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Dispersive Soft Group Space

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Abstract:
The soft set theory proposed by Molodtsov is a recent mathematical approach for modeling uncertainty and vagueness. The main aim of this paper is to introduce the concepts of soft group space of dispersive soft group space, soft fixed point. Finally, the concept of dispersive soft group space was introduced and some properties related to it .

Keyword: soft topological group, soft group space, soft orbit, soft stabilizer and soft periodic point.

1. Introduction

In 1999[4] the soft set theory introduced by Molodtsov. Although this theory has not had a long history, it has created a wide range of applications in many disciplines. Especially, this theory has been studied by mathematicians in different ways such as algebraic, topological and categorical [1,2,3,6,7,8]. In this paper soft action is defined and studied, some important examples and properties are presented. Some concepts related to the soft action such as soft stabilizers, soft orbit are defined. Finally, the concept of dispersive soft group space is introduced .

2. preliminaries

In this section, we review some main concepts and properties of soft topological group, soft action and soft group space for the sake of completeness. Let X be an initial universe set and let $\hat{E}$ be a set of parameters. Let $P(X)$ denotes the power set of X.

Definition (2.1)[5]

Let $(F,\hat{E})$ be a soft set over $\mathbb{G}$ and T be a soft topology over $\mathbb{G}$, then $(\mathbb{G},\mathrm{T},F,\hat{E})$ is called a soft topological group (Sƫg) iff $(F(\omega),\mathrm{T}_{F(\omega)})$ is a topological group on $F(\omega)$ for all $\omega \in \hat{E}$, where $\mathrm{T}_{F(\omega)}$ is the relativized topology on $F(\omega)$ induced from T_{ω}.

Definition (2.2)[5]

Let $(\mathbb{G},T,F,\hat{E})$ be a Sƫg and let $(H,\Gamma,\hat{E})$ be a Sƫs over X. A left soft (continuous) action of $(\mathbb{G},T,F,\hat{E})$ on $(H,\Gamma,\hat{E})$ is a continuous map $\varphi_{\omega}: F(\omega)\times H(\omega)\to H(\omega)$ such that for all $\omega \in \hat{E}$:

(i) $\varphi_\omega(e_{\mathbb{G}}, x) = x$,for all $x \in H(\omega)$

(ii) $\varphi_\omega(g, \varphi_\omega(g *, x)) = \varphi_\omega(gg *, x)$, for all $g, g * \in F(\omega)$ and $x \in H(\omega)$.

In similar way, a right soft action is defined.

Note that the difference between the left and right soft action is not a trivial one, however there is a one to one correspondence between them as follows:

If φ_ω is a left soft action of $(\mathbb{G}, T, F, \hat{E})$ on $\left(H, \Gamma, \hat{E}\right)$, then $\acute{\varphi}_\omega: H(\omega) \times F(\omega) \to H(\omega)$ defined by $\acute{\varphi}_\omega(x, g) = \varphi_\omega(g^{-1}, x)$ is a right soft action of $(\mathbb{G}, T, F, \hat{E})$ on $\left(H, \Gamma, \hat{E}\right)$, and similarly for right soft action. Thus, for every left soft action is a conjugate right soft action and vice versa. So, every theorem that is true of left soft action has a conjugate theorem for right soft action.

Because of this, we will usually use a left soft action.

The soft topological space $(H, \Gamma, \hat{E})$ is called "Soft Group space" which is denoted by ($S\mathbb{G}$-space).

Definition (2.3)[5]

Let $(\mathbb{G}, \mathrm{T}, \mathrm{F}, \hat{E})$ be a Stg, then for all $x \in H(\omega)$ the set $Orb_\omega(x) = \{\varphi_\omega(g, x): g \in F(\omega)\}$ for all $\omega \in \hat{E}$ is called the soft Orbit of $\boldsymbol{x}$.

Definition (2.4)

Let $(H, \Gamma, \hat{E})$ be a $S\mathbb{G}$-spac, for any point $x \in H(\omega)$, then for all $\omega \in \hat{E}$:

(i) $J_\omega(x) = \{y \in H(\omega)$: there is a net $\{g_\beta\}_{\beta \in \Omega}$ in $F(\omega)$ and there is a net $\{\chi_\beta\}_{\beta \in \Omega}$ in $H(\omega)$ with $g_\beta \to \infty$ and $\chi_\beta \to x$ such that $\varphi_\omega(g_\beta, \chi_\beta) \to y\}$ is called first soft prolongation limit set of $\boldsymbol{H(\omega)}$.

(ii) $\Lambda_\omega(x) = \{y \in H(\omega)$: there is a net $\{g_\beta\}_{\beta \in \Omega}$ in $F(\omega)$ with $g_\beta \to \infty$ such that $\varphi_\omega(g_\beta, x) \to y\}$ is called soft limit set of $\boldsymbol{H(\omega)}$.

It is clear that the set $\Lambda_\omega(x)$ is a subset of $J_\omega(x)$ for all $\omega \in \hat{E}$.

Definition (2.5)[5]

Let $(H, \Gamma, \hat{E})$ be a $S\mathbb{G}$ -space, then the set $Stab_\omega(x) = \{g \in F(\omega): \varphi_\omega(g, x) = x\}$ for all $\omega \in \hat{E}$ is called ***the soft stabilizer of*** $\boldsymbol{x}$.

3. Dispersive soft group space

In this section, we will describe the concept of dispersive soft group space and exemplify it. In addition, we will establish some characterizations about it .

Definition (3.1)

A $S\mathbb{G}$-space $(H, \Gamma, \hat{E})$ is called Dispersive soft group space (briefly $DS\mathbb{G}$ -space) if for each two points $x, y \in H(\omega)$, there are open neighborhoods M, N of x *and* y respectively such that the set $((M, N)) = \{g \epsilon F(\omega): \varphi_{\omega}(g, M) \cap N \neq \emptyset\}$ is relatively compact in $F(\omega)$.

Example (3.2)

Let $\mathbb{G} = (Z_4, +_4)$ with discrete soft topology T, $\hat{E}=\{\omega_1, \omega_2\}$ and let $(F, \hat{E})$ be a soft set over $\mathbb{G}$ such that $F(\omega_1) = Z_4$, $F(\omega_2) = \{0\}$, let (X, Γ) be a discrete Sţs over Z_4 such that $H(\omega_1) = H(\omega_2) = Z_4$. If we defined $\varphi_{\omega_1}: F(\omega_1) \times H(\omega_1) \rightarrow H(\omega_1)$ by $\varphi_{\omega_1}(x, y) = x +_4 y$ and $\varphi_{\omega_2}: F(\omega_2) \times H(\omega_2) \rightarrow H(\omega_2)$ by $\varphi_{\omega_2}(x, y) = y$.

Then $(H, \Gamma, \hat{E})$ is $DS\mathbb{G}$ -space.

Theorem (3.3)

Let $(H, \Gamma, \hat{E})$ be a $S\mathbb{G}$ -space, then the following statements are equivalents:

(i) $(H, \Gamma, \hat{E})$ is $DS\mathbb{G}$ -space.

(ii) $J_{\omega}(x) = \emptyset$ for all $x \in H(\omega)$.

(iii) $y \notin J_{\omega}(x)$ for all $x, y \in H(\omega)$.

Proof:

$(i) \rightarrow (ii)$

Suppose $J_{\omega}(x) \neq \emptyset$, then there is $y \in H(\omega)$ such that $y \in J_{\omega}(x)$.

Then, there is a net $\{g_{\beta}\}_{\beta \in \Omega}$ in $F(\omega)$ and a net $\{\chi_{\beta}\}_{\beta \in \Omega}$ in $H(\omega)$ such that $g_{\beta} \rightarrow \infty$ and $\chi_{\beta} \rightarrow x$ such that $\varphi_{\omega}(g_{\beta}, \chi_{\beta}) \rightarrow y$.

Since $(H, \Gamma, \hat{E})$ be $DS\mathbb{G}$ -space, then there are open neighborhoods M, N of x and y respectively such that $((M, N))$ is relative compact in $F(\omega)$.

Since $\chi_\beta \to x$ and $\varphi_\omega(g_\beta, \chi_\beta) \to y$, then there is $\beta_0 \in \Omega$ such that $\chi_\beta \in M$ and $\varphi_\omega(g_\beta, \chi_\beta) \in N$ for all $\beta \geq \beta_0$, hence $\varphi_\omega(g_\beta, \chi_\beta) \in \varphi_\omega(g_\beta, M)$

Therefore $g_\beta \in ((M, N))$

Then $\{g_\beta\}$ has a cluster point $g \in F(\omega)$ and this is contradiction.

Then $J_\omega(x) = \emptyset$.

$(ii) \to (iii)$

Clear

$(iii) \to (i)$

Suppose that $(H, \Gamma, \hat{E})$ be not a $DS\mathbb{G}$ -space, then there are two points $x, y \in H(\omega)$ such that for all open neighborhoods M of x and N of y, the set $((M, N))$ isn't relative compact in $F(\omega)$.

Thus, there is a net $\{g_\beta\}_{\beta \in \Omega}$ in $F(\omega)$ such that $g_\beta \to \infty$.

Since $\varphi_\omega(g_\beta, M) \cap N \neq \emptyset$ for all $\beta \in \Omega$.

Then there is $x_\beta \in M$ such that $\varphi_\omega(g_\beta, \chi_\beta) \in N$ for all $\beta \in \Omega$

Hence $\{\chi_\beta\}$ and $\{\varphi_\omega(g_\beta, \chi_\beta)\}$ are nets in $H(\omega)$ such that $\chi_\beta \to x$ and $\varphi_\omega(g_\beta, \chi_\beta) \to y$.

Then $y \in J_\omega(x)$ and this contradiction.

Hence $(H, \Gamma, \hat{E})$ is $DS\mathbb{G}$ -space.

Corollary (3.4)

If $(H, \Gamma, \hat{E})$ is $DS\mathbb{G}$ -space, then $\Lambda_\omega(x) = \emptyset$, for all $x \in H(\omega)$ and $\omega \in \hat{E}$.

Proof:

Since $\left(H, \Gamma, \hat{E}\right)$ is $DS\mathbb{G}$-space, then by theorem (4.4.3)(ii) $J_\omega(x) = \emptyset$, for all $x \in H(\omega)$.

Since $\Lambda_\omega(x) \subseteq J_\omega(x)$ for all $x \in H(\omega)$ see (4.2.14).

Then $\Lambda_\omega(x) = \emptyset$ for all $x \in H(\omega)$ and $\omega \in \hat{E}$.

Remark (3.5)

The converse of corollary (3.4) isn't be true, consider the following example:

Example (3.6)

Let $\hat{E} = \{\omega\}$, $\mathbb{G} = (Q, +)$ with relative usual topology T, let $(F, \hat{E})$ be a soft set over $\mathbb{G}$ such that $F(\omega) = Q$. If $X = \mathbb{G}$

If we defined $\varphi_\omega: F(\omega) \times H(\omega) \to H(\omega)$ by $\varphi_\omega(g, x) = g + x$.

Let $y \in \Lambda_\omega(x)$, then there is a net $\{g_\beta\}_{\beta \in \Omega}$ in Q with $g_\beta \to \infty$ such that $\varphi_\omega(g_\beta, x) \to y$.

Then $g_\beta + x \to \infty$ this contradiction, then $\Lambda_\omega(x) = \emptyset$.

But Q isn't compact, then $(H, \Gamma, \hat{E})$ isn't $DS\mathbb{G}$ -space.

Definition (3.7)

Let $(H, \Gamma, \hat{E})$ be a $S\mathbb{G}$-space. A point $x \in H(\omega)$ is called ***a fixed point*** if $Orb_\omega(x) = Stab_\omega(x) = \{x\}$ for all $\omega \in \hat{E}$.

Example (3.8)

Let $\mathbb{G} = (\mathcal{R}, .)$ with discrete topology, $\hat{E} = \{\omega_1, \omega_2\}$. Let $(F, \hat{E})$ be a soft set over $\mathbb{G}$defined by $F(\omega_1) = F(\omega_2) = H(\omega_1) = H(\omega_2) = \{1\}$, and defined $\varphi_\omega: F(\omega) \times F(\omega) \to F(\omega)$ by $\varphi_\omega(g, h) = h$, for all $\omega \in \hat{E}$, then $(H, \Gamma, \hat{E})$ is $S\mathbb{G}$ −space has fixed point $\{1\}$.

Definition (3.9)

Let $(\mathbb{G}, \Gamma, F, \hat{E})$ be a Stg, a subset U of $F(\omega)$ is called ***Syndetic*** if there is a compact subset V of $F(\omega)$ such that $F(\omega) = \varphi_\omega(U, V)$ for all $\omega \in \hat{E}$.

Definition (3.10)

Let $(H, \Gamma, \hat{E})$ be a S$\mathbb{G}$ -space, a point $x \in H(\omega)$ is called ***periodic point*** if $Stab_\omega(x)$ is syndetic in $F(\omega)$ for all $\omega \in \hat{E}$.

Example (3.11)

If $(\mathbb{G}, \mathrm{T}, F, \hat{E})$be a compact Stg, then every point in $H(\omega)$ is perodic.

Theorem (3.12)

If $(H, \Gamma, \hat{E})$ is DS$\mathbb{G}$ -space, then:

(i) There is no fixed point.

(ii) There is no periodic point.

Proof:

(i) Suppose that $x \in H(\omega)$ be a fixed point.

Since $(H, \Gamma, \hat{E})$ is $DS\mathbb{G}$ -space, then there are neighborhoods M, N of x in $H(\omega)$ such that $((M, N))$ is relatively compact in $F(\omega)$.

Since x is a fixed point, then $\big((M, N)\big) = F(\omega)$.

Then $F(\omega)$ is compact for all $\omega \in \hat{E}$, but this is contradiction.

Then $(H, \Gamma, \hat{E})$ has no fixed point.

(ii) Suppose that $x \in H(\omega)$ be a periodic point.

Then, $Stab_\omega(x)$ is syndetic in $F(\omega)$.

That is there is a compact subset U of $F(\omega)$ such that $F(\omega) = \varphi_\omega(Stab_\omega(x), U)$

Since $Stab_\omega(x)$ is compact from (4.4.15(ii)), thus $F(\omega)$ is compact for all $\omega \in \hat{E}$,this is contradiction

Then $(H, \Gamma, \hat{E})$ has no periodic point.

Theorem (3.13)

If $(H, \Gamma, \hat{E})$ be a $DS\mathbb{G}$ -space, let $(K, \hat{E})$ be a soft closed subgroup of $(F, \hat{E})$ over $\mathbb{G}$ and let Y be an open subspace of $H(\omega)$ which is an invariant under $K(\omega)$. Then Y is DK −Space.

Proof:

Let $x, y \in Y$, then $x, y \in H(\omega)$.

Since $(H, \Gamma, \hat{E})$ is $DS\mathbb{G}$ −space, then there are open neighborhoods M and N of x and y respectively in $H(\omega)$, such that $((M, N))$ is relatively compact in $F(\omega)$.

Let $U = M \cap Y$ and $V = N \cap Y$.

Since Y is an open subspace of $H(\omega)$, then we have U and V to be open neighborhoods of x and y respectively in Y.

Since $((U, V)) \subseteq ((M, N))$, then $((U, V))$ is relatively compact in $F(\omega)$.

Since $K(\omega)$ is a closed subgroup of $F(\omega)$, then $((U, V))$ is relatively compact in $K(\omega)$.

Then Y is DK −space.

Corollary (3.14)

If $(H, \Gamma, \hat{E})$ be a $DS\mathbb{G}-$ space and Y be an open subspace of $H(\omega)$ which is an invariant under $F(\omega)$, then Y is $D\mathbb{G}$ -space.

Corollary (3.15)

If $(H, \Gamma, \hat{E})$ be a $DS\mathbb{G}$ -space and $(K, \hat{E})$ be a soft closed subgroup of $(F, \hat{E})$, then $H(\omega)$ is DK $-$space.

Theorem (3.16)

Let (I_G, f_ω) be a morphism from $S\mathbb{G}$-space $(H_1, \Gamma_1, \hat{E})$ into $S\mathbb{G}$-space $(H_2, \Gamma_2, \hat{E})$ such that $f_\omega: H_1(\omega) \to H_2(\omega)$ be homeomorphism for all $\omega \in \hat{E}$. If $(H_1, \Gamma_1, \hat{E})$ is $DS\mathbb{G}$-space, then so is $(H_2, \Gamma_2, \hat{E})$.

Proof:

Let $y_1, y_2 \in H_2(\omega)$.

Since f_ω is surjective, then there are $x_1, x_2 \in H_1(\omega)$ such that $f_\omega(x_1) = y_1$, $f_\omega(x_2) = y_2$.

Since $(H_1, \Gamma_1, \hat{E})$ is $DS\mathbb{G}$-space and $x_1, x_2 \in H_1(\omega)$, then there are open neighborhoods M and N of $x_1$and x_2 respectively in $H_1(\omega)$ such that $((M, N))$ is relatively compact in $F_1(\omega)$.

Since f_ω are homeomorphism for all $\omega \in \hat{E}$, then $f_\omega(M), f_\omega(N)$ be open neighborhoods of y_1, y_2 respectively in $H_2(\omega)$.

Since f_ω be injective and equivariant functions for all $\omega \in \hat{E}$, then $g \in ((M, N)) \leftrightarrow \varphi_\omega(g, M) \cap N \neq \emptyset$

$$\leftrightarrow f_\omega(\varphi_\omega(g, M) \cap N) \neq \emptyset$$

$$\leftrightarrow f_\omega(\varphi_\omega(g, M)) \cap f_\omega(N) \neq \emptyset$$

$$\leftrightarrow \varphi_\omega(g, f_\omega(M)) \cap f_\omega(N) \neq \emptyset$$

$$\leftrightarrow g \in ((f_\omega(M), f_\omega(N))).$$

Hence $((M, N)) = ((f_\omega(M), f_\omega(N)))$.

Since $((M, N))$ is relatively compact in $F_1(\omega)$.

Then $((f_\omega(M), f_\omega(N)))$ is relatively compact in $F_2(\omega)$

Thus $(H_2, \Gamma_2, \hat{E})$ is a $DS\mathbb{G}$ -space.

Theorem (3.17)

If $(H,\Gamma,\hat{E})$ and $(K,\acute{\Gamma},\hat{E})$ are two $S\mathbb{G}$-spaces, if $(H,\Gamma,\hat{E})$ is a $DS\mathbb{G}$-space, then so is $\left(H\times K,\Gamma_{prod},\hat{E}\right)$.

Proof:

Let $(H,\Gamma,\hat{E})$ be a $DS\mathbb{G}$ -space.

Let $(h_1,k_1),(h_2,k_2)\in H(\omega)\times K(\omega)$.

Then $h_1,h_2\in H(\omega)$ and $k_1,k_2\in K(\omega)$.

Since $(H,\Gamma,\hat{E})$ is $DS\mathbb{G}$ -space, then there are open neighborhoods M,N of h_1 and $h_2$respectively in $H(\omega)$ such that $((M,N))$ is relatively compact in $F(\omega)$.

Then $M\times K(\omega)$ and $N\times K(\omega)$ are open neighborhoods of $(h_1,k_1),(h_2,k_2)$ respectively in $H(\omega)\times K(\omega)$.

Since $\left((M,N)\right)=((M\times K(\omega),N\times K(\omega))$

Then $((M\times K(\omega),N\times K(\omega))$ is relatively compact in $\acute{F}(\omega)$.

Then $\left(H\times K,\Gamma_{prod},\hat{E}\right)$ is $DS\mathbb{G}$ $-$space.

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