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Using Fuzzy Partial Ro-Transforms to Solve A system of Partial Differential Equations

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ABSTRACT

Scientific and engineering are fields rely heavily on fuzzy partial integro-differential equations. This study proposes employing the fuzzy Ro-transform transform, a novel approach to solve fuzzy partial differential equations. first and second -order fuzzy partial derivative formula has discovered, is obtained using highly generalized H-differentiability notions. An introduction to fuzzy partial Ro-transforms is followed by the development of fuzzy partial differential equations using these results. Finally, an application has been provided to demonstrate the proposed method's capabilities.

1 INTRODUCTION

Physical phenomena can be modelled using differential equations. As a result, building a precise differential equation model for such issues necessitates the use of laborious procedures that are impractical to apply. Because of this, a fuzzy math approach seems to be the best fit. Fuzzy differential equations (FDEs) have been extensively used in science and engineering over the past few decades. In [15], L. A. Zadeh presented the basic notion and arithmetic of fuzzy sets. [3,10], then [2,3,1,8] and [3,10], respectively, researched the notion of fuzzy derivatives and fuzzy integration. PDEs are mathematical equations utilized in physics and engineering, among other fields, as well as in the study of chemistry and biology. A partial differential equation is a mathematical formula when dealing with numerous variables and their derivations. When it comes to solving real-world situations, ordinary differential equations fall short of their potential. This is because witnessing events frequently involves contending with multiple variables at once. When modeling a wire's heat transfer, we must, for example, dealing with both space and time at once. Partial differential equations can be solved analytically and numerically by several authors [4,6]. If you are trying to solve linear partial differential equations, you can use fuzzy integrals because they make the original function easier to solve [1,9].

Sections in this document are arranged as follows at the end. Partition Ro-transform results for fuzzy partial derivatives in the first and second - orders are presented here, as are some key ideas about fuzzy numbers and fuzzy functions in general. Solving a system partial equation is possible with the help of these formulas**.**

2.Basic concepts

Definition 1 [7]

A fuzzy number expressed in parametric form ω is a pair $\left(\underline{\omega},\overline{\omega}\right)$ of functions.

$\underline{\omega}(\vartheta),\overline{\omega}(\vartheta)$, $0\le\vartheta\le1$, which satisfies the following requirements:

1. $\underline{\omega}(\vartheta)$ is a bounded left continuous that doesn't decrease. function in $(0,1]$, and continuing right at 0 .
2. $\overline{\omega}(\vartheta)$ is a right continuous at and a bounded non-increasing continuous left function in $(0,1]$ at 0 .
3. $\underline{\omega}(\vartheta)\le\overline{\omega}(\vartheta)$, $0\le\vartheta\le1$.

Theorem 1. [13]

Let $f\ :R\to \mathrm{E}$ (E: all sets of ambiguous numbers) and it is portrayed by $\left[\underline{f}(\varpi;\vartheta),\overline{f}(\varpi;\vartheta)\right]$. For any definite $\vartheta\in(0,1]$ suppose that $\underline{f}(\varpi;\vartheta)$ and $\overline{f}(\varpi;\vartheta)$ are Riemann − integrable functions on [d, c] for every $c\ge d$, and consider two positive functions. $\underline{M}_\vartheta$ and $\overline{M}_\vartheta$ such that $\int_d^c\left|\underline{f}(\varpi;\vartheta)\right|d\varpi\le\underline{M}_\vartheta$ and $\int_d^c\left|\overline{f}(\varpi;\vartheta)\right|d\varpi\le\overline{M_\vartheta}$ for every $c\ge d$ .Then, $f(\varpi)$ is improper fuzzy Riemann − integrable on $[a,\infty)$ Additionally, we have: $\int_d^\infty f(\varpi)\ d\varpi=\left[\int_d^\infty \underline{f}(\varpi;\vartheta)\ d\varpi,\int_d^\infty \overline{f}(\varpi;\vartheta)\ d\varpi\right]$.

Definition2. [11**]**

Suppose that $\omega,\nu\in\mathrm{E}$. Assuming there is $\rho\in\mathrm{E}$ such that $\omega+\nu=\rho$ then ρ referred to such as the Hukuhara distinction. of ω and ν and it is identify by $\omega\ominus\nu$.

Definition 3. [12]

Let $u:(d,c)\times(d,c)\to\mathrm{E}$, is side to be first-order H-differentiable at $\delta_0\in(d,c)$, with respect to δ, If such a thing exists $\frac{\partial}{\partial\delta}u(\delta_0,t)\in\mathrm{E}$ such that:

1. in all $h>0$ those who are small enough $\exists u(\delta_0+h,t)\ominus u(\delta_0,t),\exists u(\delta_0,t)\ominus u(\delta_0-h,t)$, then the following limit hold $\lim_{h\to0^+}\frac{u(\delta_0+h,t)\ominus u(\delta_0,t)}{h}=\lim_{h\to0^+}\frac{u(\delta_0,t)\ominus u(\delta_0-h,t)}{h}=\frac{\partial}{\partial\delta}u(\delta_0,t)$

Or

2. in all $h>0$ those who are small enough

$\exists u(\delta_0,t)\ominus u(\delta_0+h,t),\exists u(\delta_0-h,t)\ominus u(\delta_0,t)$ then the following limit hold

$$\lim_{h\to0^+}\frac{u(\delta_0,t)\ominus u(\delta_0+h,t)}{-h}=\lim_{h\to0^+}\frac{u(\delta_0-h,t)\ominus u(\delta_0,t)}{-h}=\frac{\partial}{\partial\delta}u(\delta_0,t)$$

Definition 4. [12]

Let $u:(d,c)\times(d,c)\to\mathrm{E}$,is side to be H-differentiable of the first-order at $t_0\in(d,c)$, with respect to t, if there exists an element $\frac{\partial}{\partial t}u(\delta,t_0)\in\mathrm{E}$ such that:

1. in all $h>0$ sufficiently small

$\exists u(\delta,t_0+h)\ominus u(\delta,t_0),\exists u(\delta,t_0)\ominus u(\delta,t_0-h)$ then the following limit hold

$$\lim_{h\to0^+}\frac{u(\delta,t_0+h)\ominus u(\delta,t_0)}{h}=\lim_{h\to0^+}\frac{u(\delta,t_0)\ominus u(\delta,t_0-h)}{h}=\frac{\partial}{\partial t}u(\delta,t_0)\ .$$

Or

2. in all $h>0$ sufficiently small

$\exists u(\delta,t_0)\ominus u(\delta,t_0+h),\exists u(\delta,t_0-h)\ominus u(\delta,t_0)$ then the following limit hold

$$\lim_{h\to0^+}\frac{u(\delta,t_0)\ominus u(\delta,t_0+h)}{-h}=\lim_{h\to0^+}\frac{u(\delta,t_0-h)\ominus u(\delta,t_0)}{-h}=\frac{\partial}{\partial t}u(\delta,t_0)\ .$$

Definition 5. Let $u=u(\varpi,t)$ be a continuous If t is a real parameter and fuzzy-valued function, the function's fuzzy Ro-transform follows u. Denote by $\Re_t(\varpi,\upsilon)$ is defined as follows:

$$\Re_t(\varpi,\upsilon)=R_t\left[u(\varpi,t)\right]=\upsilon^2\int_0^\infty e^{-\left(i\sqrt[\Omega]{\upsilon}\right)t}u(\varpi,t)\,dt=\lim_{\tau\to\infty}\upsilon^2\int_0^\tau e^{-\left(i\sqrt[\Omega]{\upsilon}\right)t}u(\varpi,t)\,dt,$$

$$\Re_t(\varpi,\upsilon)=\left[\lim_{\tau\to\infty}\upsilon^2\int_0^\tau e^{-\left(i\sqrt[\Omega]{\upsilon}\right)t}\underline{u}(\varpi,t)\,dt,\lim_{\tau\to\infty}\upsilon^2\int_0^\tau e^{-\left(i\sqrt[\Omega]{\upsilon}\right)t}\overline{u}(\varpi,t)\,dt\right].$$

whenever there are limits. The $\vartheta-$cut representation of $\Re_t(\varpi,\upsilon)$ is given as:

$$\Re_t(\varpi,\upsilon;\vartheta)=R_t\left[u(\varpi,t;\vartheta)\right]=\left[\gamma\left(\underline{u}(\varpi,t)\right),\gamma\left(\overline{u}(\varpi,t)\right)\right],$$

Theorem 2. [5] Let $u:(0,\infty)\times(0,\infty)\to E$ be a function and denote by $\left[u(\varpi,t)\right]^\vartheta=\left[\underline{u}(\varpi,t;\vartheta),\overline{u}(\varpi,t;\vartheta)\right]$. Such that $\underline{u}(\varpi,t;\vartheta)$ and $\overline{u}(\varpi,t;\vartheta)$ are first-order differentiable functions with regard to to t, then:

1. If $u(\varpi,t)$ is the first form differentiable function, then:

$$\left[\frac{\partial}{\partial t}u(\varpi,t)\right]^\vartheta=\left[\frac{\partial}{\partial t}\underline{u}(\varpi,t;\vartheta),\frac{\partial}{\partial t}\overline{u}(\varpi,t;\vartheta)\right],$$

2. If $u(\varpi,t)$ is the second form differentiable function, then :

$$\left[\frac{\partial}{\partial t}u(\varpi,t)\right]^\vartheta=\left[\frac{\partial}{\partial t}\overline{u}(\varpi,t;\vartheta),\frac{\partial}{\partial t}\underline{u}(\varpi,t;\vartheta)\right].$$

3. Formulas of fuzzy partial derivatives about first and second - orders.

Theorem 3. Let $u:(0,\infty)\times(0,\infty)\to\mathrm{E}$ continuous function with fuzzy values, and u_t the partial derivative of u with respect to t. suppose that $\upsilon^2e^{-\left(i\sqrt[\Omega]{\upsilon}\right)t}u(\varpi,t)$ and $\upsilon^2e^{-\left(i\sqrt[\Omega]{\upsilon}\right)t}u_t(\varpi,t)$ are inadequate fuzzy Riemann-integrable on $[0,\infty]$, then:

1. If u is the first form differentiable function with respect to t.

$$R_t\left[u_t(\varpi,t)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)R_t\left[u(\varpi,t)\right]\ominus\upsilon^2u(\varpi,0),$$

2. If u is the second form differentiable function with respect to t.

$$R_t\left[u_t(\varpi,t)\right]=-\upsilon^2u(\varpi,0)\ominus\left(-i\sqrt[\Omega]{\upsilon}\right)R_t\left[u(\varpi,t)\right].$$

proof: Since $u_t(\varpi,t)$ is a continuous fuzzy-valued function, then the following two situations apply:

Case 1. Since u is the first form differentiable function, for any arbitrary $\vartheta\in[0,1]$, From Theorem 4/1:

$$R_t\left[u_t(\varpi)\right]=\gamma_t\left[\underline{u}_t(\varpi,t;\vartheta)\right],\gamma_t\left[\overline{u}_t(\varpi,t;\vartheta)\right].\quad(1)$$

From Theorem ٣/1:

$$\gamma_t\left[\underline{u}_t(\varpi,\vartheta)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)\gamma_t\left[\underline{u}(\varpi,t;\vartheta)\right]-\upsilon^2\underline{u}(\varpi,0;\vartheta),$$
$$\gamma_t\left[\overline{u}_t(\varpi,\vartheta)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)\gamma_t\left[\overline{u}(\varpi,t;\vartheta)\right]-\upsilon^2\overline{u}(\varpi,0;\vartheta).\quad(2)$$

Substitute (2) in (1), then:

$$R_t\left[u_t(\varpi)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)\gamma_t\left[\underline{u}(\varpi,t;\vartheta)\right]-\upsilon^2\underline{u}(\varpi,0;\vartheta),\left(i\sqrt[\Omega]{\upsilon}\right)\gamma_t\left[\overline{u}(\varpi,t;\vartheta)\right]-\upsilon^2\overline{u}(\varpi,0;\vartheta).$$

By Theorem 1:

$$R_t\left[u_t(\varpi,t)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)R_t\left[u(\varpi,t)\right]\ominus\upsilon^2u(\varpi,0).$$

Case 2. Since u is the second form differentiable function, for any arbitrary $\vartheta\in[0,1]$ From Theorem 4/2:

$$R_t\left[u_t(\varpi)\right]=\gamma_t\left[\overline{u}_t(\varpi,t;\vartheta)\right],\gamma_t\left[\underline{u}_t(\varpi,t;\vartheta)\right].$$

By same way, then:

$$R_t\left[u_t(\varpi)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)\gamma_t\left[\overline{u}(\varpi,t;\vartheta)\right]-\upsilon^2\overline{u}(\varpi,0;\vartheta),\left(i\sqrt[\Omega]{\upsilon}\right)\gamma_t\left[\underline{u}(\varpi,t;\vartheta)\right]-\upsilon^2\underline{u}(\varpi,0;\vartheta).$$

By Theorem 1:

$$R_t\left[u_t(\varpi,t)\right]=-\upsilon^2u(\varpi,0)\ominus\left(-i\sqrt[\Omega]{\upsilon}\right)R_t\left[u(\varpi,t)\right].$$

Theorem 4. $u:(0,\infty)\times(0,\infty)\to E$ be continuous fuzzy-valued function Assume that, u,u_t are fuzzy partial derivatives of fuzzy Ro-transforms, which are continuous fuzzy-valued functions. with respect to t. about second order will be:

1. If u and u_t are the first form differentiable functions with respect to t, then:

$$R_t\left[u_{tt}(\varpi,t)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)^2R_t\left[u(\varpi,t)\right]\ominus\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)u(\varpi,0)\ominus\upsilon^2u_t(\varpi,0),$$

2. If u is the first form differentiable function with respect to t and u_t second form differentiable function with respect to t, then:

$$R_t\left[u_{tt}(\varpi,t)\right]=-\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)u(\varpi,0)\ominus\left(-i\sqrt[\Omega]{\upsilon}\right)^2R_t\left[u(\varpi,t)\right]\ominus\upsilon^2u_t(\varpi,0),$$

3. If u is the second form differentiable function with respect to t and u_t first form differentiable function with respect to t, then:

$$R_t\left[u_{tt}(\varpi,t)\right]=-\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)u(\varpi,0)\ominus\left(-i\sqrt[\Omega]{\upsilon}\right)^2R_t\left[u(\varpi,t)\right]-\upsilon^2u_t(\varpi,0),$$

4. If u and u_t are the second form differentiable functions with respect to t, then:

$$R_t\left[u_{tt}(\varpi,t)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)^2R_t\left[u(\varpi,t)\right]\ominus\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)u(\varpi,0)-\upsilon^2u_t(\varpi,0).$$

proof: We will proof two cases as following:

Case 1. Since u and u_t are the first form differentiable functions with respect to t, for any arbitrary $\vartheta\in[0,1]$, From Theorem 4/1:

$$R_t\left[u_{tt}(\varpi)\right]=\gamma_t\left[\underline{u}_{tt}(\varpi,t;\vartheta)\right],\gamma_t\left[\overline{u}_{tt}(\varpi,t;\vartheta)\right].$$

$$R\left[u_{tt}(\varpi)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)^2\gamma_t\left[\underline{u}(\varpi,t;\vartheta)\right]-\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)\underline{u}(\varpi,0;\vartheta)-\upsilon^2\underline{u}_t(\varpi,0;\vartheta),$$

$$\left(i\sqrt[\Omega]{\upsilon}\right)^2\gamma_t\left[\overline{u}(\varpi,t;\vartheta)\right]-\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)\overline{u}(\varpi,0;\vartheta)-\upsilon^2\overline{u}_t(\varpi,0;\vartheta).$$

By Theorem 1:

$$R_t\left[u_{tt}(\varpi,t)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)^2 R_t\left[u(\varpi,t)\right]\ominus\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)u(\varpi,0)\ominus\upsilon^2 u_t(\varpi,0).$$

Case 2. Since u is the first form differentiable function with respect to t and u_t is the second form differentiable function with respect to t, for any arbitrary $\vartheta\in[0,1]$, From Theorem 4/2:

$$R_t\left[u_{tt}(\varpi)\right]=\gamma_t\left[\overline{u}_{tt}(\varpi,t;\vartheta)\right],\gamma_t\left[\underline{u}_{tt}(\varpi,t;\vartheta)\right].$$

By Theorem 1:

$$R_t\left[u_{tt}(\varpi,t)\right]=-\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)u(\varpi,0)\ominus\left(-i\sqrt[\Omega]{\upsilon}\right)^2 R_t\left[u(\varpi,t)\right]\ominus\upsilon^2 u_t(\varpi,0).$$

To solve fuzzy partial differential equation about first order by use Ro-transform, we have:

Case 1: Let's think about u is the first form differentiable functions or u is the second form differentiable functions, then we get the following:

$$R\left(\frac{\partial u(\varpi,t)}{\partial\varpi}\right)=\left[\frac{\partial\underline{u}(\varpi,t;\vartheta)}{\partial\varpi},\frac{\partial\overline{u}(\varpi,t;\vartheta)}{\partial\varpi}\right],\ 0\le\vartheta\le 1.$$

Case 2: Let's think about u is the second form differentiable function, then we get the following:

$$R\left(\frac{\partial u(\varpi,t)}{\partial\varpi}\right)=\left[\frac{\partial\overline{u}(\varpi,t;\vartheta)}{\partial\varpi},\frac{\partial\underline{u}(\varpi,t;\vartheta)}{\partial\varpi}\right],\ 0\le\vartheta\le 1.$$

Case 3: Let's think about u is the first form differentiable functions, then we get the following:

$$R_t\left[u_t(\varpi,t)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)R_t\left[u(\varpi,t)\right]\ominus\upsilon^2 u(\varpi,0).$$

Case 4: Let's think about u is the first form differentiable functions, then we get the following:

$$R_t\left[u_t(\varpi,t)\right]=-\upsilon^2 u(\varpi,0)\ominus\left(-i\sqrt[\Omega]{\upsilon}\right)R_t\left[u(\varpi,t)\right].$$

In this part, we'll look at example of lateral type H-differentiability with fuzzy beginning and boundary conditions.

Example: Think about the following partial differential equation for a fuzzy system:

$$u_\varpi+w_t=1\ ;\ \text{I.C.}\quad (i)\ \ u(\varpi,0)=(1+\vartheta,1-\vartheta),w(\varpi,0)=(2+\vartheta,4-\vartheta)$$

$$w_\varpi+u_t=0\ ;\ \text{B.C.}\quad (ii)\ \ u(0,t)=(3+\vartheta,5-\vartheta)\lim_{\varpi\to\infty}\text{exist},w(\varpi,0)=(\vartheta,2-\vartheta)\lim_{\varpi\to\infty}\text{exist}.$$

Applying the Ro-transform to the final system equations both sides.

$$R_t\left[u_\varpi(\varpi,t)\right]+R_t\left[w_t(\varpi,t)\right]=R_t[1],$$
$$R_t\left[w_\varpi(\varpi,t)\right]+R_t\left[u_t(\varpi,t)\right]=0.$$

We get the following cases:

Cases (1&3) or (2&4).

$$\frac{\partial}{\partial\varpi}R_t\left[u(\varpi,t)\right]+\left(i\sqrt[\Omega]{\upsilon}\right)R_t\left[w(\varpi,t)\right]\ominus\upsilon^2 w(\varpi,0)=\frac{\upsilon^2}{i\sqrt[\Omega]{\upsilon}},$$

$$\frac{\partial}{\partial\varpi}R_t\left[w(\varpi,t)\right]+\left(i\sqrt[\Omega]{\upsilon}\right)R_t\left[u(\varpi,t)\right]\ominus\upsilon^2 u(\varpi,0)=0.$$

Then:

$$\frac{\partial}{\partial \varpi}\gamma_t\left[\underline{u}(\varpi,t)\right]+\left(i\sqrt[\Omega]{\upsilon}\right)\gamma_t\left[\underline{w}(\varpi,t)\right]-\upsilon^2\underline{w}(\varpi,0)=\frac{\upsilon^2}{i\sqrt[\Omega]{\upsilon}},$$

$$\frac{\partial}{\partial \varpi}\gamma_t\left[\bar{u}(\varpi,t)\right]+\left(i\sqrt[\Omega]{\upsilon}\right)\gamma_t\left[\bar{w}(\varpi,t)\right]-\upsilon^2\bar{w}(\varpi,0)=\frac{\upsilon^2}{i\sqrt[\Omega]{\upsilon}}.$$

$$\frac{\partial}{\partial \varpi}\gamma_t\left[\underline{w}(\varpi,t)\right]+\left(i\sqrt[\Omega]{\upsilon}\right)\gamma_t\left[\underline{u}(\varpi,t)\right]-\upsilon^2\underline{u}(\varpi,0)=0,$$

$$\frac{\partial}{\partial \varpi}\gamma_t\left[\bar{w}(\varpi,t)\right]+\left(i\sqrt[\Omega]{\upsilon}\right)\gamma_t\left[\bar{u}(\varpi,t)\right]-\upsilon^2\bar{u}(\varpi,0)=0.$$

Using initial conditions:

$$\frac{\partial}{\partial \varpi}\gamma_t\left[\underline{u}(\varpi,t)\right]+\left(i\sqrt[\Omega]{\upsilon}\right)\gamma_t\left[\underline{w}(\varpi,t)\right]=\upsilon^2(2+\vartheta)+\frac{\upsilon^2}{i\sqrt[\Omega]{\upsilon}},$$

$$\frac{\partial}{\partial \varpi}\gamma_t\left[\bar{u}(\varpi,t)\right]+\left(i\sqrt[\Omega]{\upsilon}\right)\gamma_t\left[\bar{w}(\varpi,t)\right]=\upsilon^2(4-\vartheta)+\frac{\upsilon^2}{i\sqrt[\Omega]{\upsilon}}.$$

$$\frac{\partial}{\partial \varpi}\gamma_t\left[\underline{w}(\varpi,t)\right]+\left(i\sqrt[\Omega]{\upsilon}\right)\gamma_t\left[\underline{u}(\varpi,t)\right]=\upsilon^2(1+\vartheta),$$

$$\frac{\partial}{\partial \varpi}\gamma_t\left[\bar{w}(\varpi,t)\right]+\left(i\sqrt[\Omega]{\upsilon}\right)\gamma_t\left[\bar{u}(\varpi,t)\right]=\upsilon^2(1-\vartheta).$$

Solve the last system partial equations and by taking the inverse of fuzzy Ro-transform, to get:

$$\underline{u}(\varpi,t;\vartheta)=(3+\vartheta)k(t-\varpi)-(1+\vartheta)k(t-\varpi)+(1+\vartheta),$$

$$\bar{u}(\varpi,t;\vartheta)=(5-\vartheta)k(t-\varpi)-(1-\vartheta)k(t-\varpi)+(1-\vartheta).$$

$$\underline{w}(\varpi,t;\vartheta)=t+(2+\vartheta)+(3+\vartheta)k(t-\varpi)-(1+\vartheta)k(t-\varpi),$$

$$\bar{w}(\varpi,t;\vartheta)=t+(4-\vartheta)+(5-\vartheta)k(t-\varpi)-(1-\vartheta)k(t-\varpi).$$

Cases (1&4) or (2&3).

$$\frac{\partial}{\partial \varpi}R_t\left[u(\varpi,t)\right]-\upsilon^2 w(\varpi,0)\ominus\left(-i\sqrt[\Omega]{\upsilon}\right)R_t\left[w(\varpi,t)\right]=\frac{\upsilon^2}{i\sqrt[\Omega]{\upsilon}},$$

$$\frac{\partial}{\partial \varpi}R_t\left[w(\varpi,t)\right]-\upsilon^2 u(\varpi,0)\ominus\left(-i\sqrt[\Omega]{\upsilon}\right)R_t\left[u(\varpi,t)\right]=0.$$

Then:

$$\underline{u}(\varpi,t;\vartheta)=(1+\vartheta)+\vartheta k(t-\varpi)-tk(t-\varpi)-(2+\vartheta)k(t-\varpi),$$

$$\bar{u}(\varpi,t;\vartheta)=(1-\vartheta)+(2-\vartheta)k(t-\varpi)-tk(t-\varpi)-(4-\vartheta)k(t-\varpi).$$

$$\underline{w}(\varpi,t;\vartheta)=\vartheta k(t-\varpi)-tk(t-\varpi)-(2+\vartheta)k(t-\varpi)+t+(4-\vartheta),$$

$$\bar{w}(\varpi,t;\vartheta)=(2-\vartheta)k(t-\varpi)-tk(t-\varpi)-(4-\vartheta)k(t-\varpi)+t+(2+\vartheta).$$

REFERENCES

[1] Abdul Rahman, N.A.; Ahmad, M.Z., "*Fuzzy Sumudu transform for solving fuzzy partial differential equations*", J. Nonlinear Sci. Appl. 2016, 9, 3226–3239.

[2]. Cano Y.C. and Flores H. R., "*On new solutions of fuzzy differential Equations*". Chaos, Solitons & Fract., 38(1):112–119, 2008.

[3]. Dubois D. and Prada H., "*Towards fuzzy differential calculus part 1: Integration of fuzzy mappings*". Jour. of Approx. Theory, 8(1):1–17,1982.

[4]. Elwakil, S.A., El-labany, S.K.; Zahran, M.A.; Sabry, R. “*Modified extended tanh-function method for solving nonlinear partial differential equations*”, Phys. Lett. 2002, 299, 179–188.
[5] Georgieva A., “*Double Fuzzy Sumudu Transform to Solve Partial Volterra Fuzzy Integro-Differential Equations*”, Mathematic, 2020.
[6]. Gündo˘gdu, H., Ömer G.F, “*Solving nonlinear partial differential equations by using Adomian decomposition method modified decomposition method and Laplace decomposition method*”, MANAS J. Eng. 2017, 5, 1–13.
[7]. Haydar A. K. and Hassan R. H.," *Generalization of Fuzzy Laplace Transforms of Fuzzy Fractional Derivatives about the General Fractional Order* $n-1<\beta<n$", Hindawi Publishing Corp. Mathe. Prob. in Eng, 2016.
[8]. Iguez R. R., opez L., “*Comparison results for fuzzy differential equations*”. Inform Sci.,178(6):1756–1779, 2008.
[9]. Salahshour, S.; Allahviranloo T., “*Applications of fuzzy Laplace transforms*”, Soft Comput. 2013, 17, 145–158.
[10]. Seikkala S., “*On the fuzzy initial value problem*”, Fuzzy Sets & Sys., 24(3):319–330, 1987.
[11]. Sleibi A. and Alkiffai A.N, “*Solving Ordinary Differental Equations Using Fuzzy Transformation*”, Msc. these Kufa University,2020.
[12]. Ullah S., Farooq M., Ahmad L., Abdullah S., “*Application of fuzzy Laplace transforms for solving fuzzy partial Volterra integro-differential Equations*”, arXiv, 2018.
[13]. Wu H.-C., "*The improper fuzzy Riemann integral and its numerical integration*", Inform. Sci., 111 (1998).
[14]. Wiley A. & Sons, “*Applied Engineering Analysis*”, Tai-Ran Hsu, John, 2018 (ISBN 9781119071204).
[15]. Zadeh L. A., “*Fuzzy sets*”, Inform & Cont., 8(3):338–353, 1965.