SOME RESULTS ON DIRECT PRODUCT GRAPHS

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Abstract

Let G be a simple graph. The super-connectivity of a connected graph G, $\kappa'(G)$, is the minimum number of vertices whose removal results in a disconnected graph without an isolated vertex. in this paper investigate $\kappa'(G \times K_n)$, where G is a non-complete graph.

Keywords: Direct product, super connectivity, vertex-cut.

1 Introduction

Let G_1 and G_2 be simple graphs. the direct product of G_1 and G_2, $G_1 \times G_2$, is the graph with the vertex set $V(G_1) \times V(G_2)$ and two distinct vertices (x_1, y_1), (x_2, y_2) are adjacent if and only if $x_1x_2 \in E(G_1)$ and $y_1y_2 \in E(G_2)$. Clearly, $G_1 \times G_2$ is commutative. By [18], $G_1 \times G_2$ is connected if and only if both G_1 and G_2 are connected and not both are bipartite graphs.

Let $\delta(G)$ denote the minimum degree among all vertices in the graphs G. the connectivity of G, $\kappa(G)$ is the minimum size os $S \subseteq V(G)$, where $G - S$ is disconnected or $G - S \cong K_1$. Also, the super connectivity of G, $\kappa'(G)$, is the minimum size of $S \subseteq V(G)$, where $G - S$ is disconnected with no isolated vertices.

In this paper, we propose $\kappa'(G \times K_n)$ for an arbitrary graphs G.

Proposition 1.1. *Let G be a connected graph. If G has no odd cycle, then $G \times K_2$ has exactly two components isomorphic to G.*

Theorem 1.2. *Let $S_i = V(G) \times v_i$ where $v_i \in V(K_n)$. Then $V(G \times K_n) = S_1 \cup S_2 \cup \cdots \cup S_n$.*

Theorem 1.3. *Let G be a graph with $\kappa'(G) = t < \infty$. Then $\kappa'(G \times K_n) \leq tn$.*

Theorem 1.4. *Let G be a cycle of length 5. Then $\kappa'(G \times K_n) = \min\{5n - 8, 3n\}$ for $n \geq 3$.*

Corollary 1.5. *Let G be a cycle of length 5. Then $\kappa'(G \times K_n) = 3n$ for $n \geq 4$.*

Theorem 1.6. *Let G be a bipartite graph and $\kappa'(G) = \infty$, then $\kappa'(G \times K_n) \leq m(n-2)$, where $|V(G)| = m$.*

Theorem 1.7. *Let G be a graph with girth$(G) = 3$, $|V(G)| = m$ and $\kappa'(G) = \infty$. Then $\kappa'(G \times K_n) \leq \min\{mn - 6, m(n-1) + 5, 5n + m - 8\}$.*

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