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Minimum Edge Dominating Distance Energy of a Graph

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ABSTRACT

In this paper we introduce a new kind of graph energy, the minimum edge dominating distance energy, $E_{D'_d}(G)$. The edge distance energy is defined as the sum of the absolute values of the eigenvalues of its edge distance matrix. Upper and lower bounds for $E_{D'_d}(G)$ are established. Finally, we give lower bounds of the largest eigenvalue of G (= edge dominating distance spectral radius of graph G).

KEYWORDS: Minimum edge dominating set, Edge dominating distance matrix, Edge dominating distance eigenvalues, Graph energy

1 INTRODUCTION

The concept of energy of a graph was introduced by I.Gutman [5] in the year 1978. For more details on the mathematical aspects of the theory of graph energy see [6, 2, 18, 15]. The basic properties including various upper and lower bounds for energy of a graph have been established [9, 3]. Further, incidence energy, matching energy, minimum covering energy, minimum dominating energy, Laplacian minimum dominating energy of a graph G can be found in [8, 19, 1, 10, 7, 11, 12, 17]. Recently M. R. Rajesh Kanna and et al [13, 14, 16] defined the minimum covering distance Energy, $E_{Cd}(G)$ and minimum dominating distance Energy, $E_{Dd}(G)$ of a graph which depends on its particular minimum cover C and minimum dominating set D respectively. The distance matrix of G is the square matrix of order n whose (i, j)-entries the distance (= length of the shortest path) between the vertices v_i and v_j. Detailed studies on distance energy can be found in [20, 21, 22]. Motivated by these papers, we study the minimum edge dominating distance energy MEDDE of a graph G. We compute some properties of characteristic polynomial of a $MEDD$ matrix of a graph G. Upper and lower bounds are established. The edge distance matrix of G is the square matrix of order m whose (i, j)-entry is the distance (= length of the shortest path between two edges) between the edges e_i and e_j. $Let\ \mu_1, \mu_2, \dots, \mu_m$ be the eigenvalues of the edge distance matrix of G. The edge distance energy EDE is defined by

$$EDE = \sum_{i=1}^{m} |\mu_i|$$

Let $G = (V, E)$ be a simple finit graph that is have no loops no multiple and directed edges. Graph G has n vertices, m edges with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(G) = \{e_1, e_2, \ldots, e_m\}$. A subset of D' of $E(G)$ is called an edge dominating set of G if every edge of $E - D'$ is incident to at least one edge of D'. Any edge dominating set with minimum cardinality is called a minimum edge dominating set. The edge domination number $\gamma'(G)$ of G is the minimum cardinality of an edge dominating set . Let D' be a minimum edge dominating set of a graph G. The minimum edge dominating distance matrix of G is the $m \times m$ matrix $B_{D'd} = (d_{ij})_{m\times m}$ where

$$d_{ij} = \begin{cases} 1 & if i = j \; and \; e_i \in D' \\ d(e_i, e_j) & otherwise. \end{cases} \quad (1)$$

The characteristic polynomial of $B_{D'd}(G)$ is denoted by $f_m(G,\mu) = det(\mu I - B_{D'd}(G))$. The minimum edge dominating distance eigenvalues of the graph G are the eigenvalues of $B_{D'd}(G)$ Since $B_{D'd}(G)$ is real and symmetric, its eigenvalues are real numbers and label them in non-increasing order $\mu_1, \mu_2, \ldots, \mu_m$

2. Problem formulation and Some basic properties of minimum edge dominating distance

We first compute the minimum edge dominating distance energy for the Fig 1, then we obtain some important results.

Definition 2.1. [17] Let G be a connected graph and $e_1 = (u_1, v_1)$, $e_2 = (u_2, v_2)$ be two edges of G, the distance between edges e_1 and e_2 is defined as $ed(e_1, e_2) = min\{d(u_1, u_2),\; d(u_1, v_2),\; d(v_1, v_2),$ $d(v_1, v_2)\}$. If $ed(e_1, e_2) = 0$ then these edges are called neighbour edges.

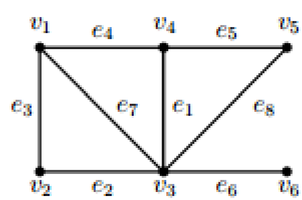

Figure 1: Graph G

Example 2. 2. The possible minimum dominating sets for the following graph G in Figure 1 are i) $D'_1 = \{e_1, e_3\}, ii) D'_2 = \{e_7, e_8\}, iii)\; D'_3 = \{e_2, e_5\}, and\; iv)\; D'_4 = \{e_8, e_3\}$

$(i) D'_1 = \{e_1, e_3\}$

$$B_{D'd}(G) = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Characteristic equation is $\mu^8 - 2\mu^7 - 9\mu^6 + 13\mu^5 + 17\mu^4 - 22\mu^3 - 1\mu^2 + 3\mu$
and Minimum edge dominating distance eigenvalues are

$$Spec(B_{D'd}(G)) = \begin{pmatrix} 3.2393 & 1.5575 & 1.0000 & 0.4498 & 0.0000 & -0.3591 & -1.6330 & -2.2546 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Minimum edge dominating distance energy, $E_{D'd}(G) = 10.4932$
$(ii) D'_2 = \{e_7, e_8\}$

$$B_{D'd}(G) = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Spec(B_{D'd}(G)) = \begin{pmatrix} 3.0461 & 1.6416 & 1.0000 & 0.7599 & 0.0000 & -0.3852 & -1.8717 & -2.1904 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Minimum edge dominating distance energy, $E_{D'd}(G) = 10.8947$

Thus minimum dominating distance energy of graph G depends on the dominating set.

Theorem 2.3. For any integer $n \geq 3$, the minimum edge dominating distance energy of star graph $k_{1,n-1}$ is equal to (1).

Proof. Let $k_{1,n-1}$be a star graph with edge set $E = \{e_1, e_2, \dots, e_m\}$ and the minimum edge dominating set D'can be a A single-member set of each of the edges that is $D' = \{e_i\}, i = 1,2, \dots, m$. Then

$$B_{D'd}(k_{1,n-1}) = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & & 0 \\ & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}_{m \times m}$$

Then its characteristic polynomial is $f_m(G,\mu) = \mu^m - \mu^{m-1} = \mu^{m-1}(\mu - 1)$. Eigen values are
$Spec(B_{D'd}(G)) = \begin{pmatrix} 1 & 0 \\ 1 & m-1 \end{pmatrix}$

So, the minimum edge dominating distance energy $E_{D'd}(G) = 1$

Theorem 2.4. Let G be a graph with verter set V, edge set E, and the minimum edge dominating distance set D'.

Let $f_m(G,\mu) = \det(\mu I - B_{D'd}(G) = c_0\mu^m + c_1\mu^{m-1} + c_2\mu^{m-2} + \cdots + c_m$ be the charactrestic polynomial of G.

Then

$(i) c_0 = 1$

$(ii) c_1 = -|D'|$

$(iii) c_2 = -\binom{|D'|}{3} - \sum_{i<j} (d_{ij})^2$

$(iv) c_3 = -\binom{|D'|}{3} + \sum_{i=1}^{m-1} \left\{ \sum_{j>i}^{m} \left[(d_{ij})^2 \sum_{k=1, k\neq i,j} d_{kk} \right] \right\} - 2 \sum_{1\le i<j<k\le m} d_{ij} d_{jk} d_{ik}$

Proof. (i) Directly from the definition of $f_m(\mathrm{G},\lambda)$, it follows that $c_0 = 1$.

(ii) Since the sum of diagonal elements of $B_{D'd}(G)$ is equal to $|D'|$, sum of determinants of all 1×1 principal submatrices of $B_{D'd}(G)$ is the trace of $B_{D'd}(G)$ which evidently equal to $|B_{D'd}(G)|$. Thus $(-1)^1 c_1 = |D'|$

(iii) $(-1)^2 c_2$ is equal to sum of detrminants of all the 2×2 principal sub matrices of $B_{D'd}(G)$, that is

$$c_2 = \sum_{1\le i<j\le m} \begin{vmatrix} d_{ii} & d_{ij} \\ d_{ji} & d_{jj} \end{vmatrix} = \sum_{1\le i<j<k\le m} (d_{ii}d_{jj} - d_{ij}d_{ji}) = \sum_{1\le i<j<k\le m} d_{ii}d_{jj} - \sum_{1\le i<j<k\le m} d_{ij}^2$$

$$= \binom{|D'|}{2} - \sum_{1\le i<j<k\le m} (d_{ij})^2$$

$$c_3 = (-1)^3 \sum_{1\le i<j<k\le m} \begin{vmatrix} d_{ii} & d_{ij} & d_{ik} \\ d_{ji} & d_{jj} & d_{jk} \\ d_{ki} & d_{kj} & d_{kk} \end{vmatrix}$$

$$= -\sum_{1\le i<j<k\le m} \left[d_{ii}(d_{jj}d_{kk} - d_{kj}d_{jk}) - d_{ij}(d_{ji}d_{kk} - d_{ki}d_{jk}) + d_{ik}(d_{ji}d_{kj} - d_{ki}d_{jj}) \right]$$

$$= -\sum_{1\le i<j<k\le m} d_{ii}d_{jj}d_{kk} + \sum_{1\le i<j<k\le m} \left[d_{ii}d_{jk}d_{kj} + d_{ij}d_{ik}d_{ki} + d_{kk}d_{ij}d_{ji} \right]$$

$$- \sum_{1\le i<j<k\le m} d_{ij}d_{jk}d_{ki} - \sum_{1\le i<j<k\le m} d_{ik}d_{kj}d_{ji}$$

$$= -\sum_{1\le i<j<k\le m} d_{ii}d_{jj}d_{kk} + \sum_{1\le i<j<k\le m} \left[d_{ii}d_{jk}^2 + d_{jj}d_{ik}^2 + d_{kk}d_{ij}^2 \right] - 2 \sum_{1\le i<j<k\le m} d_{ij}d_{jk}d_{ik}$$

$$= -\binom{|D'|}{3} + \sum_{i=1}^{m-1} \left\{ \sum_{j>i}^{m} \left[(d_{ij})^2 \sum_{k=1, k\neq i,j} d_{kk} \right] \right\} - 2 \sum_{1\le i<j<k\le m} d_{ij}d_{jk}d_{ik}$$

Theorem 2.5. let G be a simple graph with vertex set V, edge set $E = \{e_1, e_2, \dots, e_m\}$ and D' be a minimum edge dominating set. If $\mu_1, \mu_2, \dots, \mu_m$ are eigenvalues of $B_{D'd}(G)$ then

$(i) \sum_{i=1}^{m} \mu_i \, |D'|$

$(ii) \sum_{i=1}^{m} \mu_i^2 = |D'| + 2\sum_{i<j}^{m} d_{ij}$

Proof. (i) we know the sum of squares of the eigenvalues of $B_{D'd}(G)$ is just the trace of $B_{D'd}(G)$ therefore

$$\sum_{i=1}^{m} \mu_i = \sum_{i=1}^{m} = |D'|$$

(iii) similarly, the sum of squares of the eigenvalues of $B_{D'd}(G)$ is trace of $[B_{D'd}(G)]$. Therefore,

$$\sum_{i=1}^{m} \mu_i^2 = \sum_{i=1}^{m}\sum_{j=1}^{m} d_{ij}d_{ji}$$

$$= \sum_{i=1}^{m} (d_{ij})^2 + \sum_{i\neq j}^{m} d_{ij}d_{ji}$$

$$= \sum_{i=1}^{m} (d_{ij})^2 + 2\sum_{i<j} d_{ij}$$

$$= |D'| + 2\sum_{i<j} d_{ij}$$

Theorem 2.6. (Upper bound) Let G be a graph with n vertices, m edges, and let D' be a minimum edge dominating distance of G. Then

$$E_{D'd}(G) \leq \sqrt{m\left(2\sum_{i<j} (d_{ij})^2 + |D'|\right)}$$

Theorem 2.7. (Lower bound) Let G be a graph with n vertices and m edges, and let D' be a minimum edge dominating distance set of G. If $P = |\det(B_{D'd}(G))|$, then

$$[E_{D'd}(G)]^2 \leq \sqrt{|D'| + \sum_{i<j} (d_{ij})^2 + m(m-1)P^{\frac{2}{m}}}$$

Theorem 2.8. (Parity theorem) Let G be a graph with a minimum edge dominating set D'. If the minimum edge dominating distance energy $E_{D'd}(G)$ of G is a rational number, then
$E_{D'd}(G) \equiv D'(mode\ 2)$.

Definition 2.9. [4] Let G be a connected graph. Then the edge-Wiener index of G is defined as the sum of the distances in the line graph) between all pairs of edges of G, i.e.

$$W_e(G) = \sum_{\{e_i, e_j\} \subseteq E} d_{ij},$$

Theorem 2.10. if $\mu_1(G)$ is the largest minimum edge dominating distance eigenvalue of $D_{D'd}(G)$ then

$$\mu_1(G) \geq \frac{W_e(G) + |D'|}{m}$$

Where $W_e(G)$ is the edge-Wiener index of G.

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