A Suggestion for Exploring an Interval-Valued Fermatean Fuzzy Shortest Path Problem

Said Broumi
Laboratory of Information Processing, Faculty of Science Ben M'Sik, University of Hassan II, Casablanca, Morocco
broumisaid78@gmail.com

S.krishna Prabha
Department of Mathematics, PSNA college of Engineering and Technology, Dindigul, Tamil Nadu ,India
jvprbh1@gmail.com

ABSTRACT

Fermatean fuzzy set theory, a state-of-the-art mathematical method, has been developed to address the uncertainty of various real-world scenarios. The Fermatean fuzzy set was created to allow analytical management of uncertain data from common real-world decision-making situations. Due to the inadequate data available, decision-makers find it challenging to define the degree of membership (MG) and non-membership (NG) with sharp values. In these situations, intervals BG and NG are good choices. In this article, we use an interval set of values in the Fermatean fuzzy context to formulate the shortest path problem(SPP). Next, a defuzzification method using score function is proposed. To further illustrate the viability and efficacy of the suggested framework, a mathematical formulation is also presented.

KeyWords:
Fermatean fuzzy set, Shortest path problem, Interval value Fermatean fuzzy number, Score function, Intuitionistic fuzzy set, Pythagorean fuzzy set.

1. Introduction

In recent decades, decision-makers and academicians have been able to use a variety of mathematical tools to address ambiguities and uncertainties in a variety of real-world situations such as the natural sciences, medicine and engineering. In 1965, Zadeh [17] offered the Fuzzy Set (FS) as a miraculous method for dealing hesitation and vagueness. FS theory demonstrates useful applications in numerous real-life problems. Atanassov [2] launched Intuitionistic Fuzzy Set model (IFS) in 1986, where membership and non-membership (totals are always limited to 1) are used to describe all objects in IFS. Several booming studies based on the IFS hypothesis have been carried out in numerous scientific disciplines. In the majority of instances, the individual FS and IFS values that were provided to the point from the unit interval proved to be insufficient.

To address this flaw, Yager developed an idea of intonation, known as the Pythagorean Fuzzy Set (PFS), as an over simplification of IFS. However, due to some issues, the PFS approach was not accepted. In 2020, Senapati and Yager [10] devised an idea of Fermatean Fuzzy Sets (FFS) as an expansion of IFS and PFS to deal with this intricacy. For entities in FFS, the aggregate of the cubes reflecting membership and non-membership values is exactly one.FFS theory is presently performing a crucial part in many areas as it is a powerful idea for handling ambiguous and inaccurate data in Fermatean Fuzzy environment. The Fermatean Fuzzy set is a convenient way to pact with uncertainty and vagueness, as it increases the spatial volume of membership and non-membership in fuzzy and Pythagorean fuzzy set.

Kaufmann pioneered the fuzzy graph (FG) perception in 1973.In 2006, Parvathi and Karunambigai [8], developed the notion of fuzzy graphs by proposing an intuitionistic fuzzy incidence graph (IFG). In 2014, Anusuya, Sathya [1] developed an algorithm that uses similarity measure to find the optimal path in a fuzzy-weighted network. In 2015, Kumar et al [7] devised a routing problem technique employing interval-valued intuitionistic trapezoidal fuzzy numbers. Dey et al [3] explored about Fuzzy Shortest Path Problem (SPP) in 2016. SPP using Pythagorean fuzzy variable of interval values was introduced in 2022 by Jan A et al [6].Ebrahimnejad et al [4] in 2020, projected an optimization technique for unravelling SPPs with interval-valued triangular fuzzy arc weight. Singh [13] devised a Fuzzy SPP in 2020 from the viewpoint of the Entrepreneur. An enhanced A∗ exploration code for SPP in interval-valued Pythagorean fuzzy was presented by Vidhya [15] in 2022. A SPP in Fermatean fuzzy sets with interval values was devised in this research.

The article is structured as follows: The history and specific important applications described in Section 1 provide inspiration for the proposed work. Some prerequisites are portrayed in Section 2. Section 3 provides a framework for the Fermatean fuzzy SPP. In Section 4, a quantifiable example is endowed. Section 5 provides a summary of the article and future directions, as well as the benefits of the proposed work.

2. Preliminaries

Definition 2.1 [16]

A Pythagorean fuzzy set (PFS) A on a universe of discourse X, is a structure having the form as

$$A=\{\langle x, T_A(x), F_A(x)\rangle \mid x \in X\}$$

where $T_A(x): X \to [0,1]$ indicates the degree of membership and $F_A(x): X \to [0,1]$ indicates the degree of non-membership of every element $x \in X$ to the set A, respectively, with the constraints: $0 \leq \left(T_A(x)\right)^2 + \left(F_A(x)\right)^2 \leq 1$.

Senpati et al. [2, 3] suggested the idea of Fermatean fuzzy set considering more possible types of uncertainty. These are defined below,

Definition 2.2[10, 11]

A Fermatean fuzzy set ($\mathbb{FF}-set$) A on a universe of discourse X is a structure defined as,

$$A=\{\langle x, T_A(x), F_A(x)\rangle \mid x \in X\}$$

where $T_A(x): X \to [0,1]$ indicates the degree of membership, and $F_A(x): X \to [0,1]$ indicates the degree of non-membership of the element $x \in X$ to the set A, respectively, with the constraints :

$$0 \leq \left(T_A(x)\right)^3 + \left(F_A(x)\right)^3 \leq 1$$

Definition 2.3[5]

An interval valued Fermatean fuzzy set ($\mathbb{I}\mathbb{v}\mathbb{FF}-set$) A on a universe of discourse X is a structure having the form as:

$$A=\{\langle x, T_A(x), F_A(x)\rangle \mid x \in X\}$$

where $T_A^L(x), T_A^U(x), : X \to [0,1]$ indicates the degree of membership, and $F_A^L(x), F_A^U(x): X \to [0,1]$ indicates the degree of non-membership of the element $x \in X$ to the set A, correspondingly, with the constraints :

$$0 \leq \left(T_A^U(x)\right)^3 + \left(F_A^U(x)\right)^3 \leq 1$$

Definition 2. 4 [14]

A Fermatean fuzzy Graph (FFG) on a universal set X is a pair $\mathbb{G}=(\mathcal{P},\mathcal{Q})$ where $\mathcal{P}$ is Fermatean fuzzy set on X and $\mathcal{Q}$ is a Fermatean fuzzy relation on X such that:

$$\begin{cases} T_{\mathcal{Q}}(u,v) \leq \min\{T_{\mathcal{P}}(u), T_{\mathcal{P}}(v)\} \\ F_{\mathcal{Q}}(u,v) \geq \max\{F_{\mathcal{P}}(u), F_{\mathcal{P}}(v)\} \end{cases}$$

and $0 \leq T_{\mathcal{Q}}^3(u,v) + F_{\mathcal{Q}}^3(u,v) \leq 1 \quad for\ all\ u,v \in X$, where , $T_{\mathcal{Q}}: X \times X \to [0,1]$, $F_{\mathcal{Q}}: X \times X \to [0,1]$ indicates degree of membership and degree of non-membership of $\mathcal{Q}$, correspondingly. Here $\mathcal{P}$ is the Fermatean fuzzy vertex set of $\mathbb{G}$ and $\mathcal{Q}$ is the Fermatean fuzzy edge set of $\mathbb{G}$.

2.1 .Arithmetic Operations on IVFFNs

Definition 2.5 [5]

Let $\mathbb{A} = \langle[\mathbb{T}^L, \mathbb{T}^U], [\mathbb{F}^L, \mathbb{F}^U]\rangle, \mathbb{A}_1 = \langle[\mathbb{T}_1^L, \mathbb{T}_1^U], [\mathbb{F}_1^L, \mathbb{F}_1^U]\rangle$ and $\mathbb{A}_2 = \langle[\mathbb{T}_2^L, \mathbb{T}_2^U], [\mathbb{F}_2^L, \mathbb{F}_2^U]\rangle$be three interval valued fermatean fuzzy numbers and $\lambda > 0$. Then, the operations rules are described as follows;

- $\mathbb{A}_1 \oplus \mathbb{A}_2 = \langle\left[\sqrt[3]{{\mathbb{T}_1^L}^3 + {\mathbb{T}_2^L}^3 - {\mathbb{T}_1^L}^3{\mathbb{T}_2^L}^3}, \sqrt[3]{{\mathbb{T}_1^U}^3 + {\mathbb{T}_2^U}^3 - {\mathbb{T}_1^U}^3{\mathbb{T}_2^U}^3}\right], [\mathbb{F}_1^L\mathbb{F}_2^L, \mathbb{F}_1^U\mathbb{F}_2^U]\rangle$
- $\mathbb{A}_1 \otimes \mathbb{A}_2 = \langle[\mathbb{T}_1^L\mathbb{T}_2^L, \mathbb{T}_1^U\mathbb{T}_2^U], \left[\sqrt[3]{{\mathbb{F}_1^L}^3 + {\mathbb{F}_2^L}^3 - {\mathbb{F}_1^L}^3{\mathbb{F}_2^L}^3}, \sqrt[3]{{\mathbb{F}_1^U}^3 + {\mathbb{F}_2^U}^3 - {\mathbb{F}_1^U}^3{\mathbb{F}_2^U}^3}\right]\rangle$
- $\lambda\, \mathbb{A} = = \langle\left[\sqrt[3]{1 - \left(1 - {\mathbb{T}^L}^3\right)^\lambda}, \sqrt[3]{1 - \left(1 - {\mathbb{T}^U}^3\right)^\lambda}\right], \left[{\mathbb{F}^L}^\lambda, {\mathbb{F}^U}^\lambda\right]\rangle$
- $\mathbb{A}^\lambda = = \langle\left[{\mathbb{T}^L}^\lambda, {\mathbb{T}^U}^\lambda\right], \left[\sqrt[3]{1 - \left(1 - {\mathbb{F}^L}^3\right)^\lambda}, \sqrt[3]{1 - \left(1 - {\mathbb{F}^U}^3\right)^\lambda}\right]\rangle$

Definition 2.6 [5]

In order to make a comparisons between two interval valued fermatean fuzzy numbers (IVFFNs), Siravaman [5], proposed score function. Let $\mathbb{A}_1 = \langle[\mathbb{T}_1^L, \mathbb{T}_1^U], [\mathbb{F}_1^L, \mathbb{F}_1^U]\rangle$ be a, interval valued fermatean fuzzy number, then, the score function $score(\mathbb{A}_1)$ of an IVFFN are defined as follows:

$$score(\mathbb{A}_1) = \frac{1}{4}\ [\mathbb{T}^L + \mathbb{T}^U + \mathbb{T}^U(1 - \mathbb{F}^U) + \mathbb{T}^L(1 - \mathbb{F}^L)], where\ score(\mathbb{A}_1) \in [0,1]$$

2.2 Ranking of interval valued fermatean fuzzy number

Depending upon the score function of IVFFNs, the ranking method for any two IVFFNs can be defined as:

Definition 2.7 [5]

Let $\mathbb{A}_1 = \langle[\mathbb{T}_1^L, \mathbb{T}_1^U], [\mathbb{F}_1^L, \mathbb{F}_1^U]\rangle$ and $\mathbb{A}_2 = \langle[\mathbb{T}_2^L, \mathbb{T}_2^U], [\mathbb{F}_2^L, \mathbb{F}_2^U]\rangle$ be two IVFFNs. $score(\mathbb{A}_i)$ and (i = 1, 2) are the score values of $\mathbb{A}_1$ and $\mathbb{A}_2$, respectively, then

- If $score(\mathbb{A}_1) < score(\mathbb{A}_2)$, then $\mathbb{A}_1 < \mathbb{A}_2$;

- If $score(\mathbb{A}_1) > score(\mathbb{A}_2)$, then $\mathbb{A}_1 > \mathbb{A}_2$;
- If $score(\mathbb{A}_1) = score(\mathbb{A}_2)$, then $\mathbb{A}_1 = \mathbb{A}_2$;

3. Fermatean Fuzzy Shortest Path Algorithm

The shortest path issue is one of the well known graph theory conundrums. Practically every fuzzy structure in FG theory has been carefully studied in relation to the shortest path problem. The proposed new approach's importance stems from its ability to address issues that arise in interval valued fermatean fuzzy numbers. Compared to other algorithms, the one we utilized is relatively simple to use and produces results significantly more quickly. The algorithm is applicable to all varieties of fuzzy structures. This algorithmic rule is used for addressing the need for shortest path explanations, whether within the realm of artificial intelligence, transportation, embedded systems, laboratory or industrial plants, etc.

This section proposes a technique to calculate the shortest path from each node to its predecessor. This strategy can be utilized to detect the shortest path in a network in real life scenarios.

Step 1:

Prioritize v_1 and v_n as the destination's first and last nodes, respectively.

Step 2:

Considering that node 1 is not isolated from itself by any distance, let $d_1 = \langle [0,0], [1,1] \rangle$

Additionally, add the label ($\langle [0,0], [1,1] \rangle$, $-$) to the first node.

Step 3:

Find $d_j = \min\{ d_i \oplus d_{ij} \}$. For j = 2, 3, …n. use the Score function for defuzzification of IVFFNs .

$$score(\mathbb{A}_1) = \frac{1}{4} \left[\mathbb{T}^L + \mathbb{T}^U + \mathbb{T}^U(1 - \mathbb{F}^U) + \mathbb{T}^L(1 - \mathbb{F}^L) \right], where\ score(\mathbb{A}_1) \in [0,1]$$

Step 4:

If a unique distance value is encountered at i = r. hence j is thus designated as $[d_j, r]$.

If there is no unique match between the distance measurements.

It indicates that there are several IVFFN pathways leading from a node.

Use the score feature of IVFFNs to find the shortest path out of multiple options.

Step 5:

Let the destination node be labelled as $[d_n, k]$.where d_n is the shortest displacement between initial and final node.

Step 6:

Therefore, we check the label of node k to get the IVFFN shortest path from the first to the last node. Let it be$[d_n, l]$. Next, we evaluate node l's label of node l, and so forth.

To obtain the initial node, repeat the steps above.

Step 7:

Consequently, step 6 can be used to determine the IVFFN shortest path.

Flow Chart for the above algorithm is given below,

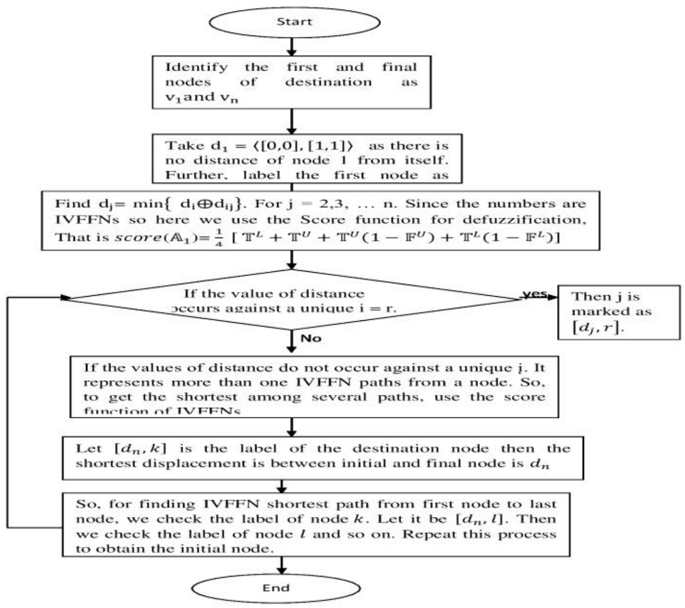

Fig.1.Flow Chart for Procedure

4. Numerical Example

Presume about a network of IVFFG shown in Fig.1.

The shortest path is computed using the proposed technique in the approach shown below.

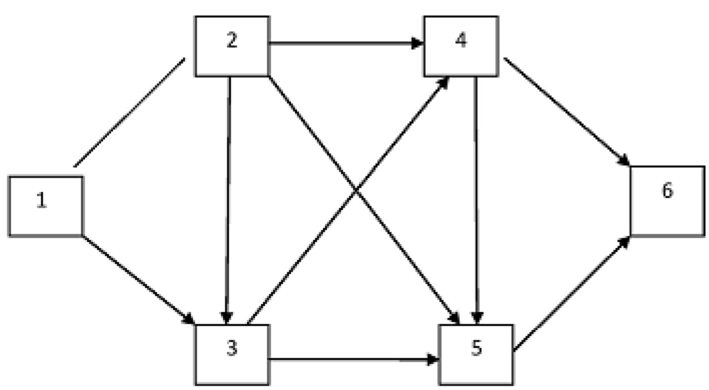

Fig. 2: IVFF network.

In Table 1, IVFFNs are utilized to illustrate the path between each pair of nodes.

Table 1: Distance between the Nodes in IVFF Network Edges.

Edges	Distance
(1, 2)	$\langle[0.4,0.6],[0.1,0.3]\rangle$
(1, 3)	$\langle[0.2,0.7],[0.1,0.5]\rangle$
(2, 3)	$\langle[0.1,0.7],[0.2,0.4]\rangle$
(2, 4)	$\langle[0.4,0.5],[0.7,0.8]\rangle$
(2, 5)	$\langle[0.5,0.6],[0.5,0.7]\rangle$
(3, 4)	$\langle[0.6,0.7],[0.4,0.6]\rangle$
(3, 5)	$\langle[0.6,0.7],[0.3,0.6]\rangle$
(4, 5)	$\langle[0.4,0.7],[0.5,0.8]\rangle$
(4, 6)	$\langle[0.3,0.5],[0.3,0.8]\rangle$
(5, 6)	$\langle[0.5,0.8],[0.5,0.6]\rangle$

Now, utilizing the methodology described, we determine the shortest path as specified:
The destination node being 6, n = 6.
If you mark the source node as ($\langle 0,1\rangle$,−)(let's say node 1) and set $d_1 = \langle[0,0],[1,1]\rangle$ to those coordinates, you can find d_j as follows.

Iteration 1:
Since node 2 has only one predecessor, we set i = 1 and j = 2, which results in d_2 as
$d_2 = \min\{d_1 \oplus d_{12}\}$
$= \min(\langle[0,0],[1,1]\rangle \oplus \langle[0.4,0.6],[0.1,0.3]\rangle)$
$= \langle[0.4,0.6],[0.1,0.3]\rangle$
When i = 1, the minimum value is attained. Thus, vertex 2 is labeled as$\langle[0.4,0.6],[0.1,0.3]\rangle, -1]$
Iteration 2:
Set i = 1, 2 and j = 3, since node 3's predecessors are 1 and 2.
$d_3 = \min\{d_1 \oplus d_{13}, d_2 \oplus d_{23}\}$
$= \min\{\langle[0,0],[1,1]\rangle \oplus \langle[0.2,0.7],[0.1,0.5]\rangle, \langle[0.4,0.6],[0.1,0.3]\rangle \oplus \langle[0.1,0.7],[0.2,0.4]\rangle\}$
$= \min\{\langle[0.2,0.7],[0.1,0.5]\rangle, \langle[0.402,0.786],[0.02,0.12]\rangle\}$
Score function enables us to identify the absolute minimum:
$S(\langle[0.2,0.7],[0.1,0.5]\rangle) = 0.357$ and
$S(\langle[0.402,0.786],[0.02,0.12]\rangle) = 0.568$.
So, the $d_3 = \langle[0.2,0.7],[0.1,0.5]\rangle$
When i = 1, the minimum value is attained. Thus, vertex 3 is labeled as$[\langle[0.2,0.7],[0.1,0.5]\rangle, 1]$.
Iteration 3:
Set i = 2, 3, and j = 4, since node 4's predecessors are 2 and 3.
$d_4 = \min\{d_2 \oplus d_{24}, d_3 \oplus d_{34}\}$
$=$
$\min\{\langle[0.4,0.6],[0.1,0.3]\rangle \oplus \langle[0.4,0.5],[0.7,0.8]\rangle, \langle[0.2,0.7],[0.1,0.5]\rangle \oplus \langle[0.6,0.7],[0.4,0.6]\rangle\}$
$= \min\{\langle[0.499,0.68],[0.07,0.24]\rangle, \langle[0.606,0.828],[0.04,0.3]\rangle\}$
Score function enables us to identify the absolute minimum:
$S(\langle[0.499,0.68],[0.07,0.24]\rangle) = 0.539$ and

$S(\langle[0.606,0.828],[0.04,0.3]\rangle) = 0.648$
Hence $d_4 = \langle[0.499,0.68],[0.07,0.24]\rangle$
When i = 2, the minimum value is attained.
Thus, vertex 4 is labeled as $[\langle[0.499,0.68],[0.07,0.24]\rangle, 2]$.
Iteration 4:
Set i = 2, 3,4 and j = 5, since node 5's predecessors are 2,3 and 4.
$d_5 = \min\{d_2 \oplus d_{25}, d_3 \oplus d_{35}, d_4 \oplus d_{45}\}$
$= \min\{\langle[0.4,0.6],[0.1,0.3]\rangle \oplus \langle[0.5,0.6],[0.5,0.7]\rangle, \langle[0.2,0.7],[0.1,0.5]\rangle \oplus \langle[0.6,0.7],[0.3,0.6]\rangle, \langle[0.499,0.68],[0.07,0.24]\rangle \oplus \langle[0.4,0.7],[0.5,0.8]\rangle\}$
$= \min\left\{\begin{matrix}\langle[0.566,0.728],[0.05,0.21]\rangle, \langle[0.606,0.828],[0.03,0.3]\rangle, \\ \langle[0.565,0.819],[0.035,0.192]\rangle\end{matrix}\right\}$
Score function enables us to identify the absolute minimum:
$S(\langle[0.566,0.728],[0.05,0.21]\rangle) = 0.601$
$S(\langle[0.606,0.828],[0.03,0.3]\rangle) = 0.650$ and
$S(\langle[0.565,0.819],[0.035,0.192]\rangle) = 0.651$.
So, the $d_5 = \langle[0.566,0.728],[0.05,0.21]\rangle$
When i = 2, the minimum value is attained. Thus, vertex 5 is labeled as $[\langle[0.566,0.728],[0.05,0.21]\rangle, 2]$.
Iteration 5:
Set i = 4,5 and j = 6, since node 6's predecessors are 4 and 5.
$d_6 = \min\{d_4 \oplus d_{46}, d_5 \oplus d_{56}\}$
$= \min\{\langle[0.499,0.68],[0.07,0.24]\rangle \oplus \langle[0.3,0.5],[0.3,0.8]\rangle, \langle[0.566,0.728],[0.05,0.21]\rangle \oplus \langle[0.5,0.8],[0.5,0.6]\rangle\}$
$= \min\{\langle[0.529,0.737],[0.021,0.192]\rangle, \langle[0.657,0.888],[0.025,0.126]\rangle\}$
Score function enables us to identify the absolute minimum:
$S([0.529,0.737],[0.021,0.192]\rangle) = 0.594$ and
$S(\langle[0.657,0.888],[0.025,0.126]\rangle) = 0.740$
So, the $d_6 = \langle[0.529,0.737],[0.021,0.192]\rangle$
When i = 4, the minimum value is attained.
Thus, vertex 6 is labeled as $[\langle[0.529,0.737],[0.021,0.192]\rangle, 4]$.
Since d_6 is the final destination.
So,The shortest displacement is specified as proceeding from vertex one to six.
$\langle[0.529,0.737],[0.021,0.192]\rangle$
The shortest way can be determined as follows:
Node 6 is labelled as $[\langle[0.529,0.737],[0.021,0.192]\rangle, 4]$.
Node 5 is labelled as $[\langle[0.566,0.728],[0.05,0.21]\rangle, 2]$.
Node 4 is labelled as $[\langle[0.499,0.68],[0.07,0.24]\rangle, 2]$.
Node 3 is labelled as as$[\langle[0.2,0.7],[0.1,0.5]\rangle, 1]$.
Consequently, the shortest path is **1→ 2 → 4 → 6** with the IVFFN value of distance being$\langle[0.529,0.737],[0.021,0.192]\rangle$,
The shortest path is depicted in Fig. 2 by the dotted line, and the paths of various nodes are shown in Table 2.

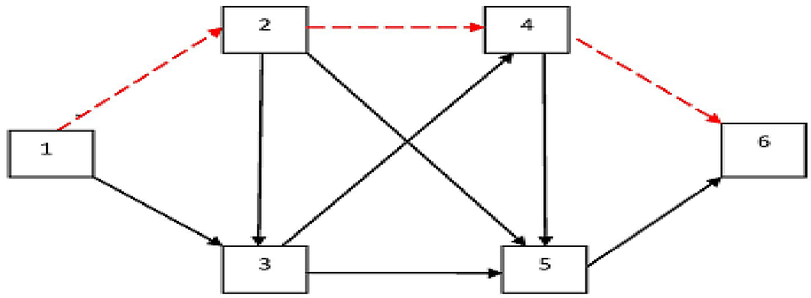

Fig. 2: Shortest Path IVFF network**.**

Table 2: Shortest Path of the above network

Nodes No.(j)	$\mathbf{d_i}$	**Shortest path from1st node to jth node**
2	$\langle[0.4,0.6],[0.1,0.3]\rangle$	$1 \longrightarrow 2$
3	$\langle[0.2,0.7],[0.1,0.5]\rangle$	$1 \longrightarrow 3$
4	$\langle[0.499,0.68],[0.07,0.24]\rangle$	$1 \longrightarrow 2 \longrightarrow 4$
5	$\langle[0.566,0.728],[0.05,0.21]\rangle$	$1 \longrightarrow 2 \longrightarrow 5$
6	$\langle[0.529,0.737],[0.021,0.192]\rangle$	$1 \longrightarrow 2 \longrightarrow 4 \longrightarrow 6$

5. Conclusion

This article introduces the conception of IVFFG, an extension of Fermatean fuzzy graph. An algorithm has been suggested to calculate the shortest path of an IVFFG. A numerical example is resolved using the suggested approach to determine the shortest path in the network out of all potential paths. Scientists who want to propose new ideas for tackling the shortest path problem will find this research very useful. Based on the results of this research, new frameworks and algorithms will be developed in future to find optimum path for a particular network in different fixed contexts under different fuzzy environments.

This strategy may be utilized to tackle a variety of fundamental issues in circuit systems, communication networks, transportation networks and other areas.

Conflicts of Interest: The authors declare no conflicts of interest.

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