LUCKY EDGE GEOMETRIC MEAN LABELING OF GRAPHS AND ITS APPLICATIONS IN ELECTRICAL NETWORKS

K.AMUTHAVALLI

Government Arts and Science College

Veppanthattai, Perambalur, India

thrcka@gmail.com;

P.SUGAPRIYA

K.Ramakrishnan College of Engineering

Samayapuram, Trichy, India

sugapriya.mat10@gmail.com

A.ATKINSWESTLEY

K.Ramakrishnan College of Engineering

Samayapuram, Trichy, India

ats.wesly@gmail.com

S. MUTHUPERUMAL

K.Ramakrishnan College of Engineering

Samayapuram, Trichy, India

muthuperumal@krce.ac.in

ABSTRACT:

A function $f:V \to N$ is said to be lucky edge geometric mean labeling if there exists a function $f^*:\mathrm{E} \to N$ such that $f^*(uv) = \left\lceil \sqrt{f(u)f(v)} \right\rceil$ (or) $\left\lfloor \sqrt{f(u)f(v)} \right\rfloor$ and the edge set E(G) of G, such that $f^*(uv) \neq f^*(vw)$ whenever *uv* and *vw* are adjacent edges. The least integer *k* for which a graph G has a lucky edge geometric mean labeling from the edge set {1, 2, 3 … *k*} is the lucky edge geometric mean number of G denoted by $\eta'_{GM}(G)$. A graph which admits lucky edge geometric mean labeling is the lucky edge geometric mean graph. In this paper, we have to investigate that Middle graph of star, Total graph of star and Central graph of star admit lucky edge geometric mean labeling and also, we obtain the lucky edge geometric mean number of these graphs. Also, we compute the power absorbed by all the resistors of electric circuit by using lucky edge geometric mean labeling.

Keywords: Central graph, Electric Networks, Lucky edge geometric mean Labeling, Middle graph, Star graph, Total graph.

1 Introduction

The concept of graph labeling was first introduced by Rosa [7] in 1967. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or edge labeling). In the intermediate years various labeling of graphs have been investigated over 1100 papers.

Labeled graphs serve as useful models in a broad range of X – ray, Crystallography, radar, astronomy, database management and coding theory. Nellai Murugan .A and Maria Irudhaya Asprin Chitra.R introduced the concept of lucky edge labeling as a function *f* is an assignment of integers to vertices such that *f(u) +f(v)* is assigned to the edge <u>*uv*</u> and *vw* are adjacent edges. The least integer k for which a graph G has a lucky edge labeling from the set $\{1, 2, ..., k\}$ is the lucky edge number of G and is denoted by $\eta(G)$. A graph which admits lucky edge labeling is the lucky edge labeled graph.

In 2014[4], they Proved P_n, C_n are lucky edge labeled graphs. In 2015, they proved planar grid graph have lucky edge labeling. In 2016[5], they proved triangular graphs are lucky edge labeled graphs and some special graphs are also lucky edge labeled graphs. In 2017, they proved subdivision of star and wheel graphs, H – super subdivision of graphs is lucky edge labeled graphs. In 2018, they proved star related graphs are lucky edge labeled graph. Motivated by these notions, we have to introduce the concept of Lucky edge geometric mean labeling.

In this paper we have to investigate that Middle graph of star, Total graph of star, Central graph of star admits lucky edge geometric mean labeling and also we obtain the lucky edge geometric mean number of these graphs. To find the power absorbed by the resistors of electric circuit by using the lucky edge geometric mean labeling.

2.1 Lucky Edge Geometric Mean Labeling

A function $f : V \to N$ is said to be lucky edge geometric mean labeling if there exists a function $f^* : \mathrm{E} \to N$ such that $f^*(uv) = \left\lceil \sqrt{f(u)\, f(v)} \right\rceil$ (or) $\left\lfloor \sqrt{f(u)\, f(v)} \right\rfloor$ and the edge set E(G) of G, such that $f^*(uv) \neq f^*(vw)$ whenever *uv* and *vw* are adjacent edges. The least integer *k* for which a graph G has a lucky edge geometric mean labeling from the edge set {1, 2, 3 … *k*} is the lucky edge geometric mean number of G denoted by $\eta'_{GM}(G)$.

2.2 Middle Graph of Star $K_{1,n}$ [6]:

Let $u, u_1, u_2, ..., u_n$ be the vertices of the star graph $K_{1,n}$ and $uu_1, uu_2, ..., uu_n$ be the edges of the star graph $K_{1,n}$.The middle graph of star $M\left[K_{1,n}\right]$ is obtained by added the vertices $v_1, v_2, ..., v_n$ corresponding to the edges $uu_1, uu_2, ..., uu_n$ of $K_{1,n}$. $(i.e,)$ $V\left[M[K_{1,n}]\right] = V\left[K_{1,n}\right] \cup E\left[K_{1,n}\right]$.

Two vertices x, y in $V\left[M[K_{1,n}]\right]$ are adjacent $\Leftrightarrow$ any of the following conditions holds,

(i) $x, y \in \left[K_{1,n}\right]$ and x, y are adjacent in $K_{1,n}$.

(ii) x is in $V\left[K_{1,n}\right]$, y is in $E\left[K_{1,n}\right]$ and x, y are incident in $K_{1,n}$.

2.3 Total Graph of Star $K_{1,n}$ [6] :

Let $u, u_1, u_2, ..., u_n$ be the vertices of the star graph $K_{1,n}$ and $uu_1, uu_2, ..., uu_n$ be the edges of the star graph $K_{1,n}$.The total graph of star $T\left[K_{1,n}\right]$ is obtained by added the vertices $v_1, v_2, ..., v_n$ corresponding to the edges $uu_1, uu_2, ..., uu_n$ of $K_{1,n}$. $(i.e,)$ $V\left[T[K_{1,n}]\right] = V\left[K_{1,n}\right] \cup E\left[K_{1,n}\right]$.

Two vertices x, y in $V\left[T[K_{1,n}]\right]$ are adjacent $\Leftrightarrow$ any of the following conditions holds,

(i) x, y are in $V\left[T[K_{1,n}]\right]$ and x is adjacent to y in $K_{1,n}$.

(ii) x, y are in a $E\left[T[K_{1,n}]\right]$ and x, y are adjacent in $K_{1,n}$.

(iii) x is in $V\left[T[K_{1,n}]\right]$, y is in $E\left[T[K_{1,n}]\right]$ and x, y are incident in $K_{1,n}$.

2.4 Central Graph of Star $K_{1,n}$ [6]:

Let $u, u_1, u_2, ..., u_n$ be the vertices of the star graph $K_{1,n}$. The central graph of star $C\left[K_{1,n}\right]$ is obtained by subdivided the edges $uu_1, uu_2, ..., uu_n$ of $K_{1,n}$ by the label the vertices $v_1, v_2, ..., v_n$.Then joining all the non-adjacent vertices of star $K_{1,n}$.

2.5 Theorem:

$M[K_{1,n}]$ is a lucky edge geometric mean graph.

Proof:

Define $f: V[M(K_{1,n})] \to N$

The Vertex labels are defined by

$f(u) = (n+1)^2$

$f(u_i) = f(v_i) = i^2, 1 \le i \le n$

Clearly, we have induced function $f^*: E[M(K_{1,n})] \to \{1,2,3, ... \}$ Such that $f^*(uv) = \left\lceil \sqrt{f(u) f(v)} \right\rceil$ (or) $\left\lfloor \sqrt{f(u) f(v)} \right\rfloor$ and the edge set $E[M(K_{1,n})]$ is a proper colouring of G, that is $f^*(uv) \neq f^*(vw)$ whenever uv and vw are adjacent edges.

The least positive integer $n^2 + n$ for which a graph $M[K_{1,n}]$ has a lucky edge geometric labeling from the set $\{1,2,3, ... , n^2 + n\}$ is the lucky edge geometric mean number of G denoted by $\eta'_{GM}(M[K_{1,n}])$.

$\therefore \eta'_{GM}(M[K_{1,n}]) = n^2 + n$

Hence $M[K_{1,n}]$ is a lucky edge geometric mean graph

2.6 Theorem:

$T[K_{1,n}]$ is a lucky edge geometric mean graph

Proof:

Define $f: V\left[T[K_{1,n}]\right] \rightarrow N$
The Vertex labels are defined by
$f(u) = (n+1)^2$
$f(u_i) = i^2, 1 \leq i \leq n$
$f(v_i) = (n+i+1)^2, 1 \leq i \leq n$

Clearly, we have induced function $f^*: E\left[T[K_{1,n}]\right] \rightarrow \{1,2,3,\dots.\}$ Such that $f^*(uv) = \left\lceil \sqrt{f(u)\,f(v)} \right\rceil$ (or) $\left\lfloor \sqrt{f(u)\,f(v)} \right\rfloor$ and the edge set $E[M(K_{1,n})]$ is a proper colouring of G, that is $f^*(uv) \neq f^*(vw)$ whenever uv and vw are adjacent edges.

The least positive integer $2n^2+3n+1$ for which a graph $T[K_{1,n}]$ has a lucky edge geometric mean labeling from the set $\{1,2,3,\dots.,2n^2+3n+1\}$ is the lucky edge geometric mean number of G denoted by $\eta'_{GM}(T[K_{1,n}])$
$\therefore \eta'_{GM}(T[K_{1,n}]) = 2n^2+3n+1$
Hence $T[K_{1,n}]$ is a lucky edge geometric mean graph

2.7 Theorem

$C[K_{1,n}]$ is a lucky edge geometric mean graph.

Proof:

Define $f: V\left[C[K_{1,n}]\right] \rightarrow N$
The Vertex labels are defined by
$f(u) = 1$
$f(u_i) = f(v_i) = (i+1)^2, 1 \leq i \leq n$

Clearly, we have induced function $f^*: E\left[C[K_{1,n}]\right] \rightarrow \{1,2,3,\dots.\}$ Such that $f^*(uv) = \left\lceil \sqrt{f(u)\,f(v)} \right\rceil$ (or) $\left\lfloor \sqrt{f(u)\,f(v)} \right\rfloor$ and the edge set $E\left[C[K_{1,n}]\right]$ is a proper colouring of G, that is $f^*(uv) \neq f^*(vw)$ whenever uv and vw are adjacent edges.

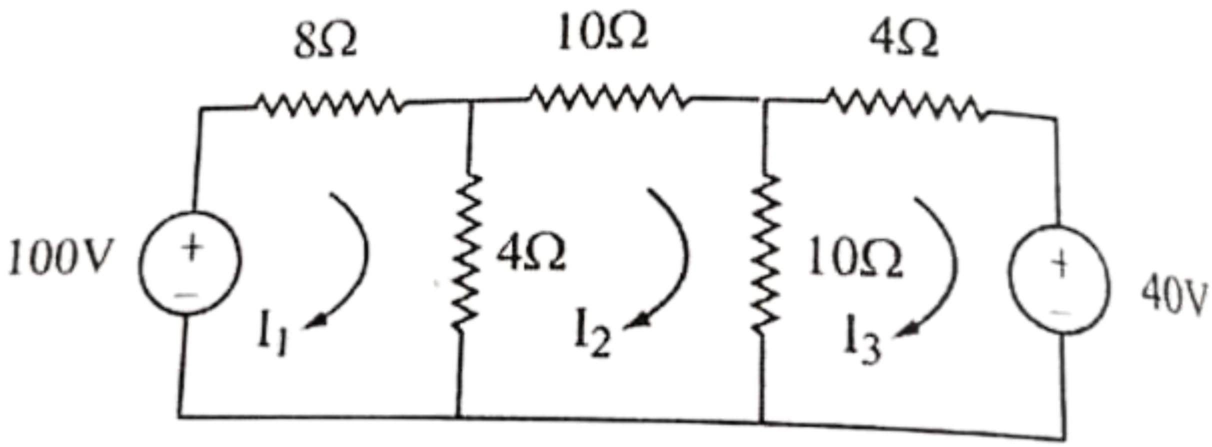
The least positive integer $(n+1)^2$for which a graph $C[K_{1,n}]$ has a lucky edge geometric mean labeling from the set $\{1,2,3,\dots.,(n+1)^2\}$ is the lucky edge geometric mean number of G denoted by $\eta'_{GM}\left(C[K_{1,n}]\right)$
$\therefore \eta'_{GM}\left(C[K_{1,n}]\right) = (n+1)^2$
Hence $C[K_{1,n}]$ is a lucky edge geometric mean graph.

2.8 Applications in Electrical Networks:

An electrical network is a collection of components and device interconnected electrically .The network components are idealized of physical device and system, in order to for them to represent several properties, they must obey the Kirchhoff's law of currents and voltage. A graph representation of electrical network in terms of line segments or arc called edges or branches and points called vertices or terminals.

2.9 Problem:

Investigate the value of power absorbed by all resistors from given below circuit by using lucky edge geometric mean labeling.



Solution:

An electrical network can be represented using graphs as each node of the network can be considered as a vertex of the graph and each branch of the network can be considered as an edge.

A graph model is used to represent a network by using the vertices and edges that cover the entire network. Given below is the graph model of the network shown above

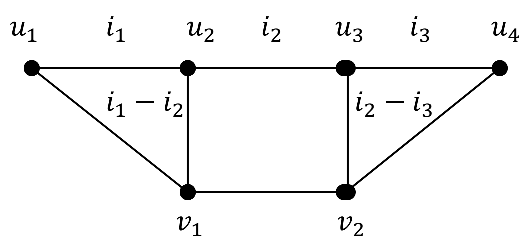


Figure 1: A Graph Model Representation and its Ordinary Labeling

Let G be a given network

$V(G) = \{u_i, v_j: 1 \leq i \leq 4, 1 \leq j \leq 2\}$

$E(G) = \{u_1u_2, u_2u_3, u_3u_4, u_1v_1, u_2v_1, u_3v_2, u_4v_2, v_1v_2\}$

Let the resistors $R_A = 8\Omega, R_B = 4\Omega, R_C = 10\Omega, R_D = 10\Omega, R_E = 4\Omega$

Define $f: V[G] \to N$

The Vertex labels are defined by

$f(u_1) = 5R_A$

$f(u_2) = \frac{R_B}{2}$

$f(u_3) = \frac{R_C}{10}$

$f(u_4) = 2R_E$

$f(v_1) = 5R_B$
$f(v_2) = R_E$

Clearly, we have induced function $f^*: E[\,G] \to \{1,2,3, \ldots .\}$ Such that $f^*(uv) = \left\lceil \sqrt{f(u)\, f(v)} \right\rceil$ (or) $\left\lfloor \sqrt{f(u)\, f(v)} \right\rfloor$ and the edge set $E[G]$ is a proper colouring of G, that is $f^*(uv) \neq f^*(vw)$ whenever uv and vw are adjacent edges.

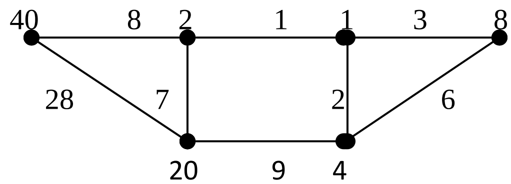


Figure 2: Geometric mean labeling of graph representation of electric network

Clearly we have, $i_1 = 8, i_2 = 1, i_3 = 3, i_1 - i_2 = 7, i_2 - i_3 = 2$

Hence the value of the power absorbed by the resistor$R_A(i_1) = 8$, $R_C(i_2) = 1$, $R_E(i_3) = 3$, $R_{\boldsymbol{B}}(i_1 - i_2) = 7$, $R_D(i_2 - i_3) = 2$

2.10 Conclusion

In this paper, we have computed the lucky edge geometric mean number of Middle graph of star, Total graph of star, Central graph of star and also to find the power absorbed by the resistors of electric circuit using the lucky edge geometric mean labeling. In future, we can compute the power absorbed by the resistors of various three phase electric circuit by using lucky edge geometric mean labeling.

REFERENCES:

[1] Aishwariya.A, "Lucky Edge Labeling of Different Graphs", International Journal of Innovative Research in Science, Engineering and Technology, Volume – 4, Issue 2 (2015), ISSN: 2319-8753, PP: 469-471.

[2] Berdewad. O.K, Deo.S.D, "Application of Graph theory in Electrical Network", International Journal of Science and Research, ISSN: 2319-7064, PP: 981-982.

[3] Poorva.V.Adhyapak, "Study of Electric circuits through Graph theory", Journal of Applied Mathematics and Statistical analysis, volume 1, Issue 3, PP: 1-4.

[4] Nellai Murugan.A, Mariya Irudhaya Asprin Chitra.R, "Lucky Edge Labeling of $P_{n,}$ C_n and corona product of $P_{n,}$ C_n". International Journal of Scientific and Innovative Mathematical Research, Volume 2, Issue 3, August 2014, 710-718.

[5] Nellai Murugan.A, Mariya Irudhaya Asprin Chitra.R, "Lucky Edge Labeling of Triangular graphs", International Journal of Mathematics Trends and Technology, ISSN: 2231-5373, Volume No: 2, 2016.

[6] Revathy.R, Amuthavalli.K, "Radio Harmonic Labeling of Some Star related graphs", AIP Conference Proceedings, 2277, 100004(2020), PP: 100004-1 – 100004-7.

[7] Rosa.A, “On Certain Valuations of the vertices of a graph.” Theory of Graphs (International Symposium, Rome, July 1966), Dunod Gordan and Breach Science publisher, Inc., New York and Dunod Paris (1967), 349-355.

[8] Shalini Rajendra Babu.A, Ramya.N, Rangarajan.K, “On Lucky Edge Labeling of Splitting graphs and snake graphs”, International Journal of Innovative Technology and Exploring Engineering, ISSN: 2278-3075, Volume – 8, Issue – 5, March 2019.

[8] Teriya.O.M, “Application of Graph theory in various field of Applied Science and Engineering”, Journal of Emerging Technologies and Innovative Research, Volume 6, Issue 6, Issue 3, ISSN: 2349-5162, PP: 516-520.

[9] Velammal.S, Arun Muthuram.M, “Application of Graph Theory in Network Analysis”, International Journal of Science Technology and Engineering, Volume 5, Issue 1, ISSN: 2349-784X, 2018.