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Further studies on edge-distance-balanced property of the generalized Petersen graphs *GP*(6*n* + 8,3)

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Abstract

A graph G is said to be edge-distance-balanced if for any edge uv of G, the number of edges closer to u than to v is equal to the number of edges closer to v than to u. Let $GP(n,k)$ be a generalized Petersen graph. It is proven that for any integers $n \geq 2$, the generalized Petersen graph $GP(6n + 8,3)$ is not edge-distance-balanced.

Key words: edge-distance-balanced graph , generalized Petersen graph, distance-balanced graph.

1 Introduction

Let G be a simple undirected graph and $V(G)$ $((E(G)))$ be it's vertex (edge) set.

The distance $d(u,v)$ between vertices u and v of G is the length of a shortest path between u and v in G.

For any edge uv in $E(G)$, let W_{uv} denote the set of all vertices of G closer to u than to v, that is

$W_{uv} = \{x \in V(G) \mid d(u,x) < d(v,x)\}$.

Similarly, let ${}_uW_v$ be the set of all vertices of G that are at the same distance to u and v, that is $uW_v = \{x \in V(G) \mid d(u,x) = d(v,x)\}$.

A graph G is called distance-balanced (DB for short) if $|Wuv| = |Wvu|$.

holds for any edge uv in $E(G)$.

Also graph G is called edge-distance-balanced [23] (EDB for short) if $|E_{uv}^G| = |E_{vu}^G|$, where

$E_{uv}^G = \{e \in E(G) \mid e$ is closer to the vertex u than the vertex $v\}$.

Similarly, we can define E_{vu}^G.

Also, consider the notion ${}_uE_v^G = \{e \in E(G) \mid$ the distance of e to both vertices a and b is the same $\}$.

For a vertex x of a connected graph G and $k \geq 0$, let $M_k(x) = \{e \in E(G) \mid d(x,e) = k\}$,

$M_k[x] = \{e \in E(G) \mid d(x,e) \leq k\}$.

For $k = 1$, we shorten these to $M(x)$ and $M[x]$. Here, denoted by $d(x,e)$ we mean the length of the shortest path between the vertex x and the edge e, i.e., the number of edges lying between the vertex x and the edge e in the shortest path.

The following result gives a different view to the definition of *EDB* graphs for regular graphs and was proven in [6].

Corollary 1 ([6], Corollary 2.3). *Let G be a regular graph of diameter d. Then G is EDB if and only if* $\sum_{k=1}^{d-1}|M_k(a)\backslash M_k[b]| = \sum_{k=1}^{d-1}|M_k(b)\backslash M_k[a]|$, *holds for every edge* $e = ab \in E(G)$.

The Petersen graph is an important graph in graph theory and has attracted much research throughout the years. Some recent research include ([24], [25]).

Kutnar et al. [17] studied the strongly distance-balanced property of the generalized petersen graphs and gave a result that:

For any integer $k \geq 2$ and $n \geq k^2 + 4k + 1$, the generalized Petersen graph $GP(n,k)$ is not strongly distance-balanced (strongly distance-balanced graph was introduced by Kutnar et al. in [18]).

Also Yang et al. [26] proved that: For any integer $k \geq 2$ and $n > 6k^2$, the generalized Petersen graph $GP(n,k)$ is not distance-balanced.

In this note, we prove the following theorem.

Theorem 1. *For any integer* $n \geq 2$, $GP(6n + 8,3)$ *is not edge-distance-balanced.*

2 Main results

Let $n \geq 3$ be a positive integer, and let $k \in \{1,...,n\text{-}1\}\backslash\{n/2\}$.

The generalized Petersen graph $GP(n,k)$ is defined to have the following vertex set and edge set: $V(GP(n,k)) = \{u_i | i \epsilon Z_n\} \cup \{v_i | i \epsilon Z_n\}$,

$E(GP(n,k)) = \{u_i u_{i+1} | i \epsilon Z_n\} \cup \{v_i v_{i+k} | i \epsilon Z_n\} \cup \{u_i v_i | i \epsilon Z_n\}$.

We call the cycle induced by the vertices $\{u_0,u_1,\cdots ,u_{n-1}\}$ the outer cycle of $GP(n,k)$, and the cycles induced by the vertices $\{v_0,v_1,\cdots ,v_{n-1}\}$ the inner cycles of $GP(n,k)$.

The edge u_iv_i $(0 \leq i \leq n - 1)$ is called a spoke of $GP(n,k)$.

Note that $GP(n,k)$ is cubic, and that it is bipartite precisely when n is even and k is odd.

It is easy to see that $GP(n,k) \sim GP(n,n - k)$.

In the following, we investigate the sets $M_1(u_0)\backslash M_1[v_0]$ and $M_1(v_0)\backslash M_1[u_0]$ of the graph $GP(6n + 8,3)$.

Lemma 1. Let $n \geq 2$ be an integer and let u_0v_0 be a spoke in $E(GP(6n + 8,3))$.

Then the following statements hold:

(i) $M_1(u_0)\backslash M_1[v_0] = \{ u_1u_2, u_1v_1, u_{-1}u_{-2}, u_{-1}v_{-1}\}$;

(ii) $M_1(v_0)\backslash M_1[u_0] = \{ u_3v_3, v_3v_6, u_{-3}v_{-3}, v_{-3}v_{-6}\}$.

Proof. By a careful inspection of the edges in the neighborhood of u_0 and v_0, we see that every edge at distance 1 from u_0 belongs to $\{ u_1u_2\ u_1v_1, u_{-1}u_{-2}, u_{-1}v_{-1}, v_0v_3, v_0v_{-3}\}$.

It is clear that the edges v_0v_3,v_0v_{-3} are in the shortest paths from u_0 such that pass through the edge u_0v_0 first and therefore are not in $M_1(u_0)\backslash M_1[v_0]$ and so (i) holds.

On the other hand the edges at distance 1 from v_0 are in $\{ u_3v_3, v_3v_6, u_{-3}v_{-3}, v_{-3}v_{-6}, u_0u_1, u_0u_{-1} \}$.

Notice that the edges u_0u_1,u_0u_{-1} are in the shortest paths from v_0 such that pass through the edge u_0v_0 first and therefore are not in $M_1(v_0)\backslash M_1[u_0]$.

This completes the proof.

□

In the next lemma we determine the sets $M_2(u_0)\backslash M_2[v_0]$ and $M_2(v_0)\backslash M_2[u_0]$ of the graph

$GP(6n + 8,3)$.

Lemma 2. *Let $n \geq 2$ be an integer and let u_0v_0 be a spoke in $E(GP(6n + 8,3))$. Then the following statements hold:*

(i) $M_2(u_0)\backslash M_2[v_0] = \{ v_1v_4, u_2v_2, v_1v_{-2}, v_{-1}v_{-4}, u_{-2}v_{-2}, v_{-1}v_2\}$;

(ii) $M_2(v_0)\backslash M_2[u_0] = \{ u_6v_6, v_6v_9, u_3u_4, u_{-6}v_{-6}, v_{-6}v_{-9}, u_{-3}u_{-4}\}$.

Proof. It can be seen that every edge at distance 2 from u_0 belongs to

$\{ v_1v_4, u_2v_2, v_1v_{-2}, v_{-1}v_{-4}, u_{-2}v_{-2}, v_{-1}v_2, u_2u_3, u_3v_3, u_{-2}u_{-3}, v_3v_6, u_{-3}v_{-3}, v_{-3}v_{-6}\}$. (1)

Two edges of them $u_2u_3, u_{-2}u_{-3}$ are at distance 2 from v_0 and hence don't belong to $M_2(u_0)\backslash M_2[v_0]$. Also the edges $u_3v_3, v_3v_6, u_{-3}v_{-3}, v_{-3}v_{-6}$ are in the shortest paths from u_0 such that pass through the edge u_0v_0 first and therefore are not in $M_2(u_0)\backslash M_2[v_0]$ and so (i) holds.

On the other hand the edges at distance 2 from v_0 are in

$\{ u_6v_6, v_6v_9, u_3u_4, u_{-6}v_{-6}, v_{-6}v_{-9}, u_{-3}u_{-4}, u_2u_3, u_1u_2\ u_1v_1, u_{-1}u_{-2}, u_{-1}v_{-1}, u_{-2}u_{-3}\}$. (2)

Notice that, the edges $u_2u_3, u_{-2}u_{-3}$ are at distance 2 from u_0 and hence don't belong to $M_2(v_0)\backslash M_2[u_0]$.

Also the edges $u_1v_1, u_1u_2, u_{-1}v_{-1}, u_{-1}u_{-2}$ are in the shortest paths from v_0 such that pass through the edge u_0v_0 first and therefore are not in $M_2(v_0)\backslash M_2[u_0]$ and the proof is completed.□

In the following, we specify the sets $M_3(u_0)\backslash M_3[v_0]$ and $M_3(v_0)\backslash M_3[u_0]$ of the graph $GP(6n + 8,3)$.

Lemma 3. *Let $n \geq 2$ be an integer and let u_0v_0 be a spoke in $E(GP(6n + 8,3))$ Then the following statements hold:*

(i) $M_3(u_0)\backslash M_3[v_0] = \{ v_4v_7, v_2v_5, v_{-4}v_{-7}, v_{-2}v_{-5}\}$;

(ii) $M_3(v_0)\backslash M_3[u_0] = \{ u_9v_9, v_9v_{12}, u_6u_7, u_5u_6, u_4u_5, u_{-9}v_{-9}, v_{-9}v_{-12}, u_{-6}u_{-7}, u_{-5}u_{-6}, u_{-4}u_{-5}\}$.

Proof. Every edge at distance 3 from u_0 belongs to

$\{ v_4v_7, v_2v_5, v_{-4}v_{-7}, v_{-2}v_{-5}, u_6v_6, v_6v_9, u_3u_4, u_{-6}v_{-6}, v_{-6}v_{-9}, u_{-3}u_{-4}, u_{-4}v_{-4}\}$.

(3)

Two edges of them $u_4v_4, u_{-4}v_{-4}$ are at distance 3 from v_0 and hence don't belong to $M_3(u_0)\backslash M_3[v_0]$.

Also the edges $u_6v_6, v_6v_9, u_3u_4, u_{-6}v_{-6}, v_{-6}v_{-9}, u_{-3}u_{-4}$ are in the shortest paths from u_0 such that pass through the edge u_0v_0 first and therefore are not in $M_3(u_0)\backslash M_3[v_0]$ and so (i) holds.

We now prove part(ii).

The edges at distance 3 from v_0 are in

$\{ u_9v_9, v_9v_{12}, u_6u_7, u_5u_6, u_4u_5, u_{-9}v_{-9}, v_{-9}v_{-12}, u_{-6}u_{-7}, u_{-5}u_{-6}, u_{-4}u_{-5}, v_1v_{-2}, v_1v_4, u_2v_2, u_4v_4, v_{-1}v_2, v_{-1}v_{-4},$

$u_{-2}v_{-2}, u_{-4}v_{-4}\}$.

(4)

Notice that, the edges $u_4v_4, u_{-4}v_{-4}$ are at distance 3 from u_0 and hence don't belong to $M_3(v_0)\backslash M_3[u_0]$. Also the edges $v_1v_{-2}, v_1v_4, u_2v_2, v_{-1}v_2, v_{-1}v_{-4}, u_{-2}v_{-2}$ are in the shortest paths from v_0 such that pass through the edge u_0v_0 first and therefore are not in $M_3(v_0)\backslash M_3[u_0]$ and this completes the proof.

□

Now we investigate the sets $M_k(u_0)\backslash M_k[v_0]$ and $M_k(v_0)\backslash M_k[u_0]$ of the graph $GP(6n + 8,3)$ of diameter d where $4 \le k \le d - 3$.

Lemma 4. *Let $n \ge 2$ be an integer and let u_0v_0 be a spoke in $E(GP(6n + 8,3))$. Then for any*

$4 \le k \le d - 3$*, the following statements hold:*

(i) $M_k(u_0)\backslash M_k[v_0] = \{v_{3k-5}v_{3k-2}, v_{3k-7}v_{3k-4}, v_{-(3k-5)}v_{-(3k-2)}, v_{-(3k-7)}v_{-(3k-4)}\}$

(ii) $M_k(v_0)\backslash M_k[u_0] = \{v_{3k}v_{3k+3}, u_{3k}v_{3k}, u_{3k-3}u_{3k-2}, u_{3k-4}u_{3k-3}, u_{3k-5}u_{3k-4}, v_{-3k}v_{-(3k+3)}, u_{-3k}v_{-3k}, u_{-(3k-3)}u_{-(3k-2)},$
$u_{-(3k-4)}u_{3k-3)}, u_{-(3k-5)}u_{-(3k-4)}\}$.

Where d is diameter of $GP(6n + 8,3)$.

Proof. There are 18 edges at distance k from u_0 such that belong to

$\{v_{3k-5}v_{3k-2}, v_{3k-7}v_{3k-4}, v_{-(3k-5)}v_{-(3k-2)}, v_{-(3k-7)}v_{-(3k-4)}, u_{3k-3}v_{3k-3}, v_{3k-3}v_{3k}, u_{3k-6}u_{3k-5}, u_{3k-7}u_{3k-6}, u_{3k-8}u_{3k-7},$
$u_{3k-7}v_{3k-7}, u_{3k-5}v_{3k-5}, u_{-(3k-3)}v_{-(3k-3)}, v_{-(3k-3)}v_{-3k}, u_{-(3k-6)}u_{-(3k-5)}, u_{-(3k-7)}u_{-(3k-6)}, u_{-(3k-8)}u_{-(3k-7)}, u_{-(3k-7)}v_{-(3k-7)}, u_{-(3k-5)}v_{-(3k-5)}\}$.
(5)

Four edges of them $u_{3k-7}v_{3k-7}, u_{3k-5}v_{3k-5}, u_{-(3k-7)}v_{-(3k-7)}, u_{-(3k-5)}v_{-(3k-5)}$ are at distance k from v_0 and hence don't belong to $M_k(u_0)\backslash M_k[v_0]$.

Also the edges $u_{3k-3}v_{3k-3}, v_{3k-3}v_{3k}, u_{3k-6}u_{3k-5}, u_{3k-7}u_{3k-6}, u_{3k-8}u_{3k-7}, u_{-(3k-3)}v_{-(3k-3)}, v_{-(3k-3)}v_{-3k},$
$u_{-(3k-6)}u_{-(3k-5)}, u_{-(3k-7)}u_{-(3k-6)}, u_{-(3k-8)}u_{-(3k-7)}$.
(6)

are in the shortest paths from u_0 such that pass through the edge u_0v_0 first and therefore are not in $M_k(u_0)\backslash M_k[v_0]$ and so (i) holds.

It can be easily checked that the edges at distance k from v_0 are in

$v_{3k}v_{3k+3}, u_{3k}v_{3k}, u_{3k-3}u_{3k-2}, u_{3k-4}u_{3k-3}, u_{3k-5}u_{3k-4}, v_{-3k}v_{-(3k+3)}, u_{-3k}v_{-3k}, u_{-(3k-3)}u_{-(3k-2)}, v_{3k-8}v_{3k-5}, v_{3k-10}v_{3k-7}$
$u_{-(3k-4)}u_{-(3k-3)}, u_{-(3k-5)}u_{-(3k-4)}. u_{3k-7}v_{3k-7}, u_{3k-5}v_{3k-5}, v_{-(3k-8)}v_{-(3k-5)}, v_{-(3k-10)}v_{-(3k-7)} u_{-(3k-7)}v_{-(3k-7)},$
$u_{-(3k-5)}v_{-(3k-5)}$ (7)

The edges $u_{3k-7}v_{3k-7}, u_{3k-5}v_{3k-5}, u_{-(3k-7)}v_{-(3k-7)}, u_{-(3k-5)}v_{-(3k-5)}$ are at distance k from u_0 and hence don't belong to $M_k(v_0)\backslash M_k[u_0]$.

Also the edges $v_{3k-8}v_{3k-5}, v_{3k-10}v_{3k-7}, v_{-(3k-8)}v_{-(3k-5)}, v_{-(3k-10)}v_{-(3k-7)}$

are in the shortest paths from v_0 such that pass through the edge u_0v_0 first and therefore are not in $M_k(v_0)\backslash M_k[u_0]$ and the proof is completed.

□

In the next lemma we investigate the sets $M_{d-2}(u_0)\backslash M_{d-2}[v_0]$ and $M_{d-2}(v_0)\backslash M_{d-2}[u_0]$ of the graph $GP(6n + 8,3)$ of diameter d.

Lemma 5. *Let $n \ge 2$ be an integer and let u_0v_0 be a spoke in $E(GP(6n + 8,3))$.*
Then the following statements hold:

(i) $M_{d-2}(u_0)\backslash M_{d-2}[v_0] = \{ v_{3d-11}v_{3d-8}, v_{-(3d-11)}v_{-(3d-8)} \}$;

(ii) $M_{d-2}(v_0)\backslash M_{d-2}[u_0]=\{u_{3d-11}u_{3d-10}, u_{3d-10}u_{3d-9}, u_{3d-9}u_{3d-8}u_{3d-10}v_{3d-10}, u_{-(3d-11)}u_{-(3d-10)}, u_{-(3d-10)}u_{-(3d-9)}, u_{-(3d-9)}u_{-(3d-8)}u_{-(3d-10)}v_{-(3d-10)} \}$.

Where d is diameter of GP(6*n* + 8,3).

Proof. Every edge at distance $d - 2$ from u_0 belongs to $v_{3d-11}v_{3d-8}$, $v_{-(3d-11)}v_{-(3d-8)}$, $u_{3d-9}v_{3d-9}$,

$v_{3d-9}v_{3d-6}$, $u_{3d-14}u_{3d-13}$, $u_{3d-13}u_{3d-12}$, $u_{3d-12}u_{3d-11}$, $u_{3d-11}v_{3d-11}$, $u_{3d-13}v_{3d-13}$, $v_{3d-13}v_{3d-10}$, $u_{-(3d-9)}v_{-(3d-9)}$,

$v_{-(3d-9)}v_{-(3d-6)}$, $u_{-(3d-14)}u_{-3(d-13)}$, $u_{-(3d-13)}u_{-(3d-12)}$, $u_{-(3d-12)}u_{-(3d-11)}$, $u_{-(3d-11)}v_{-(3d-11)}$, $u_{-(3d-13)}v_{-(3d-13)}$,

$v_{-(3d-13)}v_{-(3d-10)}$. (8)

Six edges of them

$u_{3d-11}v_{3d-11}$, $u_{3d-13}v_{3d-13}$, $v_{3d-13}v_{3d-10}$, $u_{-(3d-11)}v_{-(3d-11)}$, $u_{-(3d-13)}v_{-(3d-13)}$, $v_{-(3d-13)}v_{-(3d-10)}$. (9)

are at distance $d-2$ from v_0 and therefore don't belong to $M_{d-2}(u_0)\backslash M_{d-2}[v_0]$.

The remaining edges except two edges $v_{3d-11}v_{3d-8}$, $v_{-(3d-11)}v_{-(3d-8)}$, are in the shortest paths from u_0 such that pass through the edge u_0v_0 first and therefore are not in $M_{d-2}(u_0)\backslash M_{d-2}[v_0]$ and so (i) holds.

$u_{3d-11}u_{3d-10}, u_{3d-10}u_{3d-9}, u_{3d-9}u_{3d-8}, u_{3d-10}v_{3d-10}, u_{-(3d-11)}u_{-(3d-10)}, u_{-(3d-10)}u_{-(3d-9)}, u_{-(3d-9)}u_{-(3d-8)}$,

$u_{-(3d-10)}v_{-(3d-10)}, v_{3d-14}v_{3d-11}, v_{3d-16}v_{3d-13}, u_{3d-13}v_{3d-13}, u_{3d-11}v_{3d-11}, v_{3d-13}v_{3d-10}, v_{-(3d-14)}v_{-(3d-11)}$,

$v_{-(3d-16)}v_{-(3d-13)}$, $u_{-(3d-13)}v_{-(3d-13)}$, $u_{-(3d-11)}v_{-(3d-11)}$, $v_{-(3d-13)}v_{-(3d-10)}$. (10)

Six edges of them

$u_{3d-13}v_{3d-13}$, $u_{3d-11}v_{3d-11}$, $v_{3d-13}v_{3d-10}$, $u_{-(3d-13)}v_{-(3d-13)}$, $u_{-(3d-11)}v_{-(3d-11)}$, $v_{-(3d-13)}v_{-(3d-10)}$ (11)

are at distance $d - 2$ from u_0 and hence don't belong to $M_{d-2}(v_0)\backslash M_{d-2}[u_0]$.

Also the edges $v_{3d-14}v_{3d-11}$, $v_{3d-16}v_{3d-13}$, $v_{-(3d-14)}v_{-(3d-11)}$, $v_{-(3d-160}v_{-(3d-13)}$,

are in the shortest paths from v_0 such that pass through the edge u_0v_0 first and therefore are not in $M_{d-2}(v_0)\backslash M_{d-2}[u_0]$ and the proof is completed.

□

In the following we determine the sets $M_{d-1}(u_0)\backslash M_{d-1}[v_0]$ and $M_{d-1}(v_0)\backslash M_{d-1}[u_0]$ of the graph *GP*(6*n* + 8,3) of diameter *d*.

Lemma 6. *Let* $n \geq 2$ *be an integer and let* u_0v_0 *be a spoke in E*(*GP*(6*n* + 8,3)).

Then the following statements hold:

(i) $M_{d-1}(u_0)\backslash M_{d-1}[v_0] = \varphi$;

(ii) $M_{d-1}(v_0)\backslash M_{d-1}[u_0]= \varphi$.

Where d is diameter of GP(6*n* + 8,3).

Proof. Every edge at distance $d - 1$ from u_0 belongs to

$u_{3d-11}u_{3d-10}, u_{3d-10}u_{3d-9}, u_{3d-9}u_{3d-8}, u_{3d-10}v_{3d-10}, u_{-(3d-11)}u_{-(3d-10)}, u_{-(3d-10)}u_{-(3d-9)}, u_{-(3d-9)}u_{-(3d-8)}, u_{-(3d-10)}v_{-(3d-10)}$,

$u_{3d-8}v_{3d-8}$. (12)

The edge $u_{3d-8}v_{3d-8}$ is at distance $d-1$ from v_0 and therefore is not in $M_{d-1}(u_0)\backslash M_{d-1}[v_0]$.

Also by Lemma 5 part (ii), the remaining edges are at distance $d-2$ from v_0 and so (i) holds.

Every edge at distance $d-1$ from v_0 belongs to $\{u_{3d-8}v_{3d-8}, v_{3d-11}v_{3d-8}, v_{-(3d-11)}v_{-(3d-8)}\}$.

As above mentioned the edge $u_{3d-8}v_{3d-8}$ are at distance $d-1$ from u_0 and therefore doesn't belong to $M_{d-1}(v_0)\backslash M_{d-1}[u_0]$.

On the other hand, by Lemma 5 part (i), the remaining two edges are at distance $d-2$ from u_0 and don't belong to $M_{d-1}(v_0)\backslash M_{d-1}[u_0]$. This completes the proof .

□

We have the following immediate corollary from Lemmas 1, 2, 3, 4, 5 and 6.

Corollary 2. *Let* $n \geq 2$ *be an integer and let* (u_0,v_0) *be a spoke in* $E(GP(6n+8,3))$. *Then the following statements hold:*

(i) $|M_1(u_0)\backslash M_1[v_0]| = |M_1(v_0)\backslash M_1[u_0]| = 4$ *;*

(ii) $|M_2(u_0)\backslash M_2[v_0]| = |M_2(v_0)\backslash M_2[u_0]| = 6$ *;*

(iii) $|M_3(u_0)\backslash M_3[v_0]| = 4, |M_3(v_0)\backslash M_3[u_0]| = 10$ *;*

(iv) $|M_k(u_0)\backslash M_k[v_0]| = 4, |M_k(v_0)\backslash M_k[u_0]| = 10$ *, for* $k \in \{4,\cdots,d-3\}$*;*

(v) $|M_{d-2}(u_0)\backslash M_{d-2}[v_0]| = 2, |M_{d-2}(v_0)\backslash M_{d-2}[u_0]| = 8$ *;*

(vi) $|M_{d-1}(u_0)\backslash M_{d-1}[v_0]| = |M_{d-1}(v_0)\backslash M_{d-1}[u_0]| = 0$.

Where d is diameter of $GP(6n+8,3)$.

We are now ready to prove our main result.

Proof of Theorem 1. Let $n \geq 2$ be an integer and let (a,b) be a spoke in $(GP(6n+8,3))$.

Then we now show that $GP(6n+8,3)$ of diameter d is not *EDB*.

Since $GP(6n+8,3)$ is cubic, by Corollary 1, evidently it is enough to prove that

$$\sum_{i=1}^{d-1}|M_i(a)\backslash M_i[b]| = \sum_{i=1}^{d-1}|M_i(b)\backslash M_i[a]|,$$

Holds for every edge $e = ab \in E(G)$.By Corollary 2, the left hand side of equation (13) is equal to $4d-8$ and the right hand side of that is equal to $10d-32$ and since $d \geq 6$, the proof is now completed.

3 Conclusions

The Petersen graph is an important graph in graph theory and has attracted much research throughout the years.

Some recent researchers studied the strongly distancebalanced property of the generalized petersen graphs $GP(n,k)$.

In this paper, we determined the edge-distance-balanced property of generalized Peterson graphs for some *n*, *k*.

It is proven that for any integers $n \geq 2$, the Generalized Petersen graph $GP(6n+8, 3)$ is not edge-distance-balanced.

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