Topological Space Generated by Vertices Neighborhoods of Topological Graph

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ABSTRACT

The present work aims is to convert the topological graph to a topology by neighborhoods of the graph vertices. The resulting topology is proved as a discrete topology. A new definition for the subbase is derived and denoted by NS_{G_τ}. It contains all sets of the vertices neighborhoods. The base NB_{G_τ} is extracted from the intersection of all elements of NS_{G_τ}. Then, the neighborhood topology $N\tau_{G_\tau}$ is extracted from the union of all elements of NB_{G_τ} with some examples.

KEYWORDS: Topological graph, discrete topology, neighborhood topology.
Mathematics Subject Classification 2020: 05C69

1 INTRODUCTION

Graph Theory is the well-recognized area of discrete mathematics that deals with the study of graphs. The graphs considered here are finite, simple, and undirected. A graph $G = (V, E)$ with vertex set $V(G)$ and edge set $E(G)$. For each vertex $v \in V(G)$, the set $N_G(V) = \{u \in V \backslash uv \in E\}$ refers to the open neighborhood of v in G. See [1-17, 19, 29-31] for details of graph theoretic terminology and its applications. The discrete topology is denoted by (X, τ), where X is a non-empty set and τ is a family of all subsets of X, where $\tau = P(X)$. The sets X and $\emptyset$ belong to τ, and both are open sets. The set $\mathcal{B} \subseteq \tau$ is called a base for τ if every open set in τ is a union of members of $\mathcal{B}$ [28]. A set $\sigma \subseteq \tau$ is called a subbase of τ if every open set in $\mathcal{B}$ is a finite intersection of elements of σ. Let $\{M_i;\ i \in I\}$ be a family of the subset of X where if $I = \emptyset$, then $\bigcup_{i\in \mathrm{I}} M_i = \emptyset$ and $\bigcap_{i\in \mathrm{I}} M_i = X$ [32]. Many papers joined graph theory and topology, see [18, 20-27]. In this work, converting the topological graph to a discrete topology by adjacent vertices are studied. A new definition of subbase is introduced, containing all sets of the vertices neighborhoods. The base is extracted from the intersection of all elements of the subbase. After that, the neighborhood (discrete) topology is extracted from the base elements with some examples.

2 DEFINITION AND PROPERTIES OF TOPOLOGICAL GRAPH

In this section, many properties proved by authors in [20] for the discrete topological graph G_τ are given.

Definition 2.1[20] Let X be a non-empty set and τ be a discrete topological graph. A topological graph $\mathrm{G}_\tau = (\mathrm{V}, \mathrm{E})$ is a graph of the vertex set $\mathrm{V}(\mathrm{G}_\tau) = \{\mathrm{A};\ \mathrm{A} \in \tau \text{ and } \mathrm{A} \neq \emptyset, \mathrm{X}\}$ and the edge set defined by $\mathrm{E}(\mathrm{G}_\tau) = \{\mathrm{A\,B};\ \mathrm{A} \subseteq \mathrm{B} \text{ and } \mathrm{A} \neq \mathrm{B}\}$.

Proposition 2.2[20]: Let G_τ be a discrete topological graph on X, where $|\mathrm{X}| = 2$, then $\mathrm{G}_\tau \cong \mathrm{N}_2$.
Corollary 2.3[20]: Let G_τ be a discrete topological graph on X, where $|\mathrm{X}| = 3$. Then, $\mathrm{G}_\tau \cong \mathrm{K}_{3,3}$.

Proposition 2.4[20]: Let G_τ be a discrete topological graph on X, where $|\mathrm{X}| = 4$, then $\mathrm{G}_\tau \cong \mathrm{K}_{4,6,4}$.

Example 2.5[20]: let $|X| = 5$, then τ =

$$\left\{\begin{matrix} \emptyset, X, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{2,3\}, \{2,4\}, \{2,5\}, \{3,4\} \\ \{3,5\}, \{4,5\}, \{1,2,3\}, \{1,2,4\}, \{1,2,5\}, \{1,3,4\}, \{1,3,5\}, \{1,4,5\}, \{2,3,4\}, \{2,3,5\} \\ \{2,4,5\}, \{3,4,5\}, \{1,2,3,4\}, \{1,2,3,5\}, \{1,2,4,5\}, \{1,3,4,5\}, \{2,3,4,5\} \end{matrix}\right\}, \text{ so } =$$

$$\left\{\begin{matrix} \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{2,3\}, \{2,4\}, \{2,5\}, \{3,4\}, \{3,5\}, \\ \{4,5\}, \{1,2,3\}, \{1,2,4\}, \{1,2,5\}, \{1,3,4\}, \{1,3,5\}, \{1,4,5\}, \{2,3,4\}, \{2,3,5\}, \{2,4,5\} \\ \{3,4,5\}, \{1,2,3,4\}, \{1,2,3,5\}, \{1,2,4,5\}, \{1,3,4,5\}, \{2,3,4,5\} \end{matrix}\right\}$$

Proposition 2.6[20]: Let G_τ be a discrete topological graph on a non-empty set X, where $|X| = n$. Then $|G_\tau| = 2^n - 2$.

3 TOPOLOGICAL SPACE GENERATED BY TOPOLOGICAL GRAPH

In this section, converting the topological graph to a discrete topology are studied.

Definition 3.1. Let G_τ be a topological graph where the neighborhoods of vertex v_i for any $v_i \in V(G_\tau)$ defined as $N(v_i) = \{v_j \in V(G_\tau)$: v_j is adjacent with v_i where $i \neq j\}$. Let NS_{G_τ} be a collection of subsets of neighborhoods of V whose union equals V. such that $NS_{G_\tau}(V) = \{N(v_i)\}_{v_i \in V(G_\tau)}$, for all $v_i \in V(G_\tau), i = 1,2, \dots, 2^n - 2$. And NS_{G_τ} forms a subbase. The topology generated by $NS_{G_\tau}(V)$ is defined to be a collection of all unions of the finite intersection of elements of NS_{G_τ}.

Definition 3.2. Let NB_{G_τ} be a basis generated by finite intersection of members of $NS_{G_\tau}(V)$. Where it is defined as follows:

$NB_{G_\tau}(V) = \{A; A \subseteq V, A \text{ is a finite intersection of members of } NS_{G_\tau}\}$

Definition 3.3. Let $N\tau_{G_\tau}$ be a topology on a set V generated by $N\mathcal{B}_{G_\tau}$ and NS_{G_τ} called neighborhood topology of a graph G_τ.

Example 3.4. Let G_τ be a topological graph for $|X| = 2$ then.
Let $V(G_\tau) = \{v_1, v_2\}$, Where: $v_1 = \{1\}$ and $v_2 = \{2\}$
$N(v_1) = \emptyset$, $N(v_2) = \emptyset$, $NS_{G_\tau}(V) = \{\emptyset\}$, and $NB_{G_\tau}(V) = \{\emptyset\}$.
Hence, $N\tau_{G_\tau}$ is not topology on V see Figure 1.

Figure 1. The topological graph for $|X| = 2$.

Example 3.5. Let G_τ be a topological graph for $|X| = 3$. Thus, we extract the neighborhood topology $N\tau_{G_\tau}$ of topological graph G_τ. Let $V(G_\tau) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$. Where $v_1 = \{1\}, v_2 = \{2\}$, $v_3 = \{3\}$, $v_4 = \{1,2\}$, $v_5 = \{1,3\}$, $v_6 = \{2,3\}$, then

$N(v_1) = \{v_4, v_5\}$, $N(v_2) = \{v_4, v_6\}$, $N(v_3) = \{v_5, v_6\}$, $N(v_4) = \{v_1, v_2\}$, $N(v_5) = \{v_1, v_3\}$, and $N(v_6) = \{v_2, v_3\}$.

$NS_{G_\tau}(V) = \{\{v_4, v_5\}, \{v_4, v_6\}, \{v_5, v_6\}, \{v_1, v_2\}, \{v_1, v_3\}, \{v_2, v_3\}\}$,
By taking the intersection of sets of NS_{G_τ} we get the base as:
$NB_{G_\tau}(V) = \{\emptyset, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_6\}, \{v_4, v_5\}, \{v_4, v_6\}, \{v_5, v_6\}, \{v_1, v_2\}, \{v_1, v_3\}, \{v_2, v_3\}\}$.

By taking all unions. The neighborhood topology can be written as follows:

$N\tau_{G_\tau} = \{\emptyset, V\} \cup \{\{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_6\}, \{v_1, v_2\}, \{v_1, v_3\},$
$\{v_1, v_4\}, \{v_1, v_5\}, \{v_1, v_6\}, \{v_2, v_3\}, \{v_2, v_4\}, \{v_2, v_5\}, \{v_2, v_6\}, \{v_3, v_4\},$
$\{v_3, v_5\}, \{v_3, v_6\}, \{v_4, v_5\}, \{v_4, v_6\}, \{v_5, v_6\}, \{v_1, v_2, v_3\}, \{v_1, v_2, v_4\},$
$\{v_1, v_2, v_5\}, \{v_1, v_2, v_6\}, \{v_1, v_3, v_4\}, \{v_1, v_3, v_5\}, \{v_1, v_3, v_6\}, \{v_1, v_4, v_5\},$
$\{v_1, v_4, v_6\}, \{v_1, v_5, v_6\}, \{v_2, v_3, v_4\}, \{v_2, v_3, v_5\}, \{v_2, v_3, v_6\}, \{v_2, v_4, v_5\},$
$\{v_2, v_4, v_6\}, \{v_2, v_5, v_6\}, \{v_3, v_4, v_5\}, \{v_3, v_4, v_6\}, \{v_3, v_5, v_6\}, \{v_4, v_5, v_6\},$
$\{v_1, v_2, v_3, v_4\}, \{v_1, v_2, v_3, v_5\}, \{v_1, v_2, v_3, v_6\}, \{v_1, v_3, v_4, v_5\}, \{v_1, v_3, v_4, v_6\},$
$\{v_1, v_3, v_5, v_6\}, \{v_1, v_4, v_5, v_6\}, \{v_1, v_2, v_4, v_5\}, \{v_1, v_2, v_4, v_6\}, \{v_1, v_2, v_5, v_6\},$
$\{v_2, v_3, v_4, v_5\}, \{v_2, v_3, v_4, v_6\}, \{v_2, v_3, v_5, v_6\}, \{v_2, v_4, v_5, v_6\}, \{v_3, v_4, v_5, v_6\},$
$\{v_1, v_2, v_3, v_4, v_5\}, \{v_1, v_2, v_3, v_4, v_6\}, \{v_1, v_2, v_3, v_5, v_6\}, \{v_1, v_2, v_4, v_5, v_6\},$
$\{v_1, v_3, v_4, v_5, v_6\}, \{v_2, v_3, v_4, v_5, v_6\}\}.$

Hence, $N\tau_{G_\tau}$ is a discrete topology on X. See Figure 2.

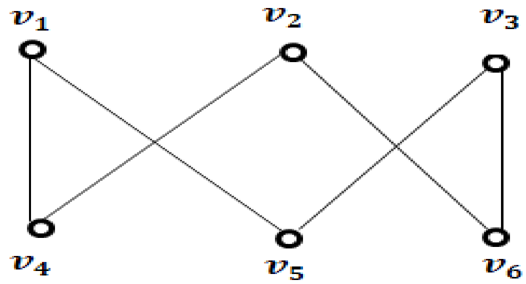

Figure 2. The topological graph for $|X| = 3$.

Example 3.6. Let G_τ be a topological graph for $|X| = 4$. We find neighborhood topology $N\tau_{G_\tau}$ of topological graph G_τ.

Let $V(G_\tau) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}\}$.

Where $v_1 = \{1\}, v_2 = \{2\}, v_3 = \{3\}, v_4 = \{4\}, v_5 = \{1,2\}, v_6 = \{1,3\},$
$v_7 = \{1,4\}, v_8 = \{2,3\}, v_9 = \{2,4\}, v_{10} = \{3,4\}, v_{11} = \{1,2,3\},$
$v_{12} = \{1,2,4\}, v_{13} = \{1,3,4\}, v_{14} = \{2,3,4\},$
$N(v_1) = \{v_5, v_6, v_7, v_{11}, v_{12}, v_{13}\}$, $N(v_2) = \{v_5, v_8, v_9, v_{11}, v_{12}, v_{14}\}$,
$N(v_3) = \{v_6, v_8, v_{10}, v_{11}, v_{13}, v_{14}\}$, $N(v_4) = \{v_7, v_9, v_{10}, v_{12}, v_{13}, v_{14}\}$,
$N(v_5) = \{v_1, v_2, v_{11}, v_{12}\}$, $N(v_6) = \{v_1, v_3, v_{11}, v_{13}\}$,
$N(v_7) = \{v_1, v_4, v_{12}, v_{13}\}$, $N(v_8) = \{v_2, v_3, v_{11}, v_{14}\}$,
$N(v_9) = \{v_2, v_4, v_{12}, v_{14}\}$, $N(v_{10}) = \{v_3, v_4, v_{13}, v_{14}\}$,
$N(v_{11}) = \{v_1, v_2, v_3, v_5, v_6, v_8\}$, $N(v_{12}) = \{v_1, v_2, v_4, v_5, v_7, v_9\}$,
$N(v_{13}) = \{v_1, v_3, v_4, v_6, v_7, v_{10}\}$, $N(v_{14}) = \{v_2, v_3, v_4, v_8, v_9, v_{10}\}$.
$NS_{G_\tau}(V) = \{\{v_5, v_6, v_7, v_{11}, v_{12}, v_{13}\}, \{v_5, v_8, v_9, v_{11}, v_{12}, v_{14}\}$
$\{v_6, v_8, v_{10}, v_{11}, v_{13}, v_{14}\}, \{v_7, v_9, v_{10}, v_{12}, v_{13}, v_{14}\}, \{v_1, v_2, v_{11}, v_{12}\},$
$\{v_1, v_3, v_{11}, v_{13}\}, \{v_1, v_4, v_{12}, v_{13}\}, \{v_2, v_3, v_{11}, v_{14}\}, \{v_2, v_4, v_{12}, v_{14}\},$
$\{v_3, v_4, v_{13}, v_{14}\}, \{v_1, v_2, v_3, v_5, v_6, v_8\}, \{v_1, v_2, v_4, v_5, v_7, v_9\},$
$\{v_1, v_3, v_4, v_6, v_7, v_{10}\}, \{v_2, v_3, v_4, v_8, v_9, v_{10}\}\}.$
$NB_{G_\tau}(V) = \{\emptyset, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_6\}, \{v_7\}, \{v_8\}, \{v_9\}, \{v_{10}\}, \{v_{11}\}$
$, \{v_{12}\}, \{v_{13}\}, \{v_{14}\}, \{v_1, v_2\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_1, v_{11}\}, \{v_1, v_{12}\}, \{v_1, v_{13}\},$
$\{v_2, v_3\}, \{v_2, v_4\}, \{v_2, v_{11}\}, \{v_2, v_{12}\}, \{v_2, v_{14}\}, \{v_3, v_4\}, \{v_3, v_{11}\}, \{v_3 v_{13}\}$
$, \{v_3, v_{14}\}, \{v_4, v_{12}\}, \{v_4, v_{13}\}, \{v_4, v_{14}\}, \{v_5, v_6\}, \{v_5, v_7\}, \{v_5, v_8\}, \{v_5, v_9\},$
$\{v_2, v_3\}, \{v_6, v_7\}, \{v_6, v_8\}, \{v_6, v_{10}\}, \{v_7, v_9\}, \{v_7, v_{10}\}, \{v_8, v_{10}\}, \{v_9, v_{10}\},$
$\{v_{11}, v_{12}\}, \{v_{11}, v_{13}\}, \{v_{11}, v_{14}\}, \{v_{12}, v_{13}\}, \{v_{12}, v_{14}\}, \{v_{13}, v_{14}\},$
$\{v_8, v_{11}, v_{14}\}, \{v_9, v_{12}, v_{14}\}\{v_{10}, v_{13}, v_{14}\}, \{v_1, v_2, v_5\}, \{v_2, v_3, v_8\},$
$\{v_1, v_4, v_7\}, \{v_2, v_4, v_9\}, \{v_3, v_4, v_{10}\}, \{v_5, v_{11}, v_{12}\}, \{v_6, v_{11}, v_{13}\},$

$\{v_7, v_{12}, v_{13}\}, \{v_1, v_2, v_{11}, v_{12}\}, \{v_1, v_3, v_{11}, v_{13}\}, \{v_1, v_4, v_{12}, v_{13}\},$
$\{v_2, v_3, v_{11}, v_{14}\}, \{v_2, v_4, v_{12}, v_{14}\}, \{v_3, v_4, v_{13}, v_{14}\},$
$\{v_5, v_6, v_7, v_{11}, v_{12}, v_{13}\}, \{v_5, v_8, v_9, v_{11}, v_{12}, v_{14}\},$
$\{v_6, v_8, v_{10}, v_{11}, v_{13}, v_{14}\}, \{v_7, v_9, v_{10}, v_{12}, v_{13}, v_{14}\}, \{v_1, v_2, v_3, v_5, v_6, v_8\},$
$\{v_1, v_2, v_4, v_5, v_7, v_9\}, \{v_1, v_3, v_4, v_6, v_7, v_{10}\}, \{v_2, v_3, v_4, v_8, v_9, v_{10}\}\}.$

By similar technique to example 3.5, we find the neighborhood topology $N\tau_{G_\tau}$. Such that the number of all sets of $N\tau_{G_\tau}(V)$ is 2^{14}.Therefore, $N\tau_{G_\tau}$ is a discrete topology on X. See Figure 3.

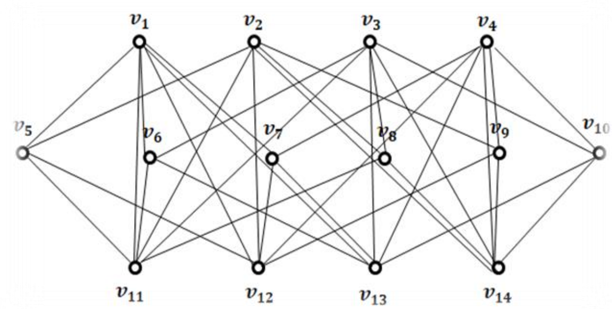

Figure 3. The topological graph for $|X| = 4$.

Example 3.7. Let G_τ be a topological graph for $|X| = 5$. To find the neighborhood topology $N\tau_{G_\tau}$ of topological graph G_τ.

Let $V(G_\tau) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}v_{15},$
$v_{16}, v_{17}, v_{18}, v_{19}, v_{20}, v_{21}, v_{22}, v_{23}, v_{24}, v_{25}, v_{26}, v_{27}, v_{28}, v_{29}, v_{30}\}$.

Where $v_1 = \{1\}, v_2 = \{2\}, v_3 = \{3\}, v_4 = \{4\}, v_5 = \{5\}, v_6 = \{1,2\}, v_7 = \{1,3\}, v_8 = \{1,4\}, v_9 = \{1,5\}, v_{10} = \{2,3\}, v_{11} = \{2,4\}, v_{12} = \{2,5\}, v_{13} = \{3,4\}, v_{14} = \{3,4\}, v_{15} = \{4,5\}, v_{16} = \{1,2,3\}, v_{17} = \{1,2,4\}, v_{18} = \{1,2,5\}, v_{19} = \{1,3,4\}, v_{20} = \{1,3,5\}, v_{21} = \{1,4,5\}, v_{22} = \{2,3,4\}, v_{23} = \{2,3,5\}, v_{24} = \{2,4,5\}, v_{25} = \{3,4,5\}, v_{26} = \{1,2,3,4\}, v_{27} = \{1,2,3,5\}, v_{28} = \{1,2,4,5\}, v_{29} = \{1,3,4,5\}, v_{30} = \{2,3,4,5\}$, then

$$V(G_\tau) = \begin{Bmatrix} v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, \\ v_{17}, v_{18}, v_{19}, v_{20}, v_{21}, v_{22}, v_{23}, v_{24}, v_{25}, v_{26}, v_{27}, v_{28}, v_{29}, v_{30} \end{Bmatrix},$$

$N(v_1) = \{v_6, v_7, v_8, v_9, v_{16}, v_{17}, v_{18}, v_{19}, v_{20}, v_{21}, v_{26}, v_{27}, v_{28}, v_{29}\},$
$N(v_2) = \{v_6, v_{10}, v_{11}, v_{12}, v_{16}, v_{17}, v_{18}, v_{22}, v_{23}, v_{24}, v_{26}, v_{27}, v_{28}, v_{30}\},$
$N(v_3) = \{v_7, v_{10}, v_{13}, v_{14}, v_{16}, v_{19}, v_{20}, v_{22}, v_{23}, v_{25}, v_{26}, v_{27}, v_{29}, v_{30}\},$
$N(v_4) = \{v_8, v_{11}, v_{13}, v_{15}, v_{17}, v_{19}, v_{21}, v_{22}, v_{24}, v_{25}, v_{26}, v_{28}, v_{29}, v_{30}\},$
$N(v_5) = \{v_9, v_{12}, v_{14}, v_{15}, v_{18}, v_{20}, v_{21}, v_{23}, v_{24}, v_{25}, v_{27}, v_{28}, v_{29}, v_{30}\},$
$N(v_6) = \{v_1, v_2, v_{16}, v_{17}, v_{18}, v_{26}, v_{27}, v_{28}\},$
$N(v_7) = \{v_1, v_3, v_{16}, v_{19}, v_{20}, v_{26}, v_{27}, v_{29}\},$
$N(v_8) = \{v_1, v_4, v_{17}, v_{19}, v_{21}, v_{26}, v_{28}, v_{29}\},$
$N(v_9) = \{v_1, v_5, v_{18}, v_{20}, v_{21}, v_{27}, v_{28}, v_{29}\},$
$N(v_{10}) = \{v_2, v_3, v_{16}, v_{22}, v_{23}, v_{26}, v_{27}, v_{30}\},$

In the similar way above we find $N(u_i), i = 11,12, \ldots ,30$.

Such that $NS_{G_\tau}(V) = \{N(u_i)\}_{u_i \in V(G_\tau)}$, for all $i = 1, 2, 3, \ldots, 30$. We find NB_{G_τ} and $N\tau_{G_\tau}$ by the same technique of Example 3.5. So, we get NB_{G_τ} which has all sets of singleton u_i where $\{u_i\} \in NB_{G_\tau}$, for all $i = 1, 2, 3, \ldots, 30$, and $V \in NB_{G_\tau}$. Since $N\tau_{G_\tau}$ is the union of all sets of NB_{G_τ}. Then, the number of all sets of $N\tau_{G_\tau}$ is 2^{30} and it is discrete topology. See Figure 4. Also, if $n > 5$ the topological space generated by topological graph is discrete topology.

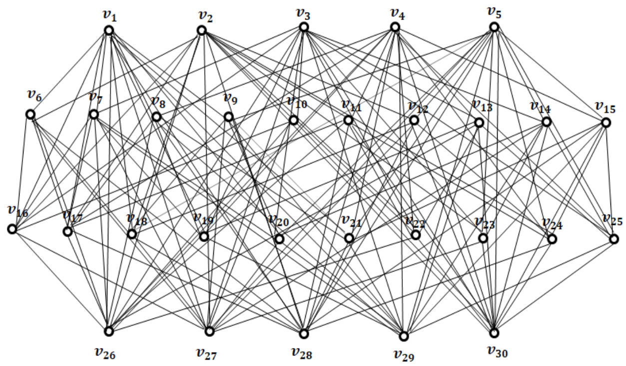

Figure 4. The topological graph for $|X| = 5$.

4 OPEN PROBLEMS

1- Converting the topological graph to the discrete topology by other ways, by the adjacent or non-adjacent edges or vertices.

2- Apply many types of domination parameters on the topological graph such as: Pitchfork domination, co-even domination, and co-independent domination.

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