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Long–term optimal vaccination strategies for controlling a pandemic: a computational modelling approach using COVID-19 data in Sri Lanka

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Abstract

Infectious diseases with pandemic potential such as COVID-19 and influenza are having a devastating impact on global health and socioeconomic activities. A major and a primary strategy to control such pandemics that is used globally is the process of vaccination. However, the primary vaccination becomes less effective after several months for most infectious diseases. As a result, regular booster doses are required for several years to improve the immune response. The government policy of vaccine administration and public response to the vaccination procedure varies from country to country, depending on the incidence of the infected population, vaccine availability and types, age–related health risks, and prioritization by population density or mobility patterns. In this study, we propose an optimal control model for regional vaccination allocation based on age distribution and mobility patterns, with the goal of achieving a trade–off between vaccination costs and the economic burden caused by the infected population. We use a compartmental model to capture the transmission dynamics, write an optimal control equation and examine the validity using computer simulations generated for COVID-19 related data in Sri Lanka.

Keywords: Optimal control, Pandemic, Vaccination, Epidemic Models
AMS Mathematical Subject Classification [2010]: 49J15, 93A30

1 Introduction

Pandemic diseases have become an untenable threat to any country around the world [1]. With the process of globalization and sophisticated methods of travelling such a disease could spread all around the world within a few days. Apart from that, it could hinder the economic and social progress of a country [2]. Responding

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to pandemic diseases presents a challenge since vaccines for them may not exist or be ineffective. As a result, producing large quantities of any new pandemic disease vaccine takes several months. To control the spread of the disease until a safe, effective vaccine is available, countries implement a variety of strategies such as strict and mild lockdowns, mandatory hospitalization, and quarantine measures for close contacts. [3, 4]. After developing vaccines against the disease, vaccination becomes the most effective control measure and the countries return to normalcy [5]. For example, during the COVID-19 outbreak a number of vaccines, including Oxford–AstraZeneca, Sinopharm BIBP, Pfizer–BioNTech and Sputnik V, have been developed and approved for emergency use by regulatory bodies in various countries including Sri Lanka. However, based on the limitation of supplies, the usage of vaccines and administering are different from country to country.

Countries with limited resources like Sri Lanka have to use combinations of vaccines in different regions to eradicate the disease during the epidemic based on availability. For instance, during the COVID-19 pandemic, Oxford–AstraZeneca, and Sinopharm are deployed in Western, Southern and North Western provinces. Even though the primary series of vaccination reduces hospitalization and mortality rates, immune protection declines over time, especially for older adults [6]. Moreover, constantly mutating RNA type virus emergence new variants with high transmissibility. Consequently, it has spread rapidly in populations with high levels of vaccine and naturally acquired infection-induced immunity. For the diseases like influenza and COVID-19, there may be a need for additional doses as an extension of the primary series for several target groups in order to enhance the immune response against the disease as the standard primary series is deemed insufficient. Therefore, it is vital to identify long–term optimal vaccination strategies for mitigating the disease as well as reducing the health care cost during a pandemic.

Several vaccination strategies have been proposed to vaccinate healthcare and aged care workers and other front-line responders who are at high risk of disease transmission, as well as socio-demographic groups who are at significantly higher risk of severe diseases, such as older adults or people with high-risk chronic health conditions [7]. On the other hand, prioritizing young people may have a greater impact on transmission because vaccines are more effective in younger people and transmission is highest in young adults because of their mobility [8]. In addition, the age structure of the regional population and inter-regional mobility patterns also contributed to the spread of the disease. In this paper, we propose an optimal control model for regional booster coverage, aiming at minimizing the trade-off between vaccination costs and the ecological burden caused by infected populations.

2 Model Development

The study conducted by Roda et al. [9] has demonstrated that the SIR model performs more adequately than the SEIR model in representing the information related to confirmed cases of disease modeling using COVID-19 data. Hence we adopt SIR (Susceptible–Infected–Recovered) model to identify the disease transmission. In the SIR model, the population (N) is divided into three disjoint classes, susceptible (S), infected (I) and recovered (R). Once the susceptible individuals become infected with the disease, they move to the infected class and once they recovered from the disease they move to the Recovered (R) class. The transmission rate β denotes the rate of virus transmission from an infected human to a susceptible human and the recovery rate, γ, is a measure of how fast the infected individuals recover. Moreover, we suppose that all newborns are susceptible and individuals are removed by death from each class at equal per–capita death rate μ. To

reduce the computational cost, we normalize the model by dividing the population in each compartment by N. The normalized model is described by the following differential equations:

$$\frac{dS}{dt} = \mu(1 - S) - \beta SI, \tag{1}$$

$$\frac{dI}{dt} = \beta SI - (\gamma + \mu)I, \tag{2}$$

$$\frac{dR}{dt} = \gamma I - \mu R. \tag{3}$$

Since the allocation of vaccine amount differs from region to region based on the age groups, we extend the model in (1) by including three vaccinated age groups for each region i. Compartment V_{1i} denote fully vaccinated people above age 60 years in ith region and V_{2i} denote the fully vaccinated people between age 30 and 60 years in ith region. Compartment V_{3i} denote fully vaccinated people below age 30 years in ith region with primary series. Further, fully vaccinated persons in ith region move to booster compartment (B_i) once they get the booster dose. Based on the immunity and the transmission probability, we can divide the infected population in ith region into three compartments: infected individuals with no vaccination (I_{1i}), fully vaccinated infected individuals (I_{2i}) and fully vaccinated infected individuals who also had the booster (I_{3i}). Removed (R_i) compartment represent the recovered and death individuals in ith region.

Let β_{kij} denote the transmission rate related to kth age group between ith and jth provinces. Here, γ_{ki}, d_{ki} and u_{ki} denote,recovery rate, death rate and vaccination proportion with booster vaccine of kth age group in ith region. Mean efficacy of the vaccine-related to kth age group in ith province is ϵ_{ki} and the immigration infected rate in ith province is λ_i. The following set of modified differential equations represents the model.

$$\frac{dS_i}{dt} = \alpha - \sum_{j=1}^{n} w_{ij} S_i \left(\beta_{1ij} I_{1j} + \beta_{2ij} I_{2j} + \beta_{3ij} I_{3j}\right) - (\mu + a_1 + a_2 + a_3) S_i, \tag{4}$$

$$\frac{dV_{1i}}{dt} = a_1 S_i - \sum_{j=1}^{n} w_{ij} V_{1i}(1-\epsilon_{1i}) \left(\beta_{1ij} I_{1j} + \beta_{2ij} I_{2j} + \beta_{3ij} I_{3j}\right) - (\mu + u_{1i}) V_{1i}, \tag{5}$$

$$\frac{dV_{2i}}{dt} = a_2 S_i - \sum_{j=1}^{n} w_{ij} V_{2i}(1-\epsilon_{2i}) \left(\beta_{1ij} I_{1j} + \beta_{2ij} I_{2j} + \beta_{3ij} I_{3j}\right) - (\mu + u_{2i}) V_{2i}, \tag{6}$$

$$\frac{dV_{3i}}{dt} = a_3 S_i - \sum_{j=1}^{n} w_{ij} V_{3i}(1-\epsilon_{3i}) \left(\beta_{1ij} I_{1j} + \beta_{2ij} I_{2j} + \beta_{3ij} I_{3j}\right) - (\mu + u_{3i}) V_{3i}, \tag{7}$$

$$\frac{dB_i}{dt} = \sum_{k=1}^{3} u_{ki} V_{ki} - \sum_{j=1}^{n} w_{ij} B_i(1-\epsilon_{4i}) \left(\beta_{1ij} I_{1j} + \beta_{2ij} I_{2j} + \beta_{3ij} I_{3j}\right) - \mu B_i, \tag{8}$$

$$\frac{dI_{1i}}{dt} = \sum_{j=1}^{n} w_{ij} S_i \left(\beta_{1ij} I_{1j} + \beta_{2ij} I_{2j} + \beta_{3ij} I_{3j}\right) - (\gamma_{1i} + d_{1i} + \mu) I_{1i}, \tag{9}$$

$$\frac{dI_{2i}}{dt} = \lambda_i + \sum_{j=1}^{n} w_{ij} \Big(\sum_{k=1}^{3} V_{ki}(1-\epsilon_{ki})\Big) \left(\beta_{1ij} I_{1j} + \beta_{2ij} I_{2j} + \beta_{3ij} I_{3j}\right) - (\gamma_{2i} + d_{2i} + \mu) I_{2i}, \tag{10}$$

$$\frac{dI_{3i}}{dt} = \sum_{j=1}^{n} w_{ij} B_i(1-\epsilon_{4i}) \left(\beta_{1ij} I_{1j} + \beta_{2ij} I_{2j} + \beta_{3ij} I_{3j}\right) - (\gamma_{3i} + d_{3i} + \mu) I_{3i}, \tag{11}$$

$$\frac{dR_i}{dt} = (\gamma_{1i} + d_{1i}) I_{1i} + (\gamma_{2i} + d_{2i}) I_{2i} + (\gamma_{3i} + d_{3i}) I_{3i} - \mu R_i. \tag{12}$$

Notice that the variables u_{ki} depend on the availability of the vaccine and the vaccination effort. Therefore, we have specified an upper bound for each variable. The control model, which involves the number of infected individuals, the cost of vaccination and the socio-economic cost are minimized subject to the differential equations (4) to (12). Our objective functional is defined as,

$$\text{Min} \quad J = \int_0^{t_f} \sum_{i=1}^{n} \left((A + C) \left(I_{1i} + I_{2i} + I_{3i}\right) + D \left(u_{1i}^2 + u_{2i}^2 + u_{3i}^2\right) \right) dt. \tag{13}$$

where t_f is the final time point for the considered time period. A, C and D are the cost per treatment of the infected individual, the cost of vaccination and the socio-economic cost associated with infected individuals. In this study, we use quadratic functions for measuring the vaccination control cost.

3 Computer simulation

For our implementation, we obtained the reported Covid-19 data from the Department of Health, Sri Lanka The country presently has vaccinated over 90 percent of those above age 20 with at least the first dose of a COVID-19 vaccine. First, we observed the dynamical behaviour of the infected population in each province with 5% daily vaccination in all ages. Population data were obtained by the report of Census of Population and Housing 2012 issued by the Department of Census and Population, Sri Lanka and the mobility data were obtained by the nation–wide public transport statistics in 2014 available from the National Transport Commission, Sri Lanka. To solve the system of differential equations numerically, we used *ODE45* solver in MATLAB which applies the variable step Runge–Kutta method.

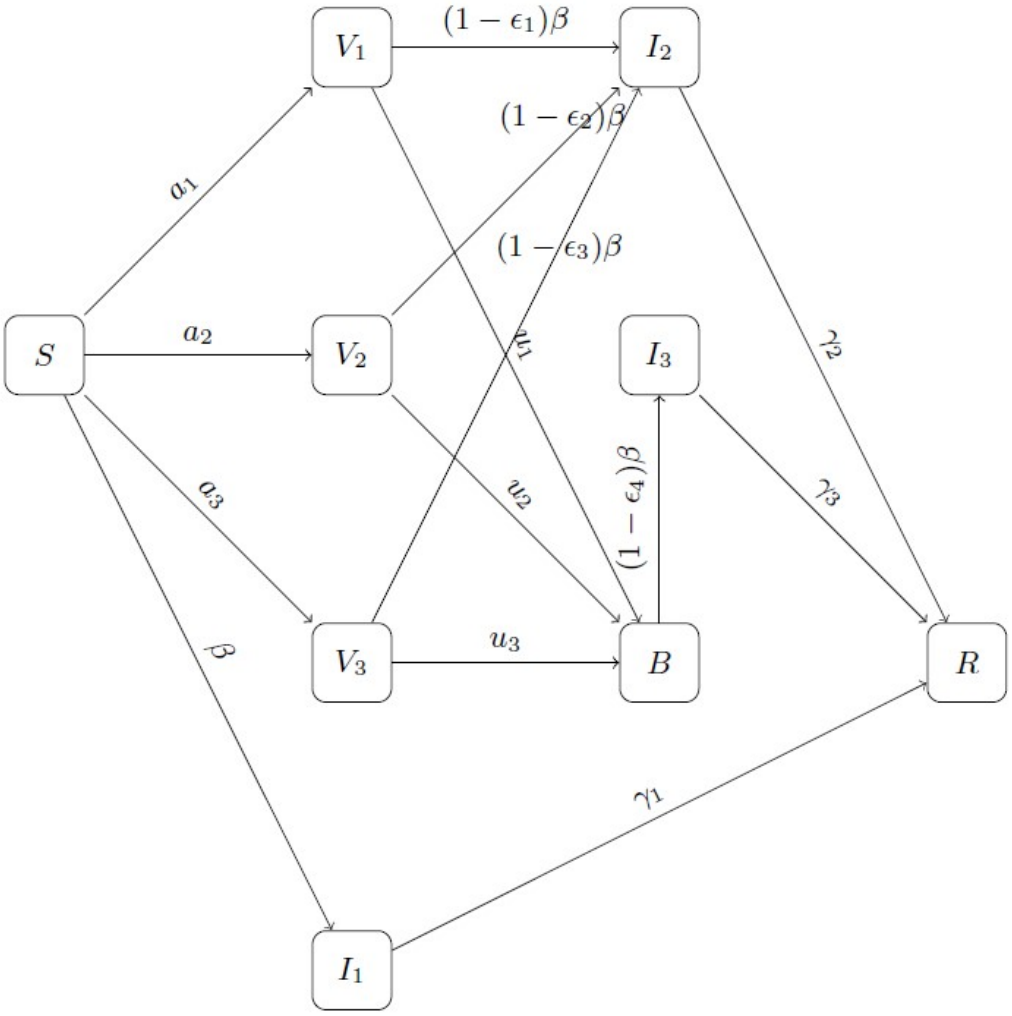

Figure 1: Schematic of an infectious disease transmission model with booster compartment for one region

From Figure 2, it can be observed that at the beginning the infected population increased slightly and after 80 days the infected population increased dramatically for each province under the fixed vaccination proportion. However, daily fixed rate vaccination in all ages in each province is not practical, specially in the countries like Sri Lanka with limited resources. Therefore, we need to identify the age–specified optimal vaccination strategies for each province.

For that purpose, we experimented with three different real–world scenarios by changing the provincial vaccination strategies and inter-provincial mobility in Sri Lanka. The first two scenarios were designed to examine the behavior of optimal control strategies with different provincial vaccination proportions. The last scenario was designed to understand the connectivity between provincial optimal control strategies with mobility.

Scenario 1: No specific constraint

In this scenario, we supposed vaccines are available to everyone. That is the upper limit for the control in each age group in each province is 1. Figure 3 illustrates the optimal vaccination for scenario 1 and Figure

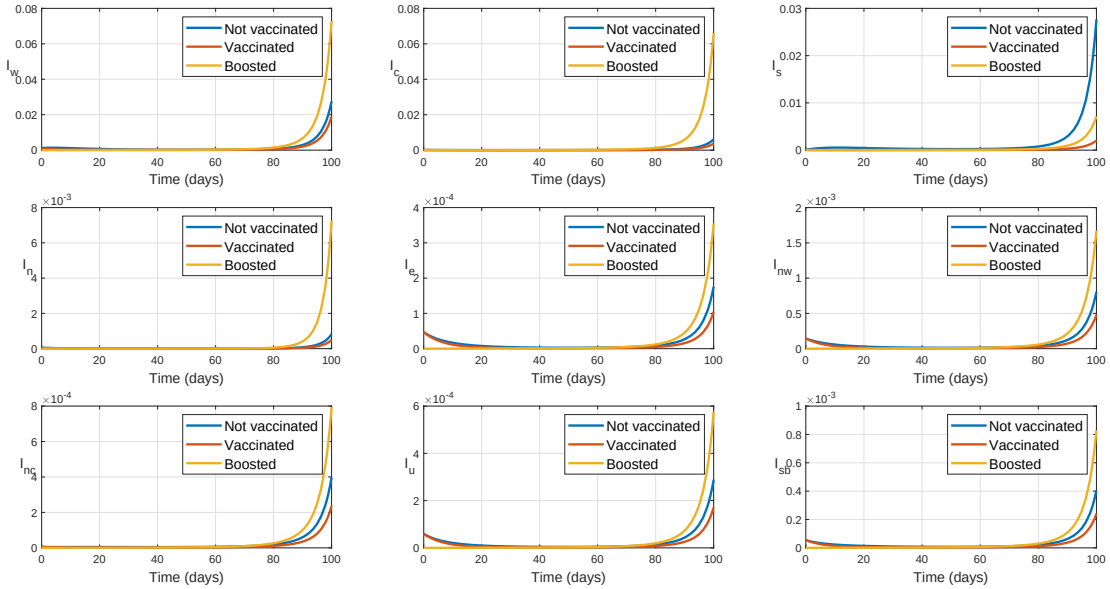

Figure 2: Infected population in (a) Western Province (b) Central Province (c) Southern Province (d) Northern Province (e) Eastern Province (f) North Western Province (g) North Central Province (h) Uva Province and (i) Sabaragamuwa Province without optimal control strategies

4 shows the dynamical behavior of the infected population in each age group change with the optimal vaccination.

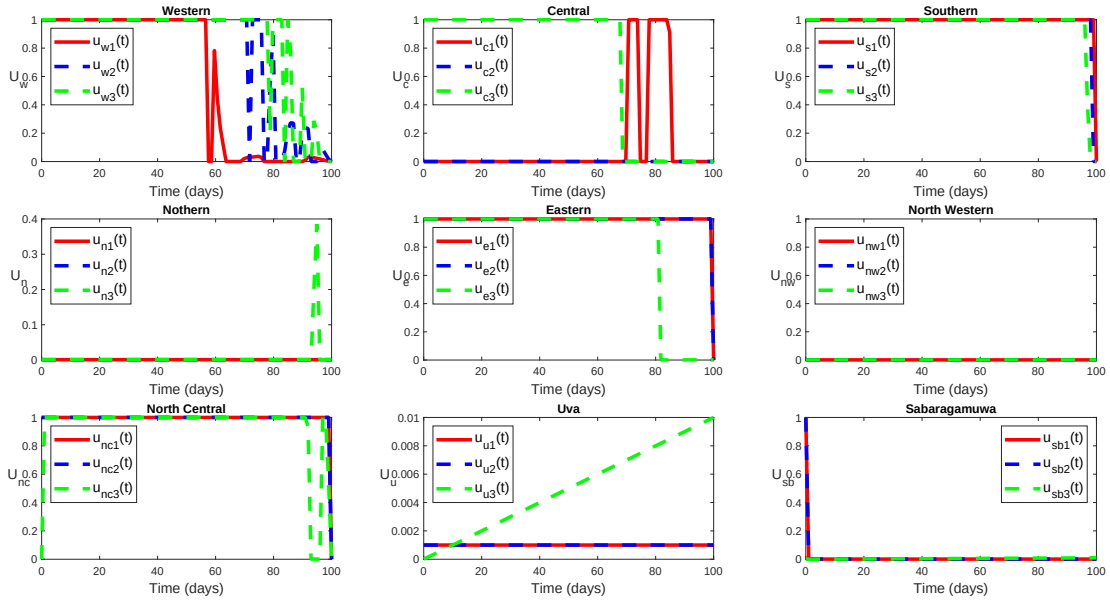

Figure 3: Optimal vaccination strategies for (a) Western Province (b) Central Province (c) Southern Province (d) Northern Province (e) Eastern Province (f) North Western Province (g) North Central Province (h) Uva Province and (i) Sabaragamuwa Province for scenario 1. Here U_{i1}, U_{i2}, U_{i3} represent optimal vaccination strategies for people above age 60, between age 30 and 60 and below age 30 in ith province respectively

The results in Figure 3 indicate that even though the vaccines are available for everyone, the vaccination

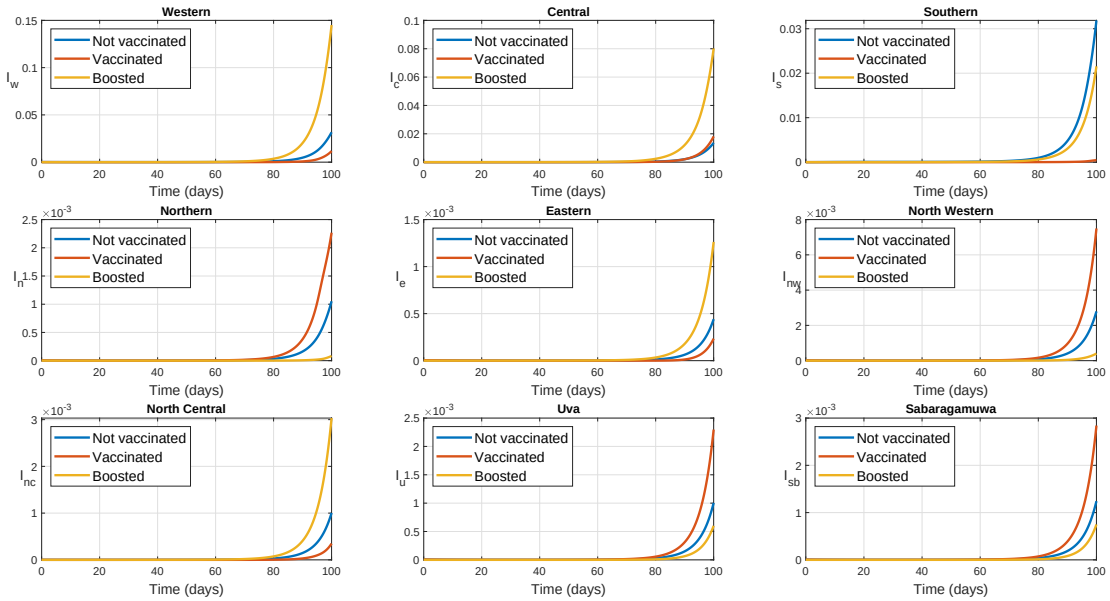

Figure 4: Infected population in (a) Western Province (b) Central Province (c) Southern Province (d) Northern Province (e) Eastern Province (f) North Western Province (g) North Central Province (h) Uva Province and (i) Sabaragamuwa Province for scenario 1

strategies should differ from province to province based on age groups. For example, vaccination programs should start with all age categories in Western, Southern and Eastern provinces. According to the simulation results, at the beginning of the period, people below 30 years should be vaccinated more than adults and after 70 days the vaccination plan should be changed to adults over age 60 in Central province. Moreover, in the Uva province vaccine proportion of people below age 30 should be increased constantly with time.

Scenario 2: Higher vaccine coverage in urban areas

This scenario is designed to identify the optimal strategies when we allocate a high amount of vaccine to the most urbanized provinces in the country. Here we suppose that the upper limit for the control in the Western province is 1 for each age category and the upper limit for the control in all the other provinces is reduced by 20%.

Figure 5 displays the optimal vaccination for scenario 2 and Figure 6 describes the dynamical behaviour of the infected population in each age group change with the optimal vaccination.

Figure 5 shows that the vaccination strategies and proportions for the second scenario differ from those for the first in some provinces. For example, under the second scenario, the vaccine proportion of people under the age of 30 in both Uva and Sabaragamuwa provinces should be gradually increased over time. However, in this scenario too, vaccination programs should begin with all age groups in the Western, Southern, and Eastern provinces with the maximum capacity plus the optimal strategy in Central Province favors the age group under 30.

Scenario 3: Optimal strategies with mobility

The final scenario is designed to examine the optimal strategies for mobility. In this scenario, inter-provincial mobility increased by 25% for each province. This type of scenario occurs during the school vacations, new year and Christmas festival seasons.

Figure 7 shows the optimal vaccination for scenario 3 and Figure 8 illustrates the dynamical behavior

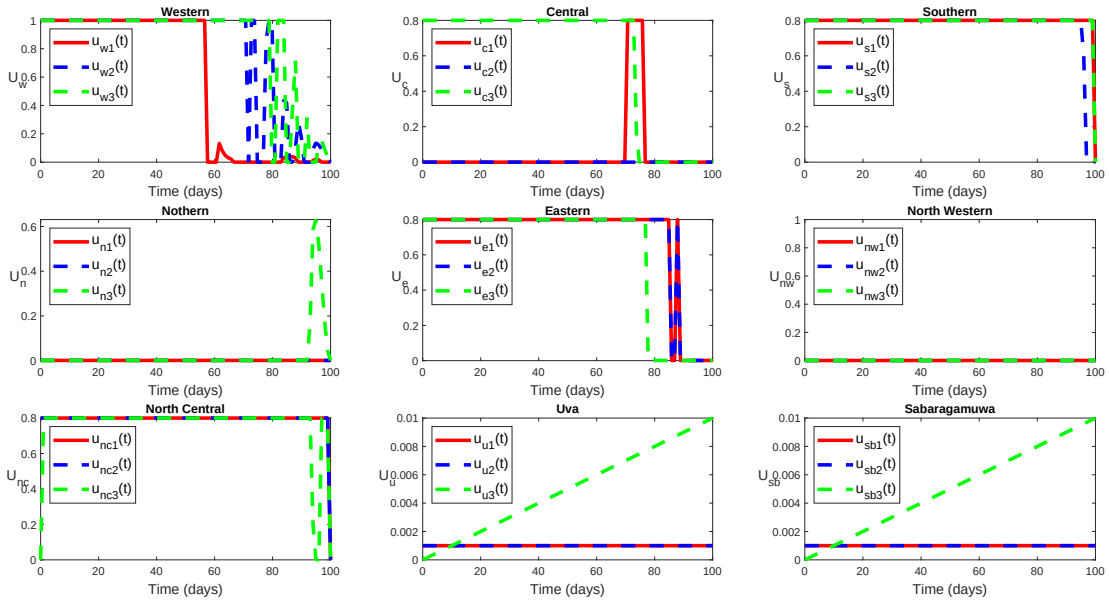

Figure 5: Optimal vaccination strategies for (a) Western Province (b) Central Province (c) Southern Province (d) Northern Province (e) Eastern Province (f) North Western Province (g) North Central Province (h) Uva Province and (i) Sabaragamuwa Province for scenario 2. Here U_{i1}, U_{i2}, U_{i3} represent optimal vaccination strategies for people above age 60, between age 30 and 60 and below age 30 in ith province respectively

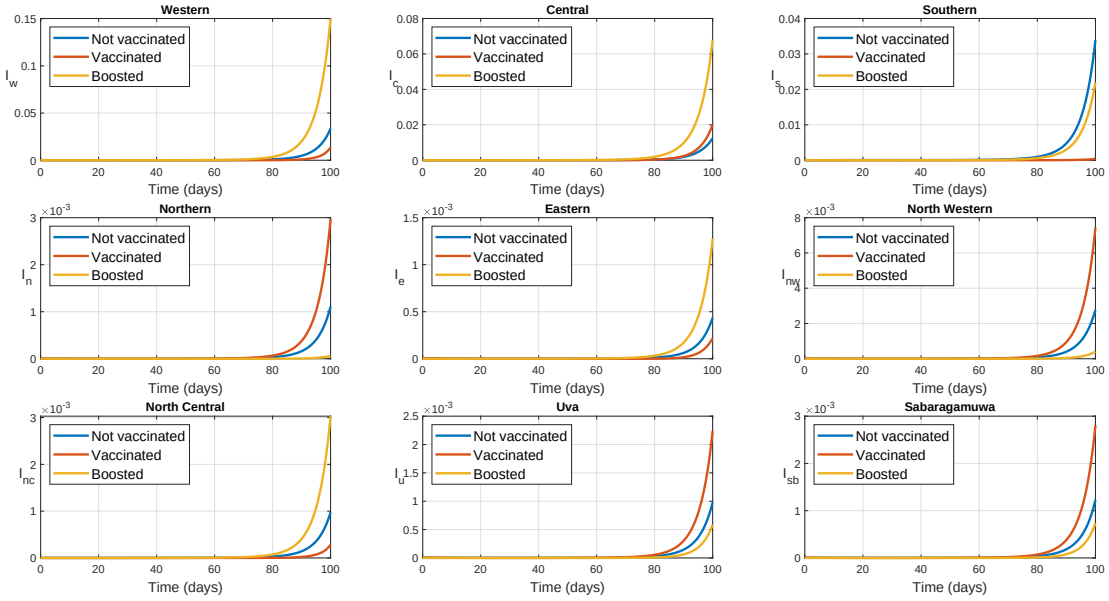

Figure 6: Infected population in (a) Western Province (b) Central Province (c) Southern Province (d) Northern Province (e) Eastern Province (f) North Western Province (g) North Central Province (h) Uva Province and (i) Sabaragamuwa Province for scenario 2

of the infected population in each age group change with the optimal vaccination. From Figure 7, it can be observed that the optimal vaccination strategies change with the changes in the mobility pattern. For instance, scenario three's optimal vaccination strategy for the central province is totally different from

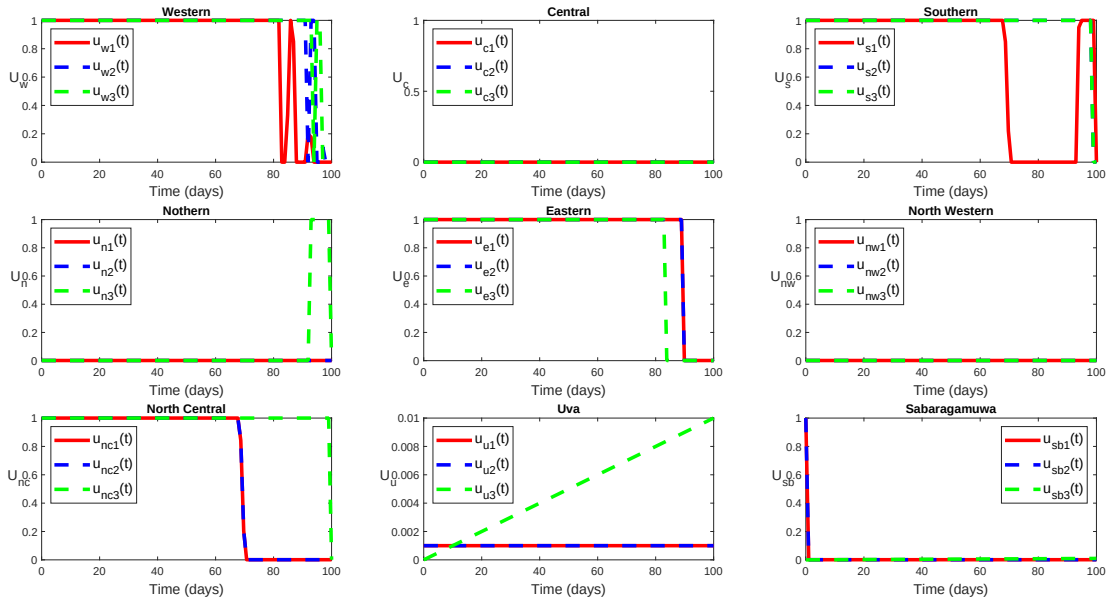

Figure 7: Optimal vaccination strategies for (a) Western Province (b) Central Province (c) Southern Province (d) Northern Province (e) Eastern Province (f) North Western Province (g) North Central Province (h) Uva Province and (i) Sabaragamuwa Province for scenario 3. Here U_{i1}, U_{i2}, U_{i3} represent optimal vaccination strategies for people above age 60, between age 30 and 60 and below age 30 in ith province respectively

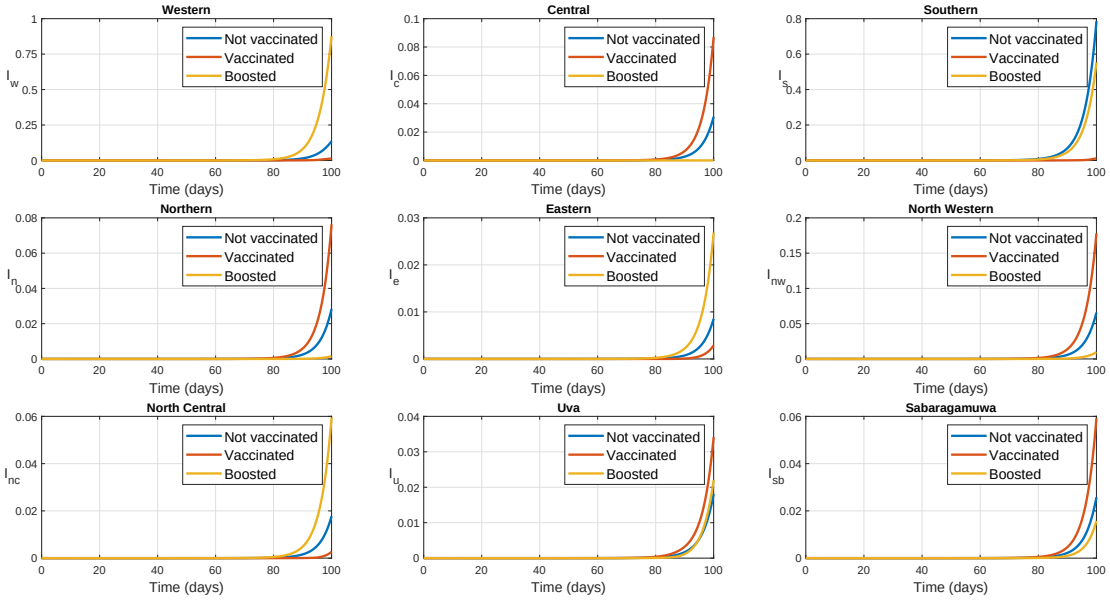

Figure 8: Infected population in (a) Western Province (b) Central Province (c) Southern Province (d) Northern Province (e) Eastern Province (f) North Western Province (g) North Central Province (h) Uva Province and (i) Sabaragamuwa Province for scenario 3

scenario one.

Moreover, the time period for vaccination and the infected population (see Figure 4, 6 and 8) differs from province to province depending on the availability of vaccines, the density of the population and the

mobility patterns. Therefore, to design vaccination strategies during a pandemic, decision-makers have to consider the availability of resources, age distribution as well as the seasonal mobility pattern of the country.

4 Conclusion

Considering the age distribution and mobility patterns, optimal control strategies can be derived for regional allocation of vaccination to minimize the trade-off between vaccination costs and economic burden during an infectious disease pandemic. In this study, we selected three scenarios to align with the availability of vaccines and priority conditions. The proposed model can be modified by including different scenarios. Then, the proposed model was tested by changing the boundary conditions to examine the validity of the model. The results reflected that the optimal vaccination strategies depend not only on the infected population but also on the availability of resources, population density and human behavior. The proposed model can be modified by different scenarios. In a more realistic setting, the number of persons who received a particular vaccine in each province and the allocation of each vaccine type for each region should be added to the model. However, this model can be implemented for any infectious disease pandemic with different controlling strategies.

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