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A New Secant Method for Minima One Variable Problems

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ABSTRACT

In single variable situations, the traditional Newton's approaches are the most popular. The new secant approach is based on the 2nd order Taylor expansion, which eliminates the necessity to compute the second derivative. Numerical tests have shown that the New secant approach is both numerical and effective when compared to the existing Newton method (N-method).

KEYWORDS: Secant Method; Minimization Method; Test Problems.

1 INTRODUCTION

Single-variable function minimizations are a type of minimization method that has just one variable. The function is assumed to be of the form $\mathfrak{F} = \mathfrak{F}(\chi)$, which is similar to determining the nonlinear equation’s root in the following explanation of Single-variable function minimizations:

$$\mathfrak{F}'(\chi^*) = 0 \tag{1}$$

since [9] states that this is a sufficient condition for the extremum of $\mathfrak{F}(\chi)$.

In reality, one of the easiest approaches for reducing functions of a single variable is to apply Newton's method to find the objective function’s minimum, which is the most appealing in terms of convergence, as shown in [11]. The following is an updated version of Newton's method for minimizing a function:

$$\chi_{k+1} = \chi_k - \frac{\mathfrak{F}'(\chi_k)}{\mathfrak{F}''(\chi_k)} \tag{2}$$

This approach is characterized as a second order method because it not only employs second derivative information about the function f, but it also has a quadratic rate of convergence to the minimum. The method's downsides include that it converges slowly when the second derivative converges to zero, and in some practical situations, obtaining this derivative analytically may not be possible or may be difficult. For more details, see [12].

Using Secant techniques is a good idea since the bulk of the methods used to discover the minimal value of Single functions do not require the compute matrix and are based on Newton's approach to determine the stationary point of a function, where the derivative is zero, as shown in [3-4]. In Newton's approach, we may get the following results by using a forward-difference approximation for the second derivative:

$$\chi_{k+1} = \chi_k - \frac{\mathfrak{F}'(\chi_k)(\chi_k - \chi_{k-1})}{\mathfrak{F}'(\chi_k) - \mathfrak{F}'(\chi_{k-1})} \tag{3}$$

This is referred to as the secant technique. It is typically significantly faster than Newton's approach. The second derivative of the function must be higher than zero for a successful convergence to the minimum, as shown in [2].

In other papers, this notion was presented and researched as a secant approach; see [1,5-8, and 13] for additional information. There is no matrix computing involved in the examined secant approaches.

To create more resilient method and examine their key aspects for single variable problems, an efficient approximation optimum secant approach is being developed. The numerical findings demonstrated the newton method's superiority.

2 DERIVATION OF THE NEW SECANT METHOD

Using 2^{nd} order Taylor series, it is possible to improve the secant approach effectively. Around the point χ_k, we have [10], the 2^{nd} order Taylor's series:

$$\mathfrak{F}(\chi) \cong \mathfrak{F}(\chi_k) + \mathfrak{F}'(\chi_k)(\chi - \chi_k) + \frac{1}{2}\mathfrak{F}''(\chi_k)(\chi - \chi_k)^2 \tag{4}$$

To get the minimum, equate the derivative by zero, with the result:

$$\mathfrak{F}'(\chi) \cong \mathfrak{F}'(\chi_k) + \mathfrak{F}''(\chi_k)(\chi - \chi_k) = 0 \tag{5}$$

We derived the following from Eqs. (5) and (4):

$$\mathfrak{F}''(\chi_k)(\chi - \chi_k)^2 \cong \mathfrak{F}(\chi_k) - \mathfrak{F}(\chi) - \frac{1}{2}\mathfrak{F}'(\chi_k)(\chi - \chi_k) \tag{6}$$

We arrive to the following estimate:

$$\mathfrak{F}''(\chi_k) = \frac{(2/3)\left(\mathfrak{F}(\chi_k) - \mathfrak{F}(\chi)\right) - (2/3)\mathfrak{F}'(\chi_k)(\chi - \chi_k)}{(\chi - \chi_k)^2} \tag{7}$$

Substituting χ_{k-1} in to χ in above equation and can be rewritten as:

$$\mathfrak{F}''(\chi_k) = \frac{(2/3)\left(\mathfrak{F}(\chi_k) - \mathfrak{F}(\chi_{k-1})\right) - (2/3)\mathfrak{F}'(\chi_k)(\chi_{k-1} - \chi_k)}{(\chi_{k-1} - \chi_k)^2} \tag{8}$$

Putting (8) in to (1), we get:

$$\chi_{k+1} = \chi_k - \frac{\mathfrak{F}'(\chi_k)(\chi_{k-1} - \chi_k)^2}{(2/3)\left(\mathfrak{F}(\chi_k) - \mathfrak{F}(\chi_{k-1})\right) - (2/3)\mathfrak{F}'(\chi_k)(\chi_{k-1} - \chi_k)} \tag{9}$$

It is a new iterative method (BBR-method). As a result, we arrive to the following algorithm:

Stage 1. Put ε, χ_0 and the function $\mathfrak{F}(\chi_0)$, set $k = 0$.
Stage 2. Let $k = k + 1$.
Stage 3. Assume $\chi_k = \chi_{k-1} - \frac{\mathfrak{F}'(\chi_{k-1})}{\mathfrak{F}''(\chi_{k-1})}$.
Stage 4. Compute $\chi_{k+1} = \chi_k - \frac{\mathfrak{F}'(\chi_k)(\chi_{k-1} - \chi_k)^2}{(2/3)\left(\mathfrak{F}(\chi_k) - \mathfrak{F}(\chi_{k-1})\right) - (2/3)\mathfrak{F}'(\chi_k)(\chi_{k-1} - \chi_k)}$.
Stage 5. If $|\chi_{k+1} - \chi_k| \leq \varepsilon$, then stop.

3 ANALYSIS OF CONVERGENCE

Optimization algorithms provide a series of approximate answers that we hope will eventually converge to the correct solution. It's feasible to establish the following truth using the BBR-secant approach.

Theorem 3.1

Let $\mathfrak{F}: I \rightarrow R$ be a sufficiently smooth function where I is an open interval and let $\chi^* \in I$ be a zero of $\mathfrak{F}$. If χ_0 is close enough to χ^*, then the BBR-iterative method has 2^{nd} order convergence.

Proof:

Given a new iterative schema as:

$$\chi_{k+1} = \chi_k - \frac{\mathfrak{F}'(\chi_k)(\chi_{k-1} - \chi_k)^2}{(2/3)\big(\mathfrak{F}(\chi_k) - \mathfrak{F}(\chi_{k-1})\big) - (2/3)\mathfrak{F}'(\chi_k)(\chi_{k-1} - \chi_k)} \quad (10)$$

Let e_{k+1} and e_k are errors k^{th} and $(k+1)^{th}$ respectively, then the new iterative schema, we have:

$$e_{k+1} = e_k - \frac{\mathfrak{F}'(\chi_k)(e_{k-1} - e_k)^2}{(2/3)\big(\mathfrak{F}(\chi_k) - \mathfrak{F}(\chi_{k-1})\big) - (2/3)\mathfrak{F}'(\chi_k)(e_{k-1} - e_k)} \quad (11)$$

Now, for some c between χ^* and χ_k, the mean value form of Taylor's theorem, we obtain:

$$\mathfrak{F}(\chi^*) = \mathfrak{F}(\chi_k) + \mathfrak{F}'(\chi_k)(\chi^* - \chi_k) + \frac{1}{2!}\mathfrak{F}''(\chi_k)(\chi^* - \chi_k)^2 + \frac{1}{3!}\mathfrak{F}'''(c)(\chi^* - \chi_k)^2 \quad (12)$$

Since the derivative of above equation equal zero, we get:

$$-\mathfrak{F}'(\chi_k) = -e_k\mathfrak{F}''(\chi_k) + \frac{e_k^2}{2}\mathfrak{F}'''(c) \quad (13)$$

Using (13) into (10) we get:

$$e_{k+1} = e_k^2 \frac{1}{2}\frac{\mathfrak{F}'''(c)}{\mathfrak{F}''(\chi_k)} \quad (14)$$

Hence new BBR-iterative schema has 2nd order convergence

4 APPLICATION EXAMPLES

The main purpose of our computational studies is to look at the performance of Algorithms for solving seven test problems from [1]. We will denote the number of iterations by (No.), Execution time (ET) and Newton method (N-method). All of the approaches in the numerical experiments employ the following parameters, $\varepsilon = 10^{-10}$. Matlab is used to write all of the applications.

Example 1. Function $\mathfrak{F}(\chi) = cos(\chi) + (\chi - 2)^2$, $\chi_0 = 2$.

Method	No.	x_k	ET
N	4	2.3542	0.57
BBR	4	2.4715	0.07

Example 2. Function $\mathfrak{F}(\chi) = e^{\chi} - 3\chi^2$, $\chi_0 = 0.25$.

Method	No.	x_k	ET
N	4	0.2045	0.26
BBR	3	0.2033	0.06

Example 3. Function $\mathfrak{F}(\chi) = e^{-\chi} + \chi^2$, $\chi_0 = 1$.

Method	No.	x_k	ET
N	5	0.3517	0.31
BBR	5	0.1291	0.09

Example 4. Function $\mathfrak{F}(\chi) = -\chi e^{-\chi}$, $\chi_0 = 0$.

Method	No.	x_k	ET
N	7	1	0.39
BBR	6	0.0971	0.07

Example 5. Function $\mathfrak{F}(\chi) = 0.65 - 0.75/(1+\chi^2) - 0.65\,\chi\, tan^{-1}(1/\chi)$, $\chi_0 = 0.1$.

Method	No.	x_k	ET
N	6	0.4809	0.79
BBR	5	0.2502	0.06

Example 6. Function $\mathfrak{F}(\chi) = 0.5\,\chi^2 - sin(\chi),\, \chi_0 = 2.$

Method	No.	x_k	ET
N	5	0.7391	0.42
BBR	3	0.7222	0.06

Example 7. Function $\mathfrak{F}(\chi) = \chi^4 + 2\,\chi^2 - \chi - 3,\, \chi_0 = 1.$

Method	No.	x_k	ET
N	7	0.2367	0.68
BBR	6	0.8782	0.10

5 CONCLUSION

For solving nonlinear equations, we developed a new iterative approach (BBR-method) based on the Taylor series formula that uses up to second terms. The effectiveness of this approach is superior to the well-known Newton's method, according to the numerical computation. Theoretical features of this technology are shown, as well as their actual performance, by thorough numerical studies .Not all techniques must converge to the global minimum, and some may fail to converge to the minimum.

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