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Graph theoretical method to study observability for biaffine systems

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ABSTRACT

In this paper we study the observability of structured biaffine system using a graph theoretic approach. We provide necessary and sufficient conditions for the observability of structured biaffine systems. These conditions well-known combinatorial techniques for large scale systems and so we can easy to check by hand for small scale systems.

KEYWORDS: Structured biaffine systems, observability, graph theory.

1 INTRODUCTION

Most of the literature of bilinear observer has dealt with special problems and applications; they come from many areas of science and engineering and could profit from a deeper understanding of the underlying mathematics. A few of them, chosen for ease of exposition and for mathematical interest, will be reported in this paper. More applications can be found in Mohler and Kolodziej [12], the collections edited by Mohler and Ruberti [15], and Mohlers books [9, 11]. Many bilinear control systems familiar in science and engineering have nonnegative state variables; their theory and applications, such as compartmental models in biology,

An exposition of observer systems often would proceed at this point in either of two ways. One is to introduce nonlinear control systems with dynamics

$$\dot{x} = f(\mathrm{x}) + \sum_{i=1}^{m} u_i(\mathrm{t})\,\mathrm{g}_i(\mathrm{x}) \qquad (1)$$

where $f, g_1, ..., g_m$ are continuously differentiable mappings $\mathbb{R}^n \to \mathbb{R}^n$. The other usual way would be to discuss time-variant linear control systems. In this paper we are interested in a third way: what happens if in the linear time-variant equation $\dot{x} = F(\mathrm{t})\mathrm{x}$ we let $F(\mathrm{t})$ be a finite linear combination of constant matrices with arbitrary time-varying coefficients That way leads to the following definition. Given constant matrices $A, B_1, ..., B_m$ in $\mathbb{R}^{n\times n}$ and controls $u \in U$ with $u(\mathrm{t}) \in \Omega \subset \mathbb{R}^m$

$$\dot{x} = Ax + \sum_{i=1}^{m} u_i(\mathrm{t})\mathrm{B}_i\mathrm{x}, \qquad (2)$$

will be called a bilinear control system on $\mathbb{R}^n$. Again the abbreviation $u := col(\mathrm{u}_1, ..., \mathrm{u}_m)$ will be convenient, and as will the list $\mathrm{B}_m := \{\mathrm{B}_1, ..., \mathrm{B}_m\}$ of control matrices. The term Ax is called the drift term. Given any of our choices of U, the differential equation (2) becomes linear time invariant and has a unique solution that satisfies (2) almost everywhere.A generalization of (2), useful beginning is to give control u as a function $\phi : \mathbb{R}^n \to \mathbb{R}^m; u = \phi(\mathrm{x})$ is then called a feedback control. In that case, the existence and uniqueness of solutions to (2) require some technical conditions on ϕ that will be addressed when the subject arises.

Time invariant bilinear control systems $\dot{x} = A(\mathrm{t})\mathrm{x} + \mathrm{u}(\mathrm{t})\mathrm{B}(\mathrm{t})\mathrm{x}$ have been discussed in Isidori and Ruberti [6] and in works on system identi_cation, but are not covered here. Bilinear systems without drift terms and with symmetric constraints

$$\dot{x} = (\sum_{i=1}^{m} u_i(\mathrm{t})\mathrm{B}_i)\mathrm{x}, \quad \Omega = -\Omega, \qquad (3)$$

are called symmetric bilinear control systems. Among bilinear control systems, they have the most complete theory. A drift term Ax is just a control term $u_0 B_0 x$ with $B_0 = A$ and the constant control $u_0 = 1$. The control theory of systems with drift terms is more difficult and incomplete;

Bilinear systems were introduced in the U.S. by Mohler; see his paper with Rink [14]. In the Russian control literature Buyakas, Barbain and others wrote about them. Mathematical work on (2) began with Kucera and gained impetus from the e_orts of Brockett [17]. Among the surveys of the earlier literature on bilinear control are Bruni et al, Mohler [9, 11], and Elliott [17] as well as the proceedings of seminars organized by Mohler and Ruberti [15, 14].

Mathematical models arise in hierarchies of approximation. One common nonlinear model, which often can be understood only qualitatively and by computer simulation, so simple approximations are needed for design and analysis. Near a state ζ with $f(\zeta) = \mathrm{a}$, the control system (1) can be linearized in the following way. Let

$$A = \frac{\partial f}{\partial x}(\zeta), \quad b_i = g_i(\zeta), \quad B_i = \frac{\partial g_i}{\partial x}(\zeta) \qquad (4)$$

$$\dot{x} = Ax + a + \sum_{i=1}^{m} u_i(\mathrm{t})(\mathrm{B}_i\mathrm{x} + \mathrm{b}_i), \quad \mathrm{x}, \mathrm{a}, \mathrm{b}_i \in \mathbb{R}^n \qquad (5)$$

to first order in x . A control system described by (5) can be called an inhomogeneous bilinear system; (5) has a_ne vector fields $(Bx + b)$, so I prefer to call it a biaffine control system.

Many of the applications mentioned to the biaffine (inhomogeneous bilinear) control systems introduced. Mohlers books [10] contain much about biaffine systems-controllability, optimal control, and applications, using control engineering methods. The general methods of stabilization given in Isidori can be used for biaffine systems for which the origin is not a fixed point This section is primarily about some aspects of the system (5) and its discrete time analogs that can be studied by embedding them as homogeneous bilinear systems in $\mathbb{R}^{n+1}$.

The observability theory of (5) began with the 1968 work of Rink and Mohler , [9],where the term "bilinear systems" was first introduced. Their method was strengthened in Khapalov and Mohler [7].

the idea is to find controls that provide $2m + 1$ equilibria of (5) at which the local uncontrolled linear dynamical systems have hyperbolic trajectories that can enter and leave neighborhoods of these equilibria (approximate control) and to find one equilibrium at which the Kalman rank condition holds.

Biaffine control systems can be studied by a Lie group method exemplified in the 1982 paper of Bonnard et al. [17].

2 GRAPH TERMINOLOGY

In this section, we deal with digraph approach for biaffin systems $(\sum_\lambda)$. Next, some useful notations and definitions are given..

2.1 Digraph definition for structured biaffine system

We now using the graph-theoretic approach to study nonlinear systems. we use the digraph is that quite close to the one presented in [8]. In this section the definition of we associate to $(\sum_\lambda)$ is noted $G(\sum_\lambda)$ and is constituted by a vertex set ν and an edge set ε i.e. $G(\sum_\lambda)=(\nu,\varepsilon)$. The vertices are associated to the state and the output components of $(\sum_\lambda)$ and the directed edges represent links between these variables.

More precisely, $\nu = X \cup Y$, where $X = \{x_1,...,x_n\}$ is the set of state vertices, $Y = \{y_1,...,y_p\}$ is the set of output vertices. The edge set is $\varepsilon = \{(\cup_{l=0}^{m} A_l - \text{edges}) \cup (C - edges)\}$, where, for $i = 0,1,...,m, A_i - edges = \{(x_j, x_i) \mid A_l(i,j) \neq 0\}$, and $C - edges = \{(x_j, y_i) \mid C(i,j) \neq 0\}$. Here $M(i,j)$ is the (i,j)th element of matrix M and (v_1, v_2) denotes a directed edge from vertex $v_1 \in \nu$ to vertex $v_2 \in \nu$. Moreover, for $i \in 0,1,...,m$, to each edge e $e \in A_i - edges$ we associate index i . Note that number i is indicated under each $A_i - edges$ in order to preserve the information about the belonging of the edges in the digraph representation. Note that we associate several indexes to an edge e if it belongs to several subsets $A_i - edges$. The following example illustrates the proposed digraph representation.

2.2 Example 1.

In Figure 1, we represent the digraph associated to the BIS defined by:

$$A_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \lambda_1 \\ 0 & 0 & \lambda_2 & 0 & 0 & 0 \\ 0 & \lambda_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_4 & 0 & 0 \\ \lambda_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad A_1 = \begin{bmatrix} \lambda_6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_7 & \lambda_8 & \lambda_9 & 0 \\ 0 & 0 & 0 & 0 & \lambda_{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_{13} & 0 & \lambda_{14} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_{15} & 0 \end{bmatrix} \qquad B = \begin{bmatrix} \lambda_{16} & 0 & 0 \\ \lambda_{17} & \lambda_{18} & \lambda_{19} \\ 0 & 0 & 0 \\ 0 & \lambda_{20} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

We study the state observability we can remove, the input vertices in the digraph so with this removal, the edges, start in our digraph from state vertices and constitute the Ai-edges. so, as we are interested in structured systems, each edge represents a free parameter which has no numerical value.

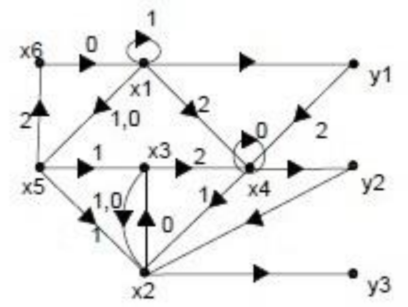

Figure 1: Digraph for the study of observability

3 PRELIMINARIES

In this paper, we explain biaffine in the form:

$$(\Sigma_\lambda) = \begin{cases} \dot{x}(t) = Ax + a + \sum_{i=1}^{m} u_i(t)(B_i x + b_i), \quad x, a, b_i \in \mathbb{R}^n \\ y(t) = Cx(t) \end{cases} \tag{6}$$

where $x(t) = (x_1(t), ..., x_n(t))^T \in \mathbb{R}^n, u(t) = (u_1(t), ..., u_m(t))^T \in \mathbb{R}^m$ and $y(t) = (y_1(t), ..., y_p(t))^T \in \mathbb{R}^p$ are respectively the state, the input and the output vectors. $i = 0,1,...,m, A_i \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times p}$ and $C \in \mathbb{R}^{n\times p}$ are matrices which elements are either fixed to zero or assumed free non-zero parameters. We can parameterize these nonzero entries by scalar real (nonzero) parameters $\mu_i, i = 1,...,k,$ forming a parameter vector $\lambda = (\mu_1, \mu_2, ..., \mu_h)^T \in \mathbb{R}^k$. If all the parameters (Σ_λ) are fixed, we obtain an admissible realization of structured system (Σ_λ). Theoretic properties of each realization can be studied according to the values of μ_i.

We say that a property is true generically [3] if it is true for almost all the realizations of structured system (Σ_λ). Here for almost all the realizations is to be understood [5] as for all parameter values $(\lambda \in \mathbb{R}^k)$ except for those in some proper algebraic variety in the parameter space.

Definition 1: Structured biaffine system (Σ_λ) is generically observable if exists an input $u(t)$ such that any pair of initial states $x^0(0)$ and $x^1(0)$ are distinguishable by observation of the corresponding outputs $y^0(t)$ and $y^1(t)$ for $t \geq 0$.

For almost all the realisations of (Σ_λ), A BAS is observable the generic observability of Biaffine for piecewise constant or continuous controls iff there exists an input $u(t)$ for which it is u-observable. This leads to the following observability criterion for Biaffine Systems.

Theorem 2. For piecewise continuous input signals $u(t)$, structured biaffine system (Σ_λ) is generically observable iff

T1: $g-rank(O(C, A_0, A_1, ..., A_m)) = n$,where

$$O(C, A_0, A_1, ..., A_m) = col(C, CA_0, CA_1, ..., CA_m, \\ CA_0^2, CA_0A_1, ..., CA_0A_m, CA_1A_0, ..., CA_m^{n-1})$$

is the observability matrix of system (Σ_λ) and where $g-rank(M)$ denotes the generic rank of matrix M [16]. The proof is based on the construction of an universal input $\bar{u}$, which distinguishes all initial states $x^1(0)$ from $x^0(0) = 0$ by concatenating at most $n+1$ constant inputs: $u = u^0 \circ u^1 \circ ... \circ u^n$. At the k^{th} stage in the construction, the set of states indistinguishable from x(0) = 0 is reduced in dimension by well chosen input value u_k .

3.1 Definitions and notations

We begin some important de_nitions concerning particular tools, For the classical de_nitions related to the digraphs [1].

- A class of biaffine systems is said to be output-connectable (or output-reachable) if for every state vertex, a path from this state vertex to at least one of the output vertices.
- In bia_ne system a path P is an $Y-topped$ path if its end vertex is an element of Y .
- Two edges e_1, e_2 are elements of ε so that $e_1 = (\mathrm{v}_1, \mathrm{v}_1')$ and $e_2 = (\mathrm{v}_2, \mathrm{v}_2')$ then edges are $v-disjoint$ if $\mathrm{v}_1 \neq \mathrm{v}_2$ and $\mathrm{v}_1' \neq \mathrm{v}_2'$. Note that edges can be $v-disjoint$ even if $\mathrm{v}_1' = \mathrm{v}_2$ or $\mathrm{v}_1 = \mathrm{v}_2'$. therefor k edges are $v-disjoint$ if edges are mutually $v-disjoint$.
- let k edges $e_1 = (\mathrm{v}_1, \mathrm{v}_1'), e_1 = (\mathrm{v}_2, \mathrm{v}_2'), ..., e_1 = (\mathrm{v}_k, \mathrm{v}_k')$. We define for system graph $i = 1, 2, ..., k, I_i$ as the set of integers j and $\mathrm{v}_j' = \mathrm{v}_i'$ i.e. $I_i = \{1 \leq \mathrm{j} \leq \mathrm{k} \mid \mathrm{v}_j' = \mathrm{v}_i'\}$
  $e_1, e_2, ..., e_k$ are $A-disjoint$ if C_1 and C_2 are satisfied, where:
  C_1: edges $e_1, e_2, ..., e_k$ in system have distinct end vertices.
  C_2: for $i = 1, 2, ..., k, (I_i = i)$ or (there exist r distinct integers $i_1, i_2, ..., i_r$ such that $e_{j1} \in A_{i1}-edges, e_{j2} \in A_{i2}-edges, ..., e_{jr} \in A_{ir}-edges$, , where $j_1, \mathrm{j}_2, ..., \mathrm{j}_r$ are all the elements of I_i).
  - in system graph k edges are $A-disjoint$ if their begin vertices are all distinct and if all the edges which have the same end vertex can be associated to distinct indexes i .

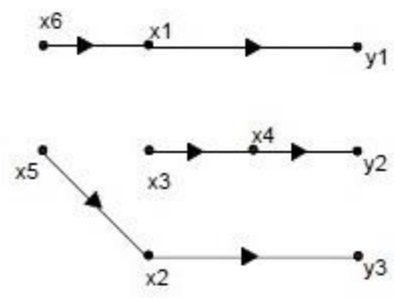

Figure 2: A set of $6\,A-disjoint$ edges

4 AFFINE GROUPS

Note that $\mathbb{R}^n$, with addition as its group operation, is an Abelian Lie group.

Definition 3. If $G \subset GL(n, \mathbb{R})$, then the semidirect product of G and $\mathbb{R}^n$ is a Lie group $, G := \mathbb{R}^n \propto G$ whose underlying set is $\{(v, X) \mid v \in \mathbb{R}^n, X \in G\}$ and whose group operation is $(\mathrm{v}, \mathrm{X}).(\mathrm{w}, \mathrm{Y}) := (\mathrm{v} + \mathrm{Xw}, \mathrm{XY})$ [17].

Corollary 4.

- This is standard representation on $\Re^{n+1}$ is

$$\hat{G} := \{\hat{\mathrm{A}} := \begin{bmatrix} A & a \\ 0_{1,n} & 1 \end{bmatrix} \mid \mathrm{A} \in \mathrm{G}, \mathrm{a} \in \mathbb{R}^n\} \subset \mathrm{GL}(\mathrm{n}+1, \mathbb{R}).$$

- The Lie group of all a_ne transformations $x \to Ax + a$ is $\mathbb{R}^n.GL(n,\mathbb{R})$, whose standard representation is called $\mathrm{Aff}(n,\mathbb{R})$.
- The Lie algebra of $\mathrm{Aff}(n,\mathbb{R})$ is the representation of $\mathbb{R}^n.\mathrm{gl}(n,\mathbb{R})$ and is explicitly

$$\mathrm{aff}(n,\mathbb{R}) = \{\hat{X} := \begin{bmatrix} X & x \\ 0_{1,n} & 0 \end{bmatrix} \mid x \in GL(n,\mathbb{R}), x \in \mathbb{R}^n\} \subset GL(n+1,\mathbb{R}).$$

$$\hat{X},\hat{Y} = \begin{bmatrix} X & x \\ 0_{1,n} & 0 \end{bmatrix}\begin{bmatrix} Y & y \\ 0_{1,n} & 0 \end{bmatrix} = \begin{bmatrix} X,Y & Xy - Yx \\ 0_{1,n} & 0 \end{bmatrix}$$

With the help of this representation, one can easily use the two canonical projections[17]

$$\pi_1 : \hat{G} \to \mathbb{R}^n \qquad \pi_2 : \hat{G} \to G$$

$$\pi_1 = \begin{bmatrix} A & a \\ 0_{1,n} & 1 \end{bmatrix} = a \qquad \pi_1 = \begin{bmatrix} A & a \\ 0_{1,n} & 1 \end{bmatrix} = A$$

Lemma 5. Suppose that $z := \mathrm{col}(x_1, x_2, ..., x_n, x_{n+1})$ is state vector and for which the hyper-plane $L := \{z \mid x_{n+1} = 1\}$ is invariant Then the affine group represent as a bilinear system[17]

$$\dot{z} = \hat{A}z + \sum_{i=1}^{m} u_i \hat{B}_i z, \quad for \quad \hat{A} = \begin{bmatrix} A & a \\ 0_{1,n} & 0 \end{bmatrix}, \hat{B} = \begin{bmatrix} B & b_i \\ 0_{1,n} & 0 \end{bmatrix} \tag{7}$$

Lemma 6. let $\dot{x} = Ax + ub$ Then For its show in $\mathrm{Aff}(n,\mathbb{R})$,have that[17]

$$z := col(x_1, x_2, ..., x_n, x_{n+1}), \quad \dot{z} = \hat{A}z + u(t)\hat{B}z, \quad z(0) = \begin{bmatrix} \zeta \\ 1 \end{bmatrix}, where$$

$$\hat{A} = \begin{bmatrix} A & 0_{n,1} \\ 0_{1,n} & 0 \end{bmatrix} \quad \hat{B}_i = \begin{bmatrix} 0_{n,n} & b \\ 0_{1,n} & 0 \end{bmatrix} \quad ad_{\hat{A}}^{k}(\hat{B}) = \begin{bmatrix} 0_{n,n} & A^k b \\ 0_{1,n} & 0 \end{bmatrix}$$

5 MAIN RESULT

In this section we provide necessary and sufficient condition for generic observability of system (Σ_λ).

Proposition 4: Structured biaffine system (Σ_λ) is generically observable iff in its associated digraph $G(\Sigma_\lambda)$

i.every state vertex is output-connectable that mean the begin vertex of an $Y-topped$ path,

ii.there exist n $A-disjo\,\mathrm{int}$ edges in $G(\Sigma_\lambda)$.

We continue by proving the sufficiency, i.e., If both the conditions (i.) and (ii.) are met, then the class of systems $[C, A]$ will be shown to be structurally observable.

In the case of Figure 2 represents 6 $A-disjo\,\mathrm{int}$ edges extracted from digraph of Figure 1. so for every the state vertices are begin vertices ,there is at least one path from state-vertex to least one of the out-put vertices.

We can study to give simpler graphic conditions based on bipartite graphs for the linear case in[16].

Corollary 7. Structured biaffine system (Σ_λ) is generically observable iff in its associated digraph $G(\Sigma_\lambda)$

i. every state vertex is the begin vertex of an $Y-topped$ path;

ii. $\text{g}-\text{rank}(\text{col}(C, A_0, A_1, ..., A_m)) = n$ or equivalently, the maximal matching of the bipartite graph associated to matrix $\text{col}(C, A_0, A_1, ..., A_m))$ is equal to n.

we aim solution has an acceptable computational burden and so is very well to large scale systems. The previous result can be seen as a generalization of the results concerning linear structured systems recalled in [16].

Proposition6 A structured linear system,

$$(\Sigma_{L,\lambda}) = \begin{cases} \dot{x}(\text{t}) = \text{Ax(t)} + \text{Bu(t)} \\ y(\text{t}) = \text{Cx(t)} \end{cases}$$

is generically observable iff in its associated graph $\text{G}(\Sigma_{L,\lambda})$

i. every state vertex is the begin vertex of an $Y-topped$ path,

ii. the maximal matching of the bipartite graph associated to matrix $col\,(\text{C}, \text{A})$ is equal to n.

6 EXAMPLE

In this section, we can results be easily explained with an example. It is clear that we using of Proposition 4 and Corollary 5 are very well known to more complex or large-scale systems using combinatorial programming techniques. Consider the biaffine systems represented in the digraph of Figure 3. This system has 21 state components. The matrix representation of this system is not given because of lack of place. Nevertheless, it can be easily deduced from the digraph. Our aim is to show the simplicity of the proposed method on a relatively large example. The proposed conditions can be checked easily by hand whereas the rank test of the observability matrix is quite difficult to do by hand. In fact, this system is observable because system is output-connect and there exist 21 $A-disjo\,\text{int}$ edges

again vertex of an $Y-topped$ paths and there exist 21 $A-disjo\,\text{int}$ edges as it is displayed in Figure 4.

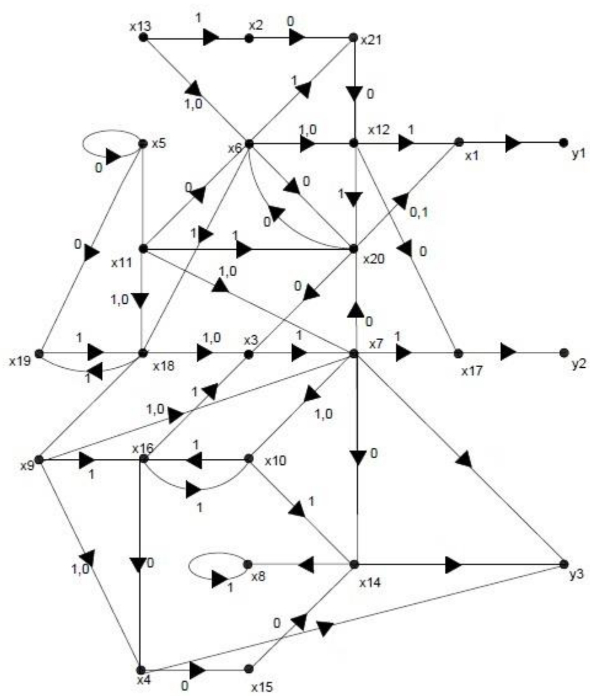

Figure 3: Illustrative example

7 Conclusion

In this paper, we have characterize a new analysis to explain the observability of bilinear systems. Using the basic de_nitions and Theorems concerning Lie algebras and study matrix Lie algebras, therefore we can slove class of nonlinear systems, finally we explain necessary and sufficient conditions for observability in the biaffine systems and so we need few information about the system that can comfortable to check techniques for small system by hand and so this way is suitable for large-scale system. Furthermore, we can use of a graph-theoretic method for makes it comfortable to obsrvation the system structure. This may be very helpful for the optimisation of sensor placement to achieve the observability of the system.

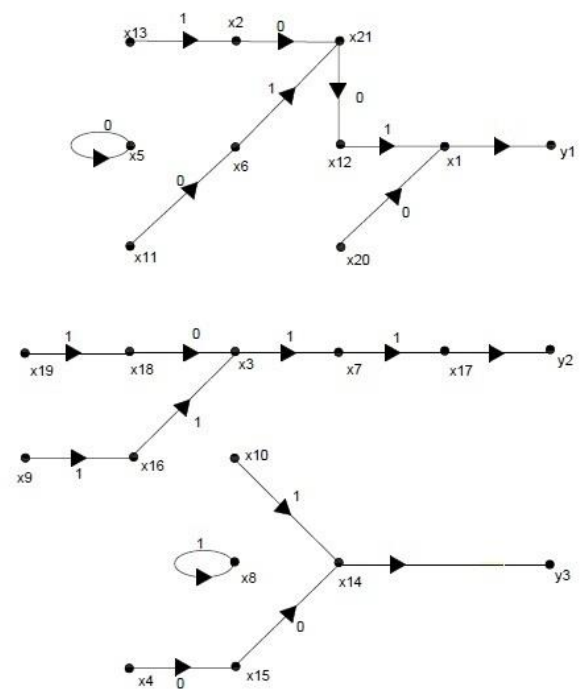

Figure 4: A set of $21\,A - disjo$ int edges

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