Iran University of Science &Technology

3TH INTERNATIONAL CONFERENCE ON COMBINATORICS, CRYPTOGRAPHY AND COMPUTATION

Proceedings of the 3rd International Conference on Combinatorics, Cryptography and Computation (I4C2018)

Energy of Some Extended Corona

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ABSTRACT

For two graphs G_1 and G_2 with n and m vertices, the corona $G_1 \circ G_2$ of G_1 and G_2 is defined as the graph obtained by taking one copy of G_1 and n copies of G_2, and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2. The neighborhood corona of $G_1$and G_2, denoted by $G_1 \star G_2$, is the graph obtained by taking one copy of G_1 and n copies of G_2, and joining every neighbour of the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2. In this paper we define extended corona, extended neighborhood corona, identity extended corona and identity extended neighbourhood corona of two graphs G_1 and G_2, which are denoted by $G_1 \bullet G_2$, $G_1 * G_2$, $I_{ex}(G_1 \circ G_2)$ and $I_{ex}(G_1 \star G_2)$ respectively. We compute an upper bound for their energy and the energy of the graph G is denoted by $\varepsilon(G)$.

KEYWORDS: corona, neighborhood corona, identity extended corona, identity extended neighbourhood corona and energy of graph.

1 INTRODUCTION

Throughout this paper, we consider only simple graphs, i.e, an undirected graph with no loops and no multiple edges. Let G be a graph with vertex set $V(G) = \{v_1, v_2, \dots, v_n\}$. The adjacency matrix of G, denoted by $A(G)$, is defined as $A(G) = (a_{ij})_{n\times n}$, where

$$a_{ij} = \begin{cases} 1, & if\ v_i v_j\ is\ an\ edge\ in\ G, \\ 0, & otherwise. \end{cases}$$

The degree of a vertex v_i in G, denoted by $\deg(v_i)$ is the number of vertices that are adjacent to v_i in G. The Laplacian matrix $L(G)$ of G is defined as $L(G) = D(G) - A(G)$ and the signless Laplacian matrix $Q(G)$ of G is given by $Q(G) = D(G) + A(G)$, where $D(G) = diag(\deg(v_1), \dots, \deg(v_n))$.

The sum $\varepsilon(G) \coloneqq \sum_{i=1}^{n} |\lambda_i(G)|$ is known as the energy of the graph G, where $\lambda_i(G)$ are the eigenvalues of $A(G)$. The concept of the energy of a graph was introduced by Gutman [3] and was recently generalized to oriented graphs as skew energy by Adiga, Balakrishnan and So in [1]. If $\lambda_i(G)$

$(\mathrm{i} = 1, 2, \dots, \mathrm{n})$ are all integers, then G is said to be an integral graph. The notion of integral graphs was first introduced by Harary and Schwenk in 1974 [14].

Let G_1 and G_2 be two graphs on disjoint sets of n and m vertices, respectively. The corona $G_1 \circ G_2$ of G_1 and G_2 is defined as the graph obtained by taking one copy of G_1 and n copies of G_2, and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2. The corona of two graphs was first introduced by Frucht and Harary in [10]. The neighborhood corona of G_1 and G_2, denoted by $G_1 \star G_2$, is the graph obtained by taking one copy of G_1 and n copies of G_2, and joining every neighbour of the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2. The neighborhood corona was introduced in [16].

In this paper, motivated by [2]; we define two types of corona namely, extended corona and extended neighborhood corona and two new types of corona namely, identity extended corona and identity extended neighborhood corona of two graphs. We compute an upper bound for their energy.

2 PRELIMINARIES

In this section, we need to state some results which will be used frequently later. Let G_1, G_2 be two graphs. G_1 is an arbitrary graph and G_2 is a r-regular graph and $V(G_1) = \{v_1, v_2, \dots, v_n\}$, $V(G_2) = \{u_1, u_2, \dots, u_m\}$ and $V(G_1 \circ G_2) = V(G_1 \star G_2) = \{v_1, v_2, \dots, v_n, v_{1_1}, \dots, v_{1_m}, v_{2_1}, \dots, v_{2_m}, \dots, v_{n_1}, \dots, v_{n_m}\}$ be vertices set of G_1, G_2 , $G_1 \circ G_2$ and $G_1 \star G_2$, where v_{i_k} is considered the same as u_k in the i^{th} copy of G_2. Let $\sigma(i) = i$ for each $\mathrm{i} \in \{1, 2, \dots, \mathrm{n}\}$ be a identity permutation. We define extended corona and extended neighborhood corona, identity extended corona, identity extended neighborhood corona of two graphs G_1 and G_2 as follows:

Definition 2.1.
The extended corona $G_1 \bullet G_2$ of two graphs G_1 and G_2 is a graph obtained by taking the corona $G_1 \circ G_2$ and joining each vertex of i^{th} copy of G_2 to every vertex of j^{th} copy of G_2, provided the vertices v_i and v_j are adjacent in G_1.

Definition 2.2.
The extended neighborhood corona $G_1 * G_2$ of two graphs G_1 and G_2 is a graph obtained by taking the neighborhood corona $G_1 \star G_2$ and joining each vertex of i^{th} copy of G_2 to every vertex of j^{th} copy of G_2, provided the vertices v_i and v_j are adjacent in G_1.

Definition 2.3.
The identity extended corona $I_{ex}(G_1 \circ G_2)$ of two graphs G_1 and G_2 is a graph obtained by taking the corona $G_1 \circ G_2$ and joining the vertex v_{i_k} of i^{th} copy of G_1 to the vertex v_{j_k} of j^{th} copy of G_2, provided the vertices v_i and v_j are adjacent in G_1.

Definition 2.4.
The identity extended neighborhood corona $I_{ex}(G_1 \star G_2)$ of two graphs G_1 and G_2 is a graph obtained by taking the neighborhood corona $G_1 \star G_2$ and joining the vertex v_{i_k} of i^{th} copy of G_1 to the vertex v_{j_k} of j^{th} copy of G_2, provided the vertices v_i and v_j are adjacent in G_1.

3 CONCLUSION

Theorem 3.1
The energy of the extended corona $G_1 \bullet G_2$:

$$\varepsilon(G_1 \bullet G_2) = \sum_{\lambda_i \in spec(G_1 \bullet G_2)} |\lambda_i(G_1 \bullet G_2)| \leq$$

$$\frac{1}{2}(m+1)\varepsilon(G_1) + n\varepsilon(G_2) - \frac{n}{2}r + \frac{1}{2}\sqrt{2n(m-1)^2|E(G_1)| + (rn)^2 + 2rn(m-1)\varepsilon(G_1) + 4nm}$$

where $E(G_1)$ is edges set of G_1.

Theorem 3.2
The energy of the extended neighborhood corona $G_1 * G_2$:

$$\varepsilon(G_1 * G_2) = \sum_{\lambda_i \in spec(G_1 * G_2)} |\lambda_i(G_1 * G_2)| \leq$$

$$\frac{1}{2}(m+1)\varepsilon(G_1) + n\varepsilon(G_2) - \frac{n}{2}r + \frac{1}{2}\sqrt{2n(m+1)^2|E(G_1)| + (rn)^2 + 2rn(m-2)\varepsilon(G_1)}$$

where $E(G_1)$ is edges set of G_1.

Theorem 3.3
The energy of the identity extended corona $I_{ex}(G_1 \circ G_2)$:

$$\varepsilon\big(I_{ex}(G_1 \circ G_2)\big) = \sum_{\lambda_i \in spec(I_{ex}(G_1 \circ G_2))} |\lambda_i(I_{ex}(G_1 \circ G_2))| \leq mE(G_1) + nE(G_2) - \frac{n}{2}r + \frac{n}{2}\sqrt{r^2 + 4m}$$

Theorem 3.4
The energy of the identity extended neighborhood corona $I_{ex}(G_1 \star G_2)$:

$$\varepsilon\big(I_{ex}(G_1 \star G_2)\big) = \sum_{\lambda_i \in spec\left(I_{ex}(G_1 \star G_2)\right)} \left|\lambda_i\big(I_{ex}(G_1 \star G_2)\big)\right| \leq$$

$$m\varepsilon(G_1) + n\varepsilon(G_2) - \frac{n}{2}r + \frac{1}{2}\sqrt{(rn)^2 + 8nm|E(G_1)|}$$

where $E(G_1)$ is edges set of G_1.

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