H}$,Then the H- transform of their convolution $\Im_1(\varepsilon) * \Im_2(\varepsilon)$ can defined by:

(5) $\quad H\{\Im_1 * \Im_2\}(\varepsilon) = \frac{1}{\varepsilon}\wp(\varepsilon)\mathrm{H}(\varepsilon)$

Proof:

$$H(\Im_1(\varepsilon) * \Im_2(\varepsilon)) = \varepsilon\int_0^\infty (\Im_1(\varepsilon) * \Im_2(\varepsilon))e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta} = \varepsilon\int_0^\infty e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}\int_0^\infty \Im_1(\delta)\Im_2(\delta-\tau)d\tau$$

$$= \varepsilon\int_0^\infty \Im_1(\delta)\int_0^\infty e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}\Im_2(\delta-\tau)d\tau = \frac{1}{\varepsilon}H_1(\varepsilon)H_2(\varepsilon).$$

Next theorem shows the relationship between H-transform and Laplace transform.

Theorem3(Duality between Laplace Transform and H-Transform): Let $\Im(\delta)$ be a differentiable function if F is Laplace transform of $\Im(\delta)$ and $H(\varepsilon)$ is H-Transform of $\Im(\delta)$, then:

(6) $\quad H(\varepsilon) = \varepsilon F\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)$

Proof:

From Definition 1: $H(\varepsilon) = H[\Im(\delta);\varepsilon] = \varepsilon\int_0^\infty e^{-\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\delta}\Im(\delta)d\delta$, since Laplace transform is denoted by $F(p) = \ell[f(t);p] = \int_0^\infty e^{-pt} f(t)dt$, Then $H(\varepsilon) = \varepsilon F\left(\frac{i}{\sqrt[2n]{\varepsilon}}\right)$.

Theorem4:Let $\Im_1(\delta), \Im_2(\delta), ..., \Im_n(\delta)$ are continuous functions have H-transforms and $\mu_1, \mu_2, ...\mu_n$ are arbitrary constants ,then linearity property is defined by

$$H\left[\mu_1\Im_1(\delta)\right]+\left[\mu_2\Im_2(\delta)\right]+...+\left[\mu_n\Im_n(\delta)\right]=\mu_1H\left[\Im_1(\delta)\right]+\mu_2H\left[\Im_2(\delta)\right]+...+\mu_nH\left[\Im_n(\delta)\right]$$

Proof:

Let $\Im_1(\delta)$ and $\Im_2(\delta)$ are continuous functions for $\delta \geq 0$:

$$H\left[\mu_1\Im_1(\delta)+\mu_2\Im_2(\delta)+...+\mu_n\Im_n(\delta)\right]=\varepsilon\int_0^\infty\left[\mu_1\Im_1(\delta)+\mu_2\Im_2(\delta)+...+\mu_n\Im_n(\delta)\right]\ \mathrm{e}^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}d\delta$$

$$=\varepsilon\int_0^\infty\left[\mu_1\Im_1(\delta)\right]\ \mathrm{e}^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}d\delta+\varepsilon\int_0^\infty\left[\mu_2\Im_2(\delta)\right]\ \mathrm{e}^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}d\delta+...+\varepsilon\int_0^\infty\left[\mu_n\Im_n(\delta)\right]\ \mathrm{e}^{-(\frac{i}{\sqrt[2]{\varepsilon}})\delta}d\delta$$

$$=\mu_1H\left[\Im_1(\delta)\right]+\mu_2H\left[\Im_2(\delta)\right]+...+\mu_nH\left[\Im_n(\delta)\right]$$

Illustrative Examples : The following examples using H-transform to solve initial value problems described by ordinary differential equation.

Example 1: Solve $y'+2y=0\ ;\ \mathrm{y}(0)=1$

Applying H-transform for both sides of original equation:

$$H\left[y'\right]+2H\left[y\right]=0$$

$$\Rightarrow\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)H\left[y\right]-\varepsilon y\left(0\right)+2H\left[y\right]=0\ ,\textit{using the initial condition, to obtain}\ :$$

$$\Rightarrow\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)H\left[y\right]+2H\left[y\right]-\varepsilon=0\Rightarrow\left(\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)+2\right)H\left[y\right]=\varepsilon\Rightarrow H\left[y\right]=\frac{\varepsilon}{\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)+2}\ .$$

Applying the inverse of H-transform for the last equation: $y=e^{-2\delta}$.

Example 2: Solve $y''+y=0\ ;\ \mathrm{y}(0)=0,\mathrm{y}'(0)=1$

Applying H-transform for both sides of original equation:

$$H\left[y''\right]+H\left[y\right]=0$$

$$\Rightarrow\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^2H\left[y\right]-\varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)y\left(0\right)-\varepsilon y'\left(0\right)+H\left[y\right]=0\ \textit{Using the initial conditions}:$$

$$\Rightarrow\left(\frac{i}{\sqrt[2n]{\varepsilon}}\right)^2H\left[y\right]+H\left[y\right]-\varepsilon=0\Rightarrow\left(\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^2+1\right)\left[y\right]=\varepsilon\Rightarrow H\left[y\right]=\frac{\varepsilon}{\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^2+1}$$

Applying inverse of H-transform for the last equation, we get: $y=\sin\varepsilon$.

4. Fuzzy H-Transform.

In the last decades, fuzzy differential equations have been used in many fields due to their numerous and important applications in a wide range of fields[7,9]. In order to keep pace with the rapid development and progress in the field of fuzzy differential equations, we presented this paper that contains a new method for solving this type of equations[5,6], In this section, we convert the H-Transform into a fuzzy SHA-transform and introduce some properties as well as fuzzy theories of the first and second order then use these formulas to solve realistic problem which is drug concentration in blood.

Definition 3[4]:A fuzzy number β in parametric form is a pair $(\underline{\beta},\overline{\beta})$ of functions $\underline{\beta}(\phi),\overline{\beta}(\phi)$, $0\le\phi\le1$, which satisfy the following requirements:

1. $\underline{\beta}(\phi)$ is a bounded non-decreasing left continuous function in $(0,1]$,and right continuous at 0 .
2. $\overline{\beta}(\phi)$ is a bounded non-increasing left continuous function in $(0,1]$,and right continuous at 0.
3. $\underline{\beta}(\phi)\le\overline{\beta}(\phi)$, $0\le\phi\le1$. For arbitrary $\beta=\underline{\beta}(\phi),\overline{\beta}(\phi)$ for $\alpha=\underline{\alpha}(\phi),\overline{\alpha}(\phi)$ and $\varphi>0$ we define addition $\beta\oplus\alpha$,subtraction $\beta\ominus\alpha$ and scalar multiplication by $\varphi>0$ as following

   (a) Addition: $\beta\oplus\alpha=\underline{\beta}(\phi)+\underline{\alpha}(\phi),\overline{\beta}(\phi)+\overline{\alpha}(\phi)$.

   (b) Subtraction: $\beta\ominus\alpha=\underline{\beta}(\phi)-\overline{\alpha}(\phi),\underline{\alpha}(\phi),\overline{\beta}(\phi)-\underline{\alpha}(\phi)$.

   (c) Scalar multiplication: $\varphi\odot\beta=\begin{cases}(\varphi\underline{\beta},\varphi\overline{\beta}) & \varphi\ge0\\ (\varphi\overline{\beta},\varphi\underline{\beta}) & \varphi<0\end{cases}$.

Definition4.[4] :Let $\beta,\alpha\in E$(E the set of all fuzzy number) If there exists $\eta\in E$ such that $\beta+\alpha=\eta$ then η is named Hukuhara difference of β , α and is identify by $\beta\ominus\alpha$.

Note: The sign "$\ominus$" always stands for Hukuhara difference.

Definition 5.[7]: Let $\varphi(\sigma):(a,b)\to E$(E the set of all fuzzy number) continuous fuzzy-valued function and $\sigma_0\in(a,b)$,it has been that φ is strongly generalized differential at σ_0 if an aspect exists an element $\varphi^{\backslash}(\sigma_0)\in E$ such that :

1- For all $\forall\ h>0$ sufficiently small $\exists\varphi(\sigma_0+h)\ominus\varphi(\varpi_0),\exists\varphi(\sigma_0)\ominus\varphi(\sigma_0-h)$ and the

$$limit\ \ is\ \ \varphi^{\backslash}(\sigma_0)=\lim_{h\to0^+}\frac{\kappa(\sigma_0+h)\ominus\varphi(\sigma_0)}{h}=\lim_{h\to0^+}\frac{\varphi(\sigma_0)\ominus\varphi(\sigma_0-h)}{h}\ .$$

Or

2- For all $\forall\ h>0$ sufficiently small $\exists\varphi(\sigma_0)\ominus\varphi(\sigma_0+h),\exists\varphi(\sigma_0-h)\ominus\varphi(\sigma_0)$ and the

$$limit\ \ is\ \ \varphi^{\backslash}(\sigma_0)=\lim_{h\to0^+}\frac{\varphi(\sigma_0)\ominus\kappa(\sigma_0+h)}{-h}=\lim_{h\to0^+}\frac{\varphi(\sigma_0-h)\ominus\varphi(\sigma_0)}{-h}\ .$$

Or

3- For all $h>0$ sufficiently small $\exists\varphi(\sigma_0+h)\ominus\varphi(\sigma_0),\exists\varphi(\sigma_0-h)\ominus\varphi(\sigma_0)$ and the limitis

$$\varphi^{\backslash}(\sigma_0)=\lim_{h\to0^+}\frac{\varphi(\sigma_0+h)\ominus\varphi(\sigma_0)}{h}=\lim_{h\to0^+}\frac{\varphi(\sigma_0-h)\ominus\varphi(\sigma_0)}{-h}\ .$$

Or

4- For all $h>0$ sufficiently small $\exists\varphi(\sigma_0)\ominus\varphi(\sigma_0+h),\exists\varphi(\sigma_0-h)\ominus\varphi(\sigma_0)$ and the limit is

$$\varphi^{\backslash}(\sigma_0) = \lim_{h\to 0^+} \frac{\varphi(\sigma_0)\ominus\varphi(\sigma_0+h)}{-h} = \lim_{h\to 0^+} \frac{\varphi(\sigma_0-h)\ominus\varphi(\sigma_0)}{h} \quad .$$

Definition 6: Let $\varphi(\sigma)$ be a continuous fuzzy-valued function Suppose that $\varepsilon\int_0^\infty e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\sigma}\varphi(\sigma)d\sigma$ is improper fuzzy Riemann-integrable on $[0,\infty)$, then $\varepsilon\int_0^\infty e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\sigma}\varphi(\sigma)d\sigma$ is called $\mathcal{H}$ -transform and it denoted by:

$$\mathcal{H}\left[\varphi(\sigma)\right] = \mathcal{H}(\varepsilon) = \varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\int_0^\infty e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\sigma}\varphi(\sigma)d\sigma$$

Sine from theorem 2 *to get* : $\varepsilon\int_0^\infty e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\sigma}\varphi(\sigma)d\sigma = \varepsilon\int_0^\infty e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\sigma}\underline{\varphi}(\sigma;\phi)d\sigma, \varepsilon\int_0^\infty e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\sigma}\overline{\varphi}(\sigma;\phi)d\sigma$

Using the definition of classic H-transform :

$$\mathcal{H}\left[\underline{\varphi}(\sigma;\phi)\right] = \varepsilon\int_0^\infty e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\sigma}\underline{\varphi}(\sigma;\phi)d\sigma,$$

$$\mathcal{H}\left[\overline{\varphi}(\sigma;\phi)\right] = \varepsilon\int_0^\infty e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\sigma}\overline{\varphi}(\sigma;\phi)d\sigma$$

So: $\mathcal{H}\left[\varphi(\sigma;\phi)\right] = H\left[\underline{\varphi}(\sigma;\phi)\right], \mathrm{H}\left[\overline{\varphi}(\sigma;\phi)\right]$

.

Theorem7. Duality Between Fuzzy Laplace – $\mathcal{H}$ transforms

If F(p) is fuzzy Laplace transform of $\varphi(\sigma)$ and $H(\varepsilon)$ is $\mathcal{H}$ -transform of $\varphi(\sigma)$ then $H(\varepsilon) = \varepsilon F\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)$ **.**

Proof: Let $\varphi(\sigma)\in E$ then: $H(\varepsilon) = \left[\varepsilon\int_0^\infty \underline{\varphi}(\sigma;\phi)e^{-\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\sigma}d\sigma \ , \ \varepsilon\int_0^\infty \overline{\varphi}(\sigma;\phi)e^{-\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\sigma}d\sigma\right]$

Since fuzzy Laplace transform denoted by: $F(p) = \left[\int_0^\infty \underline{f}(t;\vartheta)e^{-pt}\,dt \ , \ \int_0^\infty \overline{f}(t;\vartheta)e^{-pt}\,dt\right]$

$\Rightarrow H(\varepsilon) = \left[\varepsilon\int_0^\infty \underline{\varphi}(\sigma;\phi)e^{-\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\sigma}d\sigma \ , \ \varepsilon\int_0^\infty \overline{\varphi}(\sigma;\phi)e^{-\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\sigma}d\sigma\right]$ **Thus** $SHA(\varepsilon) = \varepsilon F\left(\frac{i}{\sqrt[2n]{\varepsilon}}+\varepsilon\right)$

Theorem 8. Let $\varphi_1(\sigma),\varphi_2(\sigma),...,\varphi_n(\sigma)$ be continuous fuzzy- valued functions and suppose that $\eta_1,\eta_2,...,\eta_n$ are constant, then

$$\mathcal{H}\left[(\eta_1 \text{ e } \varphi_1(\sigma))\oplus(\eta_2 \text{ e } \varphi_2(\sigma))\oplus...\oplus(\eta_2 \text{ e } \varphi_n(\sigma))\right] = \left(\eta_1 \text{ e } \mathcal{H}\left[\varphi_1(\sigma)\right]\right)\oplus\left(\eta_2 \text{ e } \mathcal{H}\left[\varphi_2(\sigma)\right]\right)$$
$$\oplus...\oplus\left(\eta_2 \text{ e } \mathcal{H}\left[\varphi_n(\sigma)\right]\right)$$

proof:

$$\tilde{H}\begin{bmatrix}(\eta_1 \text{ e } \varphi_1(\sigma))\oplus(\eta_2 \text{ e } \varphi_2(\sigma)) \\ \oplus\ldots\oplus(\eta_2 \text{ e } \varphi_n(\sigma))\end{bmatrix} = \varepsilon\int_0^\infty e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\sigma}\left[(\eta_1 \text{ e } \varphi_1(\sigma))\oplus(\eta_2 \text{ e } \varphi_2(\sigma))\oplus\ldots\oplus(\eta_2 \text{ e } \varphi_n(\sigma))\right]d\sigma$$

$$=\varepsilon\int_0^\infty e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\sigma}\left[(\eta_1\underline{\varphi_1}(\sigma))+(\eta_2\underline{\varphi_2}(\sigma))+\ldots+(\eta_n\underline{\varphi_n}(\sigma))\right],\left[(\eta_1\overline{\varphi_1}(\sigma))+(\eta_2\overline{\varphi_2}(\sigma))+\ldots+(\eta_n\overline{\varphi_n}(\sigma))\right]d\sigma$$

$$=\varepsilon\int_0^\infty e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\sigma}\begin{bmatrix}\left[(\eta_1\underline{\varphi_1}(\sigma)),(\eta_1\overline{\varphi_1}(\sigma))\right]+\left[(\eta_2\underline{\varphi_2}(\sigma)),(\eta_2\overline{\varphi_2}(\sigma))\right] \\ +\ldots+\left[(\eta_n\underline{\varphi_n}(\sigma)),(\eta_n\overline{\varphi_n}(\sigma))\right]\end{bmatrix}d\sigma$$

$$=\varepsilon\int_0^\infty e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\sigma}\left[(\eta_1\underline{\varphi_1}(\sigma)),(\eta_1\overline{\varphi_1}(\sigma))\right]d\sigma+\varepsilon\int_0^\infty e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\sigma}\left[(\eta_2\underline{\varphi_2}(\sigma)),(\eta_2\overline{\varphi_2}(\sigma))\right]d\sigma$$

$$+\ldots+\varepsilon\int_0^\infty e^{-(\frac{i}{\sqrt[2]{\varepsilon}})\sigma}\left[(\eta_n\underline{\varphi_n}(\sigma)),(\eta_n\overline{\varphi_n}(\sigma))\right]d\sigma=\left(\eta_1 \text{ e } \tilde{H}[\varphi_1(\sigma)]\right)\oplus\left(\eta_2 \text{ e } \tilde{H}[\varphi_2(\sigma)]\right)$$

$$\oplus\ldots\oplus\left(\eta_2 \text{ e } \tilde{H}[\varphi_n(\sigma)]\right)$$

Theorem 9. Assume that $\varphi^{\backslash}(\sigma)$ be continuous fuzzy-valued function and $\varphi(\sigma)$ the primitive of $\varphi^{\backslash}(\sigma)$ on $[0,\infty)$, then:

1. $\tilde{H}\left[\varphi^{\backslash}(\sigma)\right]=(\frac{i}{\sqrt[2]{\varepsilon}})\tilde{H}\left[\varphi(\sigma)\right]\ominus\varepsilon\varphi(0)$,where φ is differentiable in the first form
2. $\tilde{H}\left[\varphi^{\backslash\backslash}(\sigma)\right]=-\varepsilon\varphi(0)\ominus\left(-\frac{i}{\sqrt[2]{\varepsilon}}\right)H\left[\varphi(\sigma)\right]$, where φ is differentiable in the second form

proof: There are two scenarios that can arise due to the fact that $\varphi^{\backslash}(\sigma)$ is a continuous fuzzy-valued function.

Case 1. If φ is the first form, for any arbitrary $\phi\in[0,1]$,

$$\tilde{H}\left[\varphi^{\backslash}(\sigma)\right]=H\left[\underline{\varphi}^{\backslash}(\sigma,\phi)\right],H\left[\overline{\varphi}^{\backslash}(\sigma,\phi)\right]$$

From Theorem 1/1:

$$H\left[\underline{\varphi}^{\backslash}(\sigma)\right]=(\frac{i}{\sqrt[2]{\varepsilon}})H\left[\underline{\varphi}(\sigma,\phi)\right]-\varepsilon\left[\underline{\varphi}(0,\phi)\right]$$

$$H\left[\overline{\varphi}^{\backslash}(\sigma)\right]=(\frac{i}{\sqrt[2]{\varepsilon}})H\left[\overline{\varphi}(\sigma,\phi)\right]-\varepsilon\left[\overline{\varphi}(0,\phi)\right]$$

By Theorem 2 : $\tilde{H}\left[\varphi^{\backslash}(\sigma)\right]=(\frac{i}{\sqrt[2]{\varepsilon}})H\left[\varphi(\sigma)\right]\ominus\varepsilon\varphi(0)$

Case 2.

If φ is the second form, for any arbitrary $\phi\in[0,1]$,

$$\tilde{H}\left[\varphi(\sigma)\right]=H\left[\underline{\varphi}(\sigma,\phi)\right],H\left[\overline{\varphi}(\sigma,\phi)\right]$$

From Theorem 1/1:

$$H\left[\overline{\varphi}'(\sigma)\right]=(\frac{i}{\sqrt[2]{\varepsilon}})H\left[\overline{\varphi}(\sigma,\phi)\right]-\varepsilon(\frac{i}{\sqrt[2]{\varepsilon}})\left[\overline{\varphi}(0,\phi)\right]$$

$$H\left[\underline{\varphi}'(\sigma)\right]=(\frac{i}{\sqrt[2]{\varepsilon}})H\left[\underline{\varphi}(\sigma,\phi)\right]-\varepsilon(\frac{i}{\sqrt[2]{\varepsilon}})\left[\underline{\varphi}(0,\phi)\right]$$

By Theorem 2 : $\tilde{H}\left[\varphi'(\sigma)\right]=-\varepsilon(\frac{i}{\sqrt[2]{\varepsilon}})\varphi(0)\ominus(\frac{i}{\sqrt[2]{\varepsilon}})H\left[\varphi(\sigma)\right]$

Theorem10. Assume that, $\varphi(\sigma)$, $\varphi'(\sigma)$ are continuous fuzzy-valued functions on $[0,\infty)$, fuzzy derivative of Fuzzy H-transform about second order it will be:

1. If φ,φ' are first form then:

$$\tilde{H}\left[\varphi''(\sigma)\right]=\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^2 H\left[\varphi(\sigma)\right]\ominus\varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\varphi(0)\ominus\varepsilon\varphi'(0)$$

2. If φ is first form and φ' second form then:

$$\tilde{H}\left[\varphi''(\sigma)\right]=-\varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\varphi(0)\ominus\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^2 H\left[\varphi(\sigma)\right]\ominus\varepsilon\varphi'(0)$$

3. If φ is second form and φ' first form then:

$$\tilde{H}\left[\varphi''(\sigma)\right]=-\varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\varphi(0)\ominus\left(-\frac{i}{\sqrt[2]{\varepsilon}}\right)^2 H\left[\varphi(\sigma)\right]-\varepsilon\varphi'(0)$$

4. If φ,φ' are second form then:

$$\tilde{H}\left[\varphi''(\sigma)\right]=\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^2 H\left[\varphi(\sigma)\right]\ominus\varepsilon\left(-\frac{i}{\sqrt[2]{\varepsilon}}\right)\varphi(0)-\varepsilon\varphi'(0)$$

Proof:

1. φ , φ' are first form and for any arbitrary $\phi\in[0,1]$, then:

$\tilde{H}\left[\varphi''(\sigma)\right]=H\left[\overline{\varphi}''(\sigma,\phi)\right],H\left[\underline{\varphi}''(\sigma,\phi)\right]$ Form Theorem 1/2:

$$H\left[\underline{\varphi}''(\sigma)\right]=\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^2 H\left[\underline{\varphi}(\sigma,\phi)\right]-\varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\underline{\varphi}(0,\phi)-\varepsilon\underline{\varphi'(0,\phi)}$$

$$H\left[\overline{\varphi}''(\sigma)\right]=\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^2 H\left[\overline{\varphi}(\sigma,\phi)\right]-\varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\overline{\varphi}(0,\phi)-\varepsilon\overline{\varphi'(0,\phi)}$$

By Theorem 2: $\tilde{H}\left[\phi''(\sigma)\right]=\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^2 H\left[\varphi(\sigma)\right]\ominus\varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\varphi(0)\ominus\varepsilon\varphi'(0)$

2. φ is first form and φ' second form, then:

$\tilde{H}\left[\varphi''(\sigma)\right]=H\left[\overline{\varphi}''(\sigma,\phi)\right],H\left[\underline{\varphi}''(\sigma,\phi)\right]$ Form Theorem 1/2:

$$H\left[\underline{\varphi}''(\sigma)\right]=\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^{2}H\left[\underline{\varphi}(\sigma,\phi)\right]-\varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\underline{\varphi}(0,\phi)-\varepsilon\underline{\varphi'(0,\phi)}$$

$$H\left[\overline{\varphi}''(\sigma)\right]=\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^{2}H\left[\overline{\varphi}(\sigma,\phi)\right]-\varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\overline{\varphi}(0,\phi)-\varepsilon\overline{\varphi'(0,\phi)}$$

By Theorem 2 : $\tilde{H}\left[\phi''(\sigma)\right]=-\varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\varphi(0)\ominus\left(-\frac{i}{\sqrt[2]{\varepsilon}}\right)^{2}H\left[\varphi(\sigma)\right]\ominus\varepsilon\varphi'(0)$

3. φ is second form and φ' first form, then:

$\tilde{H}\left[\varphi''(\sigma)\right]=H\left[\overline{\varphi}''(\sigma,\phi)\right],H\left[\underline{\varphi}''(\sigma,\phi)\right]$ Form Theorem 1/2:

$$H\left[\underline{\varphi}''(\sigma)\right]=\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^{2}H\left[\underline{\varphi}(\sigma,\phi)\right]-\varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\underline{\varphi}(0,\phi)-\varepsilon\underline{\varphi'(0,\phi)}$$

$$H\left[\overline{\varphi}''(\sigma)\right]=\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^{2}H\left[\overline{\varphi}(\sigma,\phi)\right]-\varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\overline{\varphi}(0,\phi)-\varepsilon\overline{\varphi'(0,\phi)}$$

By Theorem 2 : $\tilde{H}\left[\phi''(\sigma)\right]=-\varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\varphi(0)\ominus\left(-\frac{i}{\sqrt[2]{\varepsilon}}\right)^{2}H\left[\varphi(\sigma)\right]-\varepsilon\varphi'(0)$

4. φ,φ' are second form and for any arbitrary $r\in[0,1]$, then:

$\tilde{H}\left[\varphi''(\sigma)\right]=H\left[\underline{\varphi}''(\sigma,\phi)\right],H\left[\overline{\varphi}''(\sigma,\phi)\right]$

Form Theorem 1/2:

$$H\left[\underline{\varphi}''(\sigma)\right]=\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^{2}H\left[\underline{\varphi}(\sigma,\phi)\right]-\varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\underline{\varphi}(0,\phi)-\varepsilon\underline{\varphi'(0,\phi)}$$

$$H\left[\overline{\varphi}''(\sigma)\right]=\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^{2}H\left[\overline{\varphi}(\sigma,\phi)\right]-\varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\overline{\varphi}(0,\phi)-\varepsilon\overline{\varphi'(0,\phi)}$$

By Theorem 2 : $\tilde{H}\left[\phi''(\sigma)\right]=\left(-\frac{i}{\sqrt[2]{\varepsilon}}\right)^{2}H\left[\varphi(\sigma)\right]\ominus\varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\varphi(0)-\varepsilon\varphi'(0)$

Application: The following real word example is illustrated the usage of $\tilde{H}$ -transform.

Example3[11]: The plasma concentration of a medicine at any particular moment is used to calculate the appropriate dosage of a tablet or capsule taken orally. To illustrate, if we define $\eta'(t)$ to be the rate at which a medication from an oral dosage is absorbed by the body, then the human body's $\eta(t)$ response to a unit input of drug is a function of time. It will look like this, mathematically speaking:

$$\eta'(t)=\frac{1}{t}\eta(t),\quad \eta(0)=[\underline{\eta}(0;\vartheta),\ \overline{\eta}(\vartheta;0)]$$

There are two cases :

1. If $\eta(t)$ *is the frst form then*:

fuzzy H-transform for both sides of original equation, we get:

$$\mathcal{H}[\eta^{\backslash}(t)] = \mathcal{H}[\eta(t)]$$

Last equation becomes:

$$\left(\frac{i}{\sqrt[2]{\varepsilon}}\right) H[\underline{\eta}(\mathrm{t};\phi)] - \varepsilon\underline{\eta}(0;\phi) = H[\underline{\eta}(\mathrm{t};\phi)], \left(\frac{i}{\sqrt[2]{\varepsilon}}\right) H[\overline{\eta}(\mathrm{t};\phi)] - \varepsilon\overline{\eta}(0;\vartheta) = H[\overline{\eta}(\mathrm{t};\phi)],$$

So: $H[\underline{\eta}(\mathrm{t};\vartheta)] = \dfrac{\varepsilon}{\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)-1}\underline{\eta}(0;\vartheta), H[\overline{\eta}(\mathrm{t};\vartheta)] = \dfrac{\varepsilon}{\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)-1}\overline{\eta}(0;\vartheta)$

Now we use inverse FH-Transform for last equations:

$$\underline{\eta}(\mathrm{t};\vartheta) = e^{\sigma}\underline{\eta}(0;\vartheta), \overline{\eta}(\mathrm{t};\vartheta) = e^{\sigma}\overline{\eta}(0;\vartheta)$$

2. If $\eta(\sigma)$ is second form, then:

By using FH-Transform for both sides of original equation, we get:

$$\mathcal{H}[\eta^{\backslash}(\sigma)] = \mathcal{H}[\eta(\sigma)]$$

Last equation becomes:

$$\left(\frac{i}{\sqrt[2]{\varepsilon}}\right) H[\overline{\eta}(\mathrm{t};\phi)] - \varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\overline{\eta}(0;\vartheta) = H[\overline{\eta}(\mathrm{t};\phi)],$$

$$\left(\frac{i}{\sqrt[2]{\varepsilon}}\right) H[\underline{\eta}(\mathrm{t};\phi)] - \varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\underline{\eta}(0;\phi) = H[\underline{\eta}(\mathrm{t};\phi)]$$

By solve above equation:

$$H[\underline{\eta}(\mathrm{t};\vartheta)] = \frac{\varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^2}{\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^2-1}\overline{\eta}(0;\vartheta) - \frac{\varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^2}{\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^2-1}\underline{\eta}(0;\vartheta)$$

$$H[\overline{\eta}(\mathrm{t};\vartheta)] = \frac{\varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^2}{\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^2-1}\underline{\eta}(0;\vartheta) - \frac{\varepsilon\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)}{\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)^2-1}\underline{\eta}(0;\vartheta)$$

Now we use inverse FH-Transform for last equations:

$$\underline{\eta}(0;\vartheta) = \cosh\ \mathrm{t}\overline{\eta}(0;\vartheta) - \sinh\ \mathrm{t}\underline{\eta}(0;\vartheta), \overline{\eta}(0;\vartheta) = \cosh\ \mathrm{t}\underline{\eta}(0;\vartheta) - \sinh\ \mathrm{t}\overline{\eta}(0;\vartheta)$$

4-System Analysis and Design

In this part, various image processing approaches were used to achieve Automatic Border (Types of Diseases) Identification, and then it will be classified with the of fuzzy H-transform. The developed system works on RGB color images with size 128 x 128 pixel. The system is built with four primary processes these are explained in figure (1-1).

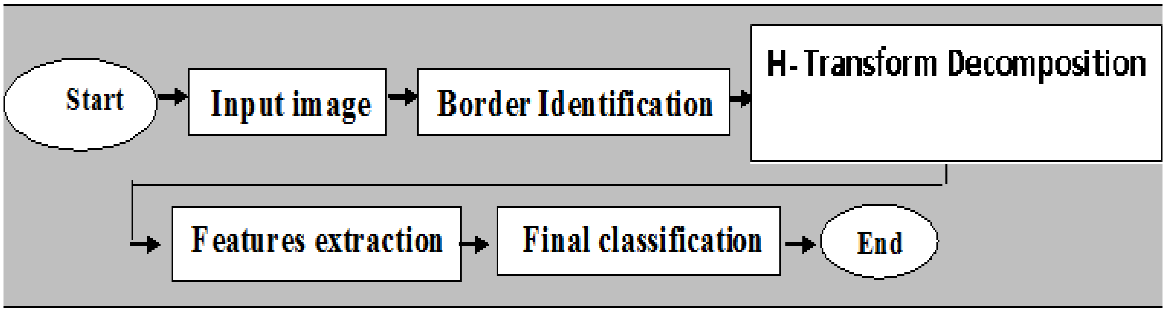
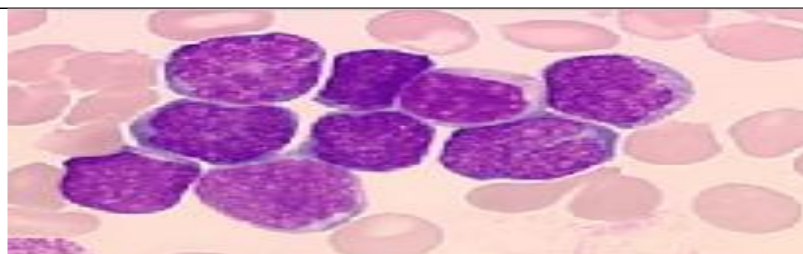
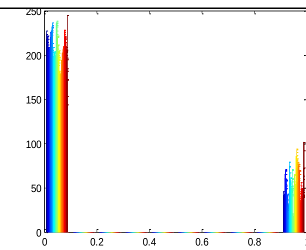
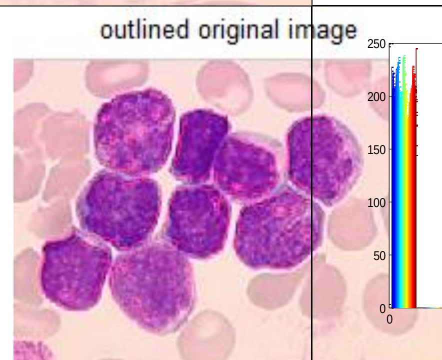
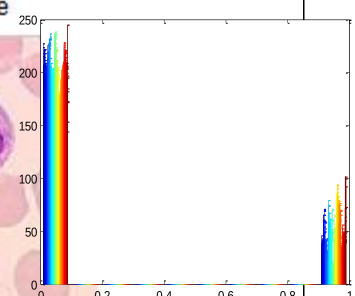


Figure (1-1)Main stages for the Automatic Border (Leukemia) Identification

Clustering approaches were presented as a means of distinguishing lymphoblasts from normal blood cells. Once an image has been preprocessed, feature extraction can be performed to gain insight into the image. There is no other purpose served by the pre-processing processes. It provides no essential details about the image. Since the image isn't immediately clear after acquisition, contrast augmentation is required. After an RGB image has been transformed to the HSI color model, a clustering approach is used to create individual segments. To clean up an image, a median filter is applied. Once features have been extracted, images can be sorted using clustering methods.

The original image			
Fuzzy H-transform			

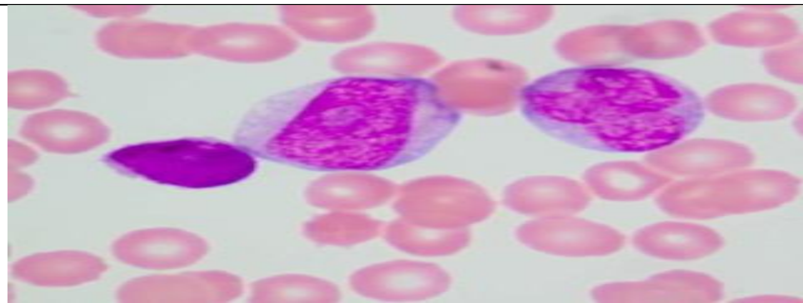
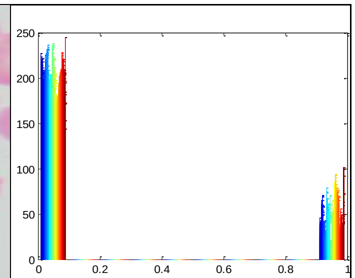
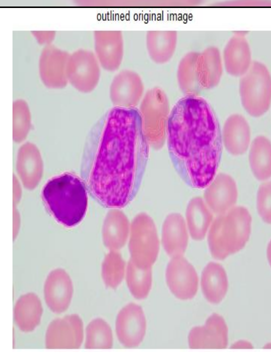
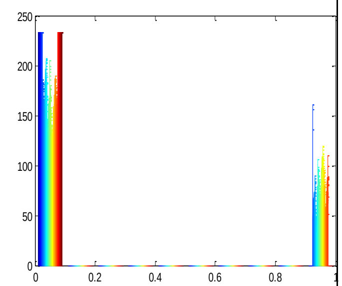
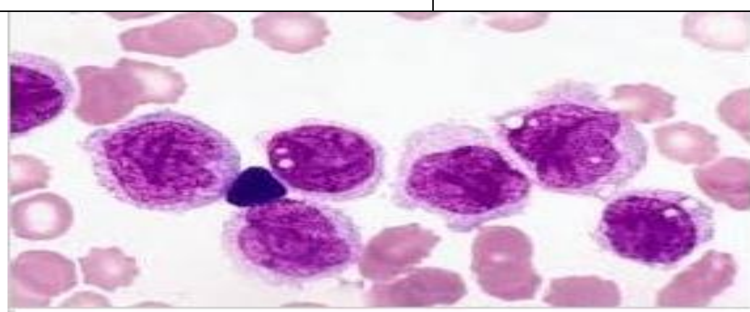
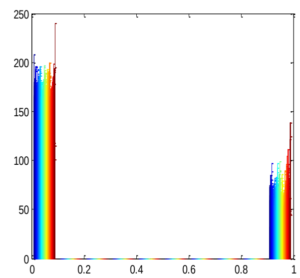
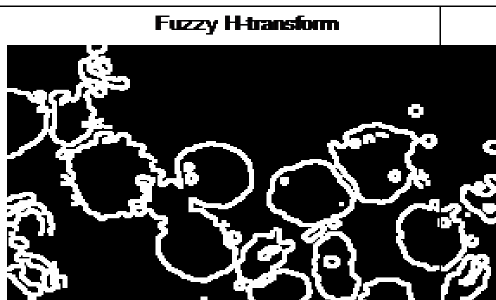
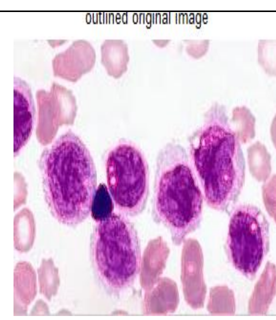
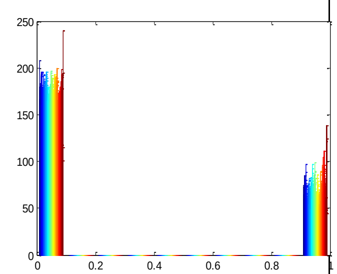
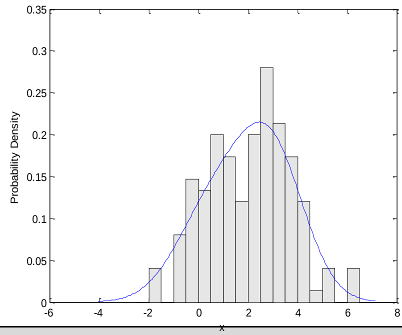
The original image		
Fuzz y H-transform	 	
The original image		
Fuzz y H-transform	 	

Figure (1-2)Main stages for the Automatic Border (Leukemia) Identification

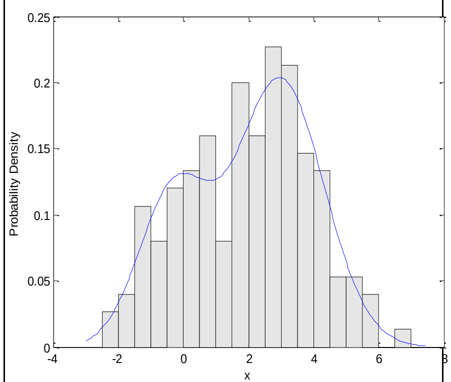
The case two statistics

The original image1



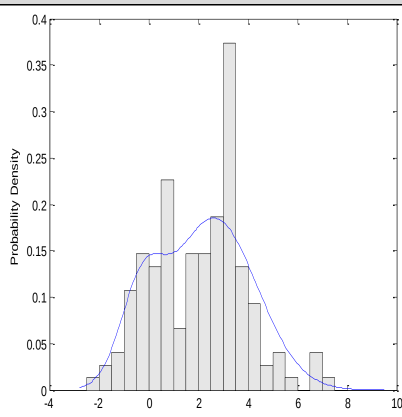
entropy	**8.989**
median	**155**
mean	**175**
Contrast	**1.4005**
Correlation	**0.888**
Energy	**3.77**
Homogeneity	**0.1170**
linearity	**1,57704**

Fuzzy H-transform



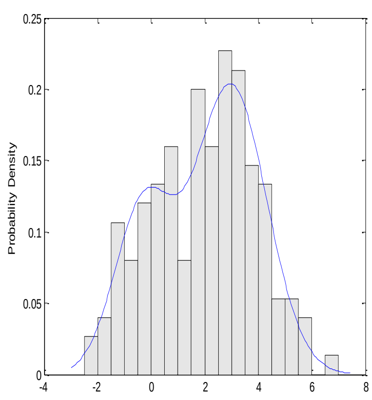
entropy	**8.411**
median	**160**
mean	**170**
Contrast	**1.4937**
Correlation	**0.0167**
Energy	**2.77**
Homogeneity	**0.02270**
linearity	**1,66604**

The original image2



entropy	**1.44**
median	**140**
mean	**183**
Contrast	**8.400**
Correlation	**0.1991**
Energy	**8.771**
Homogeneity	**0.000370**
linearity	**0,87804**

Fuzzy H-transform



entropy	**8.411**
median	**160**
mean	**170**
Contrast	**1.4937**
Correlation	**0.0167**
Energy	**2.77**
Homogeneity	**0.02270**
linearity	**1,66604**

Table (1-1)Main stages for the Automatic Border (Leukemia) Identification

5- Conclusion:

This research proposes the methodology of classifying Leukemia disease and dividing it into four stages using a new technique called (*H*-transform), where the data set consists of 30 images. It gives excellent results, as it gives preference to choosing the optimal solution for the proposed conversion, unlike the rest of the transformations, which do not provide such solutions. The conversion efficiency was also tested by distinguishing through statistical parameters, as is evident in the tables for each case, adding to the speed of results in terms of time and calculations

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