Proceedings of the 3nd International Conference on Combinatorics, Cryptography and Computation (I4C2018)

ON THE LEXICOGRAPHIC PRODUCT OF GRAPHS

REZVAN VARMAZYAR
DEPARTMENT OF MATHEMATICS, KHOY BRANCH,, ISLAMIC AZAD UNIVERSITY, KHOY 58168-44799, IRAN
varmazyar@iaukhoy.ac.ir

ABSTRACT

Let G_1 and G_2 be two graphs. The lexicographic product of G_1 and G_2, G_1 o G_2, has vertex set $V(G_1 \text{ o } G_2) = V(G_1) \times V(G_2)$ and edge set $E(G_1 \text{ o } G_2) = \{(a, b)(c, d) \backslash ac$ e $E(G_1)$ *or* $a = c$ *and* bd e $E(G_2)\}$. In this talk we give some properties of G_1 o G_2.

KEYWORDS: lexicographic product, super vertex-cut, super connectivity.

1 INTRODUCTION

For the graph terminology and notation we refer to [4]. Throughout this article, a graph always means a finite undirected with no loops and no multiple edges.

Let G be a graph and S be a non empty subset of $V(G)$. If $G - S$ is disconnected then S is called vertex-cut. The connectivity of G, $k(G)$, is defined as the minimum cardinality of S such that $G - S$ is disconnected or a graph with a single vertex. Also, a vertex-cut S is called a super vertex-cut if $G - S$ contains no isolated vertices. The corresponding index of super connectivity of G, $k_1(G)$, is defined as

$$k_1 = mm\{|S|;\ S \text{ is a super vertex} - \text{cut}\}.$$

2 SOME PROPERTIES OF LEXICOGRAPHIC PRODUCT

Proposition 2.1. Let G_1 be a totally disconnected graph and G_2 be a connected graph. Then G_1 o G_2 has $|G_1|$ components and each component is isomorphic to G_2.

Proposition 2.2. Let G_1 be k-regular and G_2 be k'-regular graphs. Then G_1 o G_2 is ($k' + k|G_2|$) -regular.

Proposition 2.3. Let G_1 and G_2 be graphs. Then

$$E(G_1 \text{ o } G_2) = |G_1||E(G_2)| + |G_2||E(G_1)|.$$

Theorem 2.4. Let G_1 be a connected graph. Then G_1 is a minor of G_1 o G_2, for every graph G_2.

Corollary *2.5.* Let G_1 be a connected graph and G_2 be a totally disconnected graph. Then G_1 o G_2 is an m—partite graph where m is the clique number of G_1. Furthermore, if G_1 is complete then G_1 o G_2 is a complete m—partite graph .

3. SUPER-CONNECTIVITY OF Gi o G_2

Let G_1 be a connected graph (not complete) with $k(G_1) = n$ and a minimum vertex-cut $X = \{x_1, \ldots, x_n\}$. It is easy to check that if $V(G_2) = \{y_1, \ldots, yt]$, then

$$X = \{(x_1, y_1), \ldots, (x_1, y_t), (x_n, y_1), \ldots, (x_n, y_t)\}$$ is a minimum vertex-cut in G_1 o G_2.

Proposition *3.1. Let G_1 and G_2 be connected graphs. Then*

$$k_1(G_1 \text{ o } G_2) = k(G_1 \text{ o } G_2).$$

Proposition *3.2. Let G_1 be a connected graph and G_2 be a disconnected graph. If $k1(G1) = k(G1)$ then $k(G_1 \text{ o } G_2) = k(G_1 \text{ o } G_2)$.*

Proposition *3.3. Let G_1 be a connected graph and G_2 be a disconnected graph. If $k(G_1) < k_1(G_1) < \infty$ then*

$$k_1(G \text{ o } G) = k(G \text{ o } G) + (|k_1(G1)| - |k(G1)|)m_0(G2).$$

Proposition *3.4. Let G_1 be a connected graph with minimum vertex-cut X and G_2 be a disconnected graph. If $k_1(G_1)$ = to then*

$$k_1(G \text{ o } G) = k(G \text{ o } G) + mo(G - X)mo(G2).$$

Corollary *3.5.* Let G_1 be a connected graph and G_2 be a disconnected graph without single vertex. If $k_1(G_1)$ = *to* then $k_1(G_1 \text{ o } G_2) = k(G_1 \text{ o } G_2)$

REFERENCES

[1] W. S. CHiue and B. S. Shieh, On connectivity of the cartesian product of two graphs, Appl Math Comput 102, 1, (1999, 129-137.

[2] M. A. Fiol, J. fabrega and M. Escudero, Short paths and connectivity in graphs and digraphs, Ars Combin 29, (1990), 17-31.

[3] M. Lu anJ. M. Xu, Super connectivity of line graphs, Acta Math Appl Sinica, (2006), no. 1, 43-48.

[4] J. M. Xu, Theory and application of graphs , kluwer, Dordrecht, (2003).