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Algebraic structures of $QIFSN(G)$

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Abstract

The purpose of this paper is to define the concepts of intersection, normality, direct product, homomorphism and anti-homomorphism of $QIFSN(G)$ and discuss their properties of them.

Keywords: Norms, Q-intuitionistic fuzzy subgroups, direct product and intersection, homomorphisms.

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1 Introduction

The concept of fuzzy sets was proposed by Zadeh [23]. The theory of fuzzy sets has several applications in real-life situations, and many scholars have researched fuzzy set theory. After the introduction of the concept of fuzzy sets, several research studies were conducted on the generalizations of fuzzy sets. The idea of intuitionistic fuzzy sets suggested by Atanassov [2, 3] is one of the extensions of fuzzy sets with better applicability. Applications of intuitionistic fuzzy sets appear in various fields, including medical diagnosis, optimization problems, and multicriteria decision making. The concept of fuzzy group was introduced by Rosenfled [21] and Anthony and Sherwood [1] gave the definition of fuzzy subgroup based on t-norm. Solairaju and Nagarajan [22] introduced the notion of Q- fuzzy groups. The First author by using norms, investigated some properties of fuzzy algebraic structures [4-20] specially in [4, 5, 6, 7, 8, 13] by using norms, defined and investigated some properties of Q-fuzzy subgroups, anti Q-fuzzy subgroups and Q-intuitionistic fuzzy subgroups($QIFSN(G)$). In this paper we define the intersection and direct product of two members of $QIFSN(G)$ and we prove that they will be $QIFSN(G)$. Next we introduce the concept of normality of $QIFSN(G)$ as $NQIFSN(G)$ and investigate some properties of them. Finaly we consider the image and pre image of $QIFSN(G)$ and $NQIFSN(G)$ under homomorphisms and anti-homomorphisms of groups.

[1]speaker

2 Preliminaries

This section contains some basic definitions and preliminary results which will be needed in the sequal. For more details we refer to [4, 5, 6, 7, 8, 13].

Definition 2.1. A group is a non-empty set G on which there is a binary operation $(a, b) \to ab$ such that
(1) if a and b belong to G then ab is also in G (closure),
(2) $a(bc) = (ab)c$ for all $a, b, c \in G$ (associativity),
(3) there is an element $e \in G$ such that $ae = ea = a$ for all $a \in G$ (identity),
(4) if $a \in G$, then there is an element $a^{-1} \in G$ such that $aa^{-1} = a^{-1}a = e$ (inverse).

One can easily check that this implies the unicity of the identity and of the inverse. A group G is called abelian if the binary operation is commutative, i.e., $ab = ba$ for all $a, b \in G$.

Remark 2.2. There are two standard notations for the binary group operation: either the additive notation, that is $(a, b) \to a + b$ in which case the identity is denoted by 0, or the multiplicative notation, that is $(a, b) \to ab$ for which the identity is denoted by e.

Proposition 2.3. *Let G be a group. Let H be a non-empty subset of G. The following are equivalent:*
(1) H is a subgroup of G.
(2) $x, y \in H$ implies $xy^{-1} \in H$ for all x, y.

Definition 2.4. Let $(G, .), (H, .)$ be any two groups. The function $f : G \to H$ is called a homomorphism (anti-homomorphism) if $f(xy) = f(x)f(y)(f(xy) = f(y)f(x))$, for all $x, y \in G$.

Definition 2.5. Let G be an arbitrary group with a multiplicative binary operation and identity e. A fuzzy subset of G, we mean a function from G into $[0, 1]$. The set of all fuzzy subsets of G is called the $[0, 1]$-power set of G and is denoted $[0, 1]^G$.

Definition 2.6. For sets X, Y and Z, $f = (f_1, f_2) : X \to Y \times Z$ is called a complex mapping if $f_1 : X \to Y$ and $f_2 : X \to Z$ are mappings.

Definition 2.7. Let X be a nonempty set. A complex mapping $A = (\mu_A, \nu_A) : X \to [0, 1] \times [0, 1]$ is called an intuitionistic fuzzy set (in short, IFS) in X such that $\mu_A, \nu_A \in [0, 1]^X$ and for all $x \in X$ we have $(\mu_A(x) + \nu_A(x)) \in [0, 1]$. In particular $\emptyset_X$ and U_X denote the intuitionistic fuzzy empty set and intuitionistic fuzzy whole set in X defined by $\emptyset_X(x) = (0, 1)$ and $U_X(x) = (1, 0)$, respectively. We will denote the set of all $IFSs$ in X as $IFS(X)$.

Definition 2.8. Let X be a nonempty set and let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be $IFSs$ in X. Then
(1) Inclusion: $A \subseteq B$ iff $\mu_A \leq \mu_B$ and $\nu_A \geq \nu_B$.
(2) Equality:$A = B$ iff $A \subseteq B$ and $B \subseteq A$.

Definition 2.9. A t-norm T is a function $T : [0, 1] \times [0, 1] \to [0, 1]$ having the following four properties:
(T1) $T(x, 1) = x$ (neutral element)
(T2) $T(x, y) \leq T(x, z)$ if $y \leq z$ (monotonicity)
(T3) $T(x, y) = T(y, x)$ (commutativity)
(T4) $T(x, T(y, z)) = T(T(x, y), z)$ (associativity),
for all $x, y, z \in [0, 1]$.

Corollary 2.10. *Let T be a t-norm. Then for all $x \in [0,1]$*
(1) $T(x,0) = 0$.
(2) $T(0,0) = 0$.

Example 2.11. (1) Standard intersection t-norm

$$T_m(x,y) = \min\{x,y\}.$$

(2) Bounded sum t-norm

$$T_b(x,y) = \max\{0, x+y-1\}.$$

(3) algebraic product t-norm

$$T_p(x,y) = xy.$$

(4) Drastic t-norm

$$T_D(x,y) = \begin{cases} y & \text{if } x = 1 \\ x & \text{if } y = 1 \\ 0 & \text{otherwise.} \end{cases}$$

(5) Nilpotent minimum t-norm

$$T_{nM}(x,y) = \begin{cases} \min\{x,y\} & \text{if } x+y > 1 \\ 0 & \text{otherwise.} \end{cases}$$

(6) Hamacher product t-norm

$$T_{H_0}(x,y) = \begin{cases} 0 & \text{if } x = y = 0 \\ \frac{xy}{x+y-xy} & \text{otherwise.} \end{cases}$$

The drastic t-norm is the pointwise smallest t-norm and the minimum is the pointwise largest t-norm:

$$T_D(x,y) \leq T(x,y) \leq T_{\min}(x,y)$$

for all $x, y \in [0,1]$.

Definition 2.12. A t-conorm C is a function $C : [0,1] \times [0,1] \to [0,1]$ having the following four properties:
(C1) $C(x,0) = x$
(C2) $C(x,y) \leq C(x,z)$ if $y \leq z$
(C3) $C(x,y) = C(y,x)$
(C4) $C(x, C(y,z)) = C(C(x,y), z)$,
for all $x, y, z \in [0,1]$.

Corollary 2.13. *Let C be a C-conorm. Then for all $x \in [0,1]$*
(1) $C(x,1) = 1$.
(2) $C(0,0) = 0$.

Example 2.14. (1) Standard union t-conorm

$$C_m(x,y) = \max\{x,y\}.$$

(2) Bounded sum t-conorm

$$C_b(x,y) = \min\{1, x+y\}.$$

(3) Algebraic sum t-conorm

$$C_p(x,y) = x+y-xy.$$

(4) Drastic t-conorm

$$C_D(x,y) = \begin{cases} y & \text{if } x=0 \\ x & \text{if } y=0 \\ 1 & \text{otherwise,} \end{cases}$$

dual to the drastic t-norm.

(5) Nilpotent maximum t-conorm , dual to the nilpotent minimum T-norm:

$$C_{nM}(x,y) = \begin{cases} \max\{x,y\} & \text{if } x+y<1 \\ 1 & \text{otherwise.} \end{cases}$$

(6) Einstein sum (compare the velocity-addition formula under special relativity)

$$C_{H_2}(x,y) = \frac{x+y}{1+xy}$$

is a dual to one of the Hamacher t-norms. Note that all t-conorms are bounded by the maximum and the drastic t-conorm:

$$C_{\max}(x,y) \leq C(x,y) \leq C_D(x,y)$$

for any t-conorm C and all $x,y \in [0,1]$.

Recall that t-norm $T($ t-conorm $C)$ is idempotent if for all $x \in [0,1]$, $T(x,x) = x(C(x,x) = x)$.

Lemma 2.15. *Let C be a t-conorm. Then*

$$T(T(x,y), T(w,z)) = T(T(x,w), T(y,z)),$$

$$C(C(x,y), C(w,z)) = C(C(x,w), C(y,z)),$$

for all $x,y,w,z \in [0,1]$.

Definition 2.16. Let $(G,.)$ be a group and Q be a non empty set. A bifuzzy (intuitionistic) fuzzy set $A = (\mu_A, \nu_A) \in IFS(G \times Q)$ is said to be a Q-intuitionistic fuzzy subgroup of G with respect to norms (t-norm T and t-conorm C) if the following conditions are satisfied:

(1)

$$A(xy,q) = (\mu_A(xy,q), \nu_A(xy,q)) \supseteq A(T(\mu_A(x,q), \mu_A(y,q)), C(\nu_A(x,q), \nu_A(y,q))),$$

(2)

$$A(x^{-1},q) = (\mu_A(x^{-1},q), \nu_A(x^{-1},q)) \supseteq A(x,q) = (\mu_A(x,q), \nu_A(x,q))$$

which mean:

(a) $\mu_A(xy,q) \geq T(\mu_A(x,q), \mu_A(y,q))$,

(b) $\nu_A(xy,q) \leq C(\nu_A(x,q), \nu_A(y,q))$,

(c) $\mu_A(x^{-1}, q) \geq \mu_A(x, q)$,
(d) $\nu_A(x^{-1}, q) \leq \nu_A(x, q)$,
for all $x, y \in G$ and $q \in Q$. Throughout this paper the set of all Q-intuitionistic fuzzy subgroups of G with respect to norms (t-norm T and t-conorm C) will be denoted by $QIFSN(G)$.

Proposition 2.17. *Let $A = (\mu_A, \nu_A) \in QIFSN(G)$ such that T and C be idempotent. Then*

$$A(e_G, q) \supseteq A(x, q)$$

for all $x \in G$ and $q \in Q$.

Proposition 2.18. *Let T and C be idempotent. Then*

$$A = (\mu_A, \nu_A) \in QIFSN(G)$$

if and only if

$$A(xy^{-1}, q) \supseteq A(T(\mu_A(x, q), \mu_A(y, q)), C(\nu_A(x, q), \nu_A(y, q)))$$

for all $x, y \in G$ and $q \in Q$.

3 Intersection, direct product and group homomorphisms of $QIFSN(G)$

Definition 3.1. Let $A = (\mu_A, \nu_A) \in QIFSN(G)$ and $B = (\mu_B, \nu_B) \in QIFSN(G)$. The intersection of A and B is defined by

$$\begin{aligned}(A \cap B)(x, q) &= ((\mu_A, \nu_A) \cap (\mu_B, \nu_B))(x, q) \\ &= (\mu_{A\cap B}, \nu_{A\cap B})(x, q) \\ &= (T(\mu_A(x, q), \mu_B(x, q)), C(\nu_A(x, q), \nu_B(x, q)))\end{aligned}$$

for all $x \in G$ and $q \in Q$.

Proposition 3.2. *Let $A = (\mu_A, \nu_A) \in QIFSN(G)$ and $B = (\mu_B, \nu_B) \in QIFSN(G)$. Then $A \cap B \in QIFSN(G)$.*

Proof. Let $x, y \in G, q \in Q$. Then

$$\begin{aligned}\mu_{A\cap B}(xy, q) &= T(\mu_A(xy, q), \mu_B(xy, q)) \\ &\geq T(T(\mu_A(x, q), \mu_A(y, q)), T(\mu_B(x, q), \mu_B(y, q))) \\ &= T(T(\mu_A(x, q), \mu_B(x, q)), T(\mu_A(y, q), \mu_B(y, q))) \ \ \textit{(Lemma 2.15)} \\ &= T(\mu_{A\cap B}(x, q), \mu_{A\cap B}(y, q))\end{aligned}$$

thus

$$\mu_{A\cap B}(xy, q) \geq T(\mu_{A\cap B}(x, q), \mu_{A\cap B}(y, q)). \qquad (a)$$

$$\begin{aligned}\nu_{A\cap B}(xy,q) &= C(\nu_A(xy,q),\nu_B(xy,q))\\ &\leq C(C(\nu_A(x,q),\nu_A(y,q)),C(\nu_B(x,q),\nu_B(y,q)))\\ &= C(C(\nu_A(x,q),\nu_B(x,q)),C(\nu_A(y,q),\nu_B(y,q))) \quad \textit{(Lemma 2.15)}\\ &= C(\nu_{A\cap B}(x,q),\nu_{A\cap B}(y,q))\end{aligned}$$

then

$$\nu_{A\cap B}(xy,q) \leq C(\nu_{A\cap B}(x,q),\nu_{A\cap B}(y,q)). \qquad (b)$$

(c)

$$\mu_{A\cap B}(x^{-1},q) = T(\mu_A(x^{-1},q),\mu_B(x^{-1},q)) \geq T(\mu_A(x,q),\mu_B(x,q)) = \mu_{A\cap B}(x,q).$$

(d)

$$\nu_{A\cap B}(x^{-1},q) = C(\nu_A(x^{-1},q),\nu_B(x^{-1},q)) \leq C(\nu_A(x,q),\nu_B(x,q)) = \nu_{A\cap B}(x,q).$$

Then from (a) to (d) we obtain that $A\cap B = (\mu_{A\cap B},\nu_{A\cap B}) \in QIFSN(G)$. □

Definition 3.3. Let $A = (\mu_A,\nu_A) \in QIFSN(G)$. We say that A is a normal if $A(xyx^{-1},q) = A(y,q)$ for all $x,y \in G$ and $q \in Q$. We denote by $NQIFSN(G)$ the set of all normal Q-intuitionistic fuzzy subgroups of G with respect to norms (t-norm T and t-conorm C).

Proposition 3.4. *Let $A = (\mu_A,\nu_A) \in NQIFSN(G)$ and $B = (\mu_B,\nu_B) \in NQIFSN(G)$. Then $A\cap B \in NQIFSN(G)$.*

Proof. Let $x,y,\in G$ and $q \in Q$. Then

$$\mu_{A\cap B}(xyx^{-1},q) = T(\mu_A(xyx^{-1},q),\mu_B(xyx^{-1},q)) = T(\mu_A(y,q),\mu_B(y,q)) = \mu_{A\cap B}(y,q)$$

and

$$\nu_{A\cap B}(xyx^{-1},q) = C(\nu_A(xyx^{-1},q),\nu_B(xyx^{-1},q)) = C(\nu_A(y,q),\nu_B(y,q)) = \nu_{A\cap B}(y,q).$$

Thus

$$\begin{aligned}(A\cap B)(xyx^{-1},q) &= (\mu_{A\cap B},\nu_{A\cap B})(xyx^{-1},q)\\ &= (\mu_{A\cap B}(xyx^{-1},q),\nu_{A\cap B}(xyx^{-1},q))\\ &= (\mu_{A\cap B}(y,q),\nu_{A\cap B}(y,q))\\ &= (A\cap B)(y,q)\end{aligned}$$

and then $A\cap B \in NQIFSN(G)$. □

Corollary 3.5. *Let $I_n = \{1,2,...,n\}$. If $\{\mu_i \mid i \in I_n\} \subseteq NQIFSN(G)$, Then*

$$\mu = \cap_{i\in I_n}\mu_i \in NQIFSN(G).$$

Definition 3.6. Let $(G,.),(H,.)$ be any two groups such that $A=(\mu_A,\nu_A)\in QIFSN(G)$ and $B=(\mu_B,\nu_B)\in QIFSN(H)$. The product of A and B, is defined as

$$\begin{aligned}(A\times B)((x,y),q)&=(\mu_A,\nu_A)\times(\mu_B,\nu_B)((x,y),q)\\&=(\mu_{A\times B},\nu_{A\times B})((x,y),q)\\&=(T(\mu_A(x,q),\mu_B(y,q)),C(\nu_A(x,q),\nu_B(y,q)))\end{aligned}$$

for all $x\in G, y\in H, q\in Q$.

Note that $\mu_{A\times B},\nu_{A\times B}\in[0,1]^{(G\times H)\times Q}$ and throughout this paper, H denotes an arbitrary group with identity element e_H.

Proposition 3.7. *Let $A=(\mu_A,\nu_A)\in QIFSN(G)$ and $B=(\mu_B,\nu_B)\in QIFSN(H)$. Then $A\times B\in QIFSN(G\times H)$.*

Proof. Let $(x_1,y_1),(x_2,y_2)\in G\times H$ and $q\in Q$. Then

$$\begin{aligned}\mu_{A\times B}((x_1,y_1)(x_2,y_2),q)&=(\mu_{A\times B})((x_1x_2,y_1y_2),q)\\&=T(\mu_A(x_1x_2,q),\mu_B(y_1y_2,q))\\&\geq T(T(\mu_A(x_1,q),\mu_A(x_2,q)),T(\mu_B(y_1,q),\mu_B(y_2,q)))\\&=T(T(\mu_A(x_1,q),\mu_B(y_1,q)),T(\mu_A(x_2,q),\mu_B(y_2,q)))\quad\textit{(Lemma 2.15)}\\&=T(\mu_{A\times B}((x_1,y_1),q),\mu_{A\times B}((x_2,y_2),q))\end{aligned}$$

hence

$$\mu_{A\times B}((x_1,y_1)(x_2,y_2),q)\geq T(\mu_{A\times B}((x_1,y_1),q),\mu_{A\times B}((x_2,y_2),q)).\qquad\textit{(a)}$$

$$\begin{aligned}\nu_{A\times B}((x_1,y_1)(x_2,y_2),q)&=(\nu_{A\times B})((x_1x_2,y_1y_2),q)\\&=C(\nu_A(x_1x_2,q),\nu_B(y_1y_2,q))\\&\leq C(C(\nu_A(x_1,q),\nu_A(x_2,q)),C(\nu_B(y_1,q),\nu_B(y_2,q)))\\&=C(C(\nu_A(x_1,q),\nu_B(y_1,q)),C(\nu_A(x_2,q),\nu_B(y_2,q)))\quad\textit{(Lemma 2.15)}\\&=C(\nu_{A\times B}((x_1,y_1),q),\nu_{A\times B}((x_2,y_2),q))\end{aligned}$$

then

$$\nu_{A\times B}((x_1,y_1)(x_2,y_2),q)\leq C(\nu_{A\times B}((x_1,y_1),q),\nu_{A\times B}((x_2,y_2),q)).\qquad\textit{(b)}$$

$$\begin{aligned}\mu_{A\times B}((x_1,y_1)^{-1},q)&=\mu_{A\times B}((x_1^{-1},y_1^{-1}),q)\\&=T(\mu_A(x_1^{-1},q),\mu_B(y_1^{-1},q))\\&\geq T(\mu_A(x_1,q),\mu_B(y_1,q))\\&=\mu_{A\times B}((x_1,y_1),q).\end{aligned}$$

so

$$\mu_{A\times B}((x_1,y_1)^{-1},q)\geq \mu_{A\times B}((x_1,y_1),q). \qquad (c)$$

$$\begin{aligned}\nu_{A\times B}((x_1,y_1)^{-1},q)&=\nu_{A\times B}((x_1^{-1},y_1^{-1}),q)\\&=C(\nu_A(x_1^{-1},q),\nu_B(y_1^{-1},q))\\&\leq C(\nu_A(x_1,q),\nu_B(y_1,q))\\&=\nu_{A\times B}((x_1,y_1),q)\end{aligned}$$

then

$$\nu_{A\times B}((x_1,y_1)^{-1},q)\leq \nu_{A\times B}((x_1,y_1),q). \qquad (d)$$

Hence (a) to (d) give us that $A\times B=(\mu_{A\times B},\nu_{A\times B})\in QIFSN(G\times H)$. □

Proposition 3.8. *Let $A=(\mu_A,\nu_A)\in IFS(G\times Q)$ and $B=(\mu_B,\nu_B)\in IFS(H\times Q)$. If T and C be idempotent and $A\times B\in QIFSN(G\times H)$, then at least one of the following two statements must hold.*
(1) $B(e_H,q)\supseteq A(x,q)$, for all $x\in G$ and $q\in Q$,
(2) $A(e_G,q)\supseteq B(y,q)$, for all $y\in H$ and $q\in Q$.

Proof. Let none of the statements (1) and (2) holds, then we can find $g\in G$ and $h\in H$ such that $A(g,q)\supset B(e_H,q)$ and $B(h,q)\supset A(e_G,q)$. Then we obtain that $\mu_A(g,q)>\mu_B(e_H,q)$ and $\nu_A(g,q)<\nu_B(e_H,q)$ and $\mu_B(h,q)>\mu_A(e_G,q)$ and $\nu_B(h,q)<\nu_A(e_G,q)$. Now

$$\begin{aligned}(A\times B)((g,h),q)&=(\mu_{A\times B},\nu_{A\times B})((g,h),q)\\&=(T(\mu_A(g,q),\mu_B(h,q)),C(\nu_A(g,q),\nu_B(h,q)))\\&\supset(T(\mu_B(e_H,q),\mu_A(e_G,q)),C(\nu_B(e_H,q),\nu_A(e_G,q)))\\&=(T(\mu_A(e_G,q),\mu_B(e_H,q)),C(\nu_A(e_G,q),\nu_B(e_H,q)))\\&=(\mu_{A\times B}((e_G,e_H),q),\nu_{A\times B}((e_G,e_H),q))\\&=(A\times B)((e_G,e_H),q)\end{aligned}$$

then

$$(A\times B)((g,h),q)\supset(A\times B)((e_G,e_H),q)$$

and it is contradiction with $A\times B\in QIFSN(G\times H)$ as Propositions 2.17 and 3.7. This complets the proof. □

Proposition 3.9. *Let $A=(\mu_A,\nu_A)\in IFS(G\times Q)$ and $B=(\mu_B,\nu_B)\in IFS(H\times Q)$. Let T and C be idempotent and $A\times B\in QIFSN(G\times H)$. Then we have the following statements.*
(1) If $A(x,q)\subseteq B(e_H,q)$, then $A\in QIFSN(G)$ for all $x\in G$ and $q\in Q$.
(2) If $B(x,q)\subseteq A(e_G,q)$, then $B\in QIFSN(H)$ for all $x\in H$ and $q\in Q$.
(3) Either $A\in QIFSN(G)$ or $B\in QIFSN(H)$.

Proof. (1) Let

$$A(x,q)=(\mu_A(x,q),\nu_A(x,q))\subseteq B(e_H,q)=(\mu_B(e_H,q),\nu_B(e_H,q)) \quad (\star)$$

for all $x\in G, q\in Q$.
From $(\star)$ we have that $\mu_A(x,q)\leq \mu_B(e_H,q)$ so we can obtain that $\mu_A(xy^{-1},q)\leq \mu_B(e_H,q)$ and $\mu_A(y,q)\leq \mu_B(e_H,q)$ for all $x,y\in G, q\in Q$ so then

$$\begin{aligned}
\mu_A(xy^{-1},q) &= T(\mu_A(xy^{-1},q),\mu_B(e_H,q))\\
&= T(\mu_A(xy^{-1},q),\mu_B(e_He_H,q))\\
&= \mu_{A\times B}((xy^{-1},e_He_H),q)\\
&= \mu_{A\times B}((x,e_H)(y^{-1},e_H),q)\\
&\geq T(\mu_{A\times B}((x,e_H),q),\mu_{A\times B}((y^{-1},e_H),q))\\
&\geq T(\mu_{A\times B}((x,e_H),q),\mu_{A\times B}((y,e_H),q))\\
&= T(T(\mu_A(x,q),\mu_B(e_H,q)),T(\mu_A(y,q),\mu_B(e_H,q)))\\
&\geq T(T(\mu_A(x,q),\mu_A(x,q)),T(\mu_A(y,q),\mu_A(y,q)))\\
&= T(\mu_A(x,q),\mu_A(y,q))
\end{aligned}$$

and then

$$\mu_A(xy^{-1},q)\geq T(\mu_A(x,q),\mu_A(y,q)). \qquad (a)$$

Also from $(\star)$ we get that $\nu_A(x,q)\geq \nu_B(e_H,q)$ so we can obtain that $\nu_A(xy^{-1},q)\geq \nu_B(e_H,q)$ and $\nu_A(y,q)\geq \nu_B(e_H,q)$ for all $x,y\in G, q\in Q$ so then

$$\begin{aligned}
\nu_A(xy^{-1},q) &= C(\nu_A(xy^{-1},q),\nu_B(e_H,q))\\
&= C(\nu_A(xy^{-1},q),\nu_B(e_He_H,q))\\
&= \nu_{A\times B}((xy^{-1},e_He_H),q)\\
&= \nu_{A\times B}((x,e_H)(y^{-1},e_H),q)\\
&\leq C(\nu_{A\times B}((x,e_H),q),\nu_{A\times B}((y^{-1},e_H),q))\\
&\leq C(\nu_{A\times B}((x,e_H),q),\nu_{A\times B}((y,e_H),q))\\
&= C(C(\nu_A(x,q),\nu_B(e_H,q)),C(\nu_A(y,q),\nu_B(e_H,q)))\\
&\leq C(C(\nu_A(x,q),\nu_A(x,q)),C(\nu_A(y,q),\nu_A(y,q)))\\
&= C(\nu_A(x,q),\nu_A(y,q))
\end{aligned}$$

and thus

$$\nu_A(xy^{-1},q)\leq C(\nu_A(x,q),\nu_A(y,q)). \qquad (b)$$

Now (a) and (b) give us that

$$A(xy^{-1},q)=(\mu_A(xy^{-1},q),\nu_A(xy^{-1},q))\supseteq (T(\mu_A(x,q),\mu_A(y,q)),C(\nu_A(x,q),\nu_A(y,q)))$$

and by using Proposition 2.18 we obtain that $A=(\mu_A,\nu_A)\in QIFSN(G)$.
(2) Let

$$B(x,q)=(\mu_B(x,q),\nu_B(x,q))\subseteq A(e_G,q)=(\mu_A(e_G,q),\nu_A(e_G,q)) \quad (*)$$

for all $x \in H, q \in Q$. Then $\mu_B(x,q) \leq \mu_A(e_G,q)$ then $\mu_B(xy^{-1},q) \leq \mu_A(e_G,q)$ and $\mu_B(y,q) \leq \mu_A(e_G,q)$ for all $x, y \in H, q \in Q$. Then

$$\begin{aligned}
\mu_B(xy^{-1},q) &= T(\mu_B(xy^{-1},q), \mu_A(e_G,q)) \\
&= T(\mu_B(xy^{-1},q), \mu_A(e_Ge_G,q)) \\
&= T(\mu_A(e_Ge_G,q), \mu_B(xy^{-1},q)) \\
&= \mu_{A\times B}((e_Ge_G, xy^{-1}),q) \\
&= \mu_{A\times B}((e_G,x)(e_G,y^{-1}),q) \\
&\geq T(\mu_{A\times B}((e_G,x),q), \mu_{A\times B}((e_G,y^{-1}),q)) \\
&= T(\mu_{A\times B}((e_G,x),q), \mu_{A\times B}((e_G,y),q)) \\
&= T(T(\mu_A(e_G,q), \mu_B(x,q)), T(\mu_A(e_G,q), \mu_B(y,q))) \\
&\geq T(T(\mu_B(x,q), \mu_B(x,q)), T(\mu_B(y,q), \mu_B(y,q))) \\
&= T(\mu_B(x,q), \mu_B(y,q))
\end{aligned}$$

and thus

$$\mu_B(xy^{-1},q) \geq T(\mu_B(x,q), \mu_B(y,q)). \qquad (c)$$

Also from $(*)$ we will have that $\nu_B(x,q) \geq \nu_A(e_G,q)$ then $\nu_B(xy^{-1},q) \geq \nu_A(e_G,q)$ and $\nu_B(y,q) \geq \nu_A(e_G,q)$ for all $x, y \in H, q \in Q$. Then

$$\begin{aligned}
\nu_B(xy^{-1},q) &= C(\nu_B(xy^{-1},q), \nu_A(e_G,q)) \\
&= C(\nu_B(xy^{-1},q), \nu_A(e_Ge_G,q)) \\
&= C(\nu_A(e_Ge_G,q), \nu_B(xy^{-1},q)) \\
&= \nu_{A\times B}((e_Ge_G, xy^{-1}),q) \\
&= \nu_{A\times B}((e_G,x)(e_G,y^{-1}),q) \\
&\leq C(\nu_{A\times B}((e_G,x),q), \nu_{A\times B}((e_G,y^{-1}),q)) \\
&= C(\nu_{A\times B}((e_G,x),q), \nu_{A\times B}((e_G,y),q)) \\
&= C(C(\nu_A(e_G,q), \nu_B(x,q)), C(\nu_A(e_G,q), \nu_B(y,q))) \\
&\leq C(C(\nu_B(x,q), \nu_B(x,q)), C(\nu_B(y,q), \nu_B(y,q))) \\
&= C(\nu_B(x,q), \nu_B(y,q))
\end{aligned}$$

and so

$$\nu_B(xy^{-1},q) \leq C(\nu_B(x,q), \nu_B(y,q)). \qquad (d)$$

Now as (c) and (d) we give that

$$B(xy^{-1},q) = (\mu_B(xy^{-1},q), \nu_B(xy^{-1},q)) \supseteq (T(\mu_B(x,q), \mu_B(y,q)), C(\nu_B(x,q), \nu_B(y,q)))$$

and Proposition 2.18 gives us $B = (\mu_B, \nu_B) \in QIFSN(H)$.
(3) Straight forward. □

Definition 3.10. Let Let $(G,.),(H,.)$ be any two groups and $\varphi : G \to H$ be a morphism such that $A = (\mu_A, \nu_A) \in IFS(G \times Q)$ and $B = (\mu_B, \nu_B) \in IFS(H \times Q)$.
For all $x \in G, y \in H, q \in Q$ we define

$$\varphi(A)(y,q) = (\varphi(\mu_A)(y,q), \varphi(\nu_A)(y,q))$$
$$= \begin{cases} (\sup\{\mu_A(x,q) \mid (x,q) \in G \times Q, \varphi(x) = y\}, \inf\{\nu_A(x) \mid (x,q) \in G \times Q, \varphi(x) = y\}) & \text{if } \varphi^{-1}(y) \neq \emptyset \\ (0,1) & \text{if } \varphi^{-1}(y) = \emptyset. \end{cases}$$

Also $\varphi^{-1}(B)(x,q) = (\varphi^{-1}(\mu_B)(x,q), \varphi^{-1}(\nu_B)(x,q)) = (\mu_B(\varphi(x,q)), \nu_B(\varphi(x,q)))$.

Proposition 3.11. *Let φ be an epimorphism from group G into group H. If $A = (\mu_A, \nu_A) \in QIFSN(G)$ then $\varphi(A) \in QIFSN(H)$.*

Proof. Let $h_1, h_2 \in H$ and $q \in Q$. Then

$$\begin{aligned}\varphi(\mu_A)(h_1h_2,q) &= \sup\{\mu_A(g_1g_2,q) \mid g_1,g_2 \in G, \varphi(g_1) = h_1, \varphi(g_2) = h_2\} \\ &\geq \sup\{T(\mu_A(g_1,q), \mu_A(g_2,q)) \mid g_1,g_2 \in G, \varphi(g_1) = h_1, \varphi(g_2) = h_2\} \\ &= T((\sup\{\mu_A(g_1,q) \mid g_1 \in G, \varphi(g_1) = h_1\}), (\sup\{\mu_A(g_2,q) \mid g_2 \in G, \varphi(g_2) = h_2\})) \\ &= T(\varphi(\mu_A)(h_1,q), \varphi(\mu)(h_2,q))\end{aligned}$$

and then

$$\varphi(\mu_A)(h_1h_2,q) \geq T(\varphi(\mu_A)(h_1,q), \varphi(\mu_A)(h_2,q)). \ (a)$$

Also

$$\begin{aligned}\varphi(\nu_A)(h_1h_2,q) &= \inf\{\nu_A(g_1g_2,q) \mid g_1,g_2 \in G, \varphi(g_1) = h_1, \varphi(g_2) = h_2\} \\ &\leq \inf\{C(\nu_A(g_1,q), \nu_A(g_2,q)) \mid g_1,g_2 \in G, \varphi(g_1) = h_1, \varphi(g_2) = h_2\} \\ &= C((\inf\{\nu_A(g_1,q) \mid g_1 \in G, \varphi(g_1) = h_1\}), (\inf\{\nu_A(g_2,q) \mid g_2 \in G, \varphi(g_2) = h_2\})) \\ &= C(\varphi(\nu_A)(h_1,q), \varphi(\nu_A)(h_2,q))\end{aligned}$$

and so

$$\varphi(\nu_A)(h_1h_2,q) \leq C(\varphi(\nu_A)(h_1,q), \varphi(\nu_A)(h_2,q)). \ (b)$$

Later

$$\begin{aligned}\varphi(\mu_A)(h_1^{-1},q) &= \sup\{\mu_A(g_1^{-1},q) \mid g_1 \in G, \varphi(g_1^{-1}) = h_1^{-1}\} \\ &\geq \sup\{\mu(g_1,q) \mid g_1 \in G, \varphi(g_1,q) = h_1\} \\ &= \varphi(\mu_A)(h_1,q)\end{aligned}$$

and

$$\varphi(\mu_A)(h_1^{-1},q) \geq \varphi(\mu_A)(h_1,q). \ (c)$$

Finally

$$\begin{aligned}\varphi(\nu_A)(h_1^{-1},q) &= \inf\{\nu_A(g_1^{-1},q) \mid g_1 \in G, \varphi(g_1^{-1}) = h_1^{-1}\} \\ &\leq \inf\{\mu(g_1,q) \mid g_1 \in G, \varphi(g_1,q) = h_1\} \\ &= \varphi(\nu_A)(h_1,q)\end{aligned}$$

and

$$\varphi(\nu_A)(h_1^{-1},q) \leq \varphi(\nu_A)(h_1,q). \ (d)$$

Therefore fram (a) to (d) we obtain that $\varphi(A) \in QIFSN(H)$. □

Proposition 3.12. *Let φ be a homorphism from group G into group H. If $B = (\mu_B, \nu_B) \in QIFSN(H)$, then $\varphi^{-1}(B) \in QIFSN(G)$.*

Proof. Let $g_1, g_2 \in G$ and $q \in Q$. Then

$$\begin{aligned}
\varphi^{-1}(\mu_B)(g_1g_2,q) &= \mu_B(\varphi(g_1g_2),q) \\
&= \mu_B(\varphi(g_1)\varphi(g_2),q) \\
&\geq T(\mu_B(\varphi(g_1),q), \mu_B(\varphi(g_2),q)) \\
&= T(\varphi^{-1}(\mu_B)(g_1,q), \varphi^{-1}(\mu_B)(g_2,q))
\end{aligned}$$

then

$$\varphi^{-1}(\mu_B)(g_1g_2,q) \geq T(\varphi^{-1}(\mu_B)(g_1,q), \varphi^{-1}(\mu_B)(g_2,q)). \ (a)$$

Also

$$\begin{aligned}
\varphi^{-1}(\nu_B)(g_1g_2,q) &= \nu_B(\varphi(g_1g_2),q) \\
&= \nu_B(\varphi(g_1)\varphi(g_2),q) \\
&\leq C(\nu_B(\varphi(g_1),q), \nu_B(\varphi(g_2),q)) \\
&= C(\varphi^{-1}(\nu_B)(g_1,q), \varphi^{-1}(\nu_B)(g_2,q))
\end{aligned}$$

so

$$\varphi^{-1}(\nu_B)(g_1g_2,q) \leq C(\varphi^{-1}(\nu_B)(g_1,q), \varphi^{-1}(\nu_B)(g_2,q)). \ (b)$$

Moreover

$$\varphi^{-1}(\mu_B)(g_1^{-1},q) = \mu_B(\varphi(g_1^{-1}),q) = \mu_B(\varphi^{-1}(g_1),q) \geq \mu_B(\varphi(g_1),q) = \varphi^{-1}(\mu_B)(g_1,q)$$

and

$$\varphi^{-1}(\mu_B)(g_1^{-1},q) \geq \varphi^{-1}(\mu_B)(g_1,q). \ (c)$$

Finally

$$\varphi^{-1}(\nu_B)(g_1^{-1},q) = \nu_B(\varphi(g_1^{-1}),q) = \nu_B(\varphi^{-1}(g_1),q) \leq \nu_B(\varphi(g_1),q) = \varphi^{-1}(\nu_B)(g_1,q)$$

and

$$\varphi^{-1}(\nu_B)(g_1^{-1},q) \leq \varphi^{-1}(\nu_B)(g_1,q). \ (d)$$

Thus from (a) to (d) we give that $\varphi^{-1}(B) \in QIFSN(G)$. □

Proposition 3.13. *Let φ be an anti-homorphism from group G into group H. If $B = (\mu_B, \nu_B) \in QIFSN(H)$, then $\varphi^{-1}(B) \in QIFSN(G)$.*

Proof. Let $g_1, g_2 \in G$ and $q \in Q$. Then

$$\begin{aligned}\varphi^{-1}(\mu_B)(g_1g_2, q) &= \mu_B(\varphi(g_1g_2), q)\\ &= \mu_B(\varphi(g_2)\varphi(g_1), q)\\ &\geq T(\mu_B(\varphi(g_2), q), \mu_B(\varphi(g_1), q))\\ &= T(\mu_B(\varphi(g_1), q), \mu_B(\varphi(g_2), q))\\ &= T(\varphi^{-1}(\mu_B)(g_1, q), \varphi^{-1}(\mu_B)(g_2, q))\end{aligned}$$

then

$$\varphi^{-1}(\mu_B)(g_1g_2, q) \geq T(\varphi^{-1}(\mu_B)(g_1, q), \varphi^{-1}(\mu_B)(g_2, q)). \ (a)$$

Also

$$\begin{aligned}\varphi^{-1}(\nu_B)(g_1g_2, q) &= \nu_B(\varphi(g_1g_2), q)\\ &= \nu_B(\varphi(g_2)\varphi(g_1), q)\\ &\leq C(\nu_B(\varphi(g_2), q), \nu_B(\varphi(g_1), q))\\ &= C(\nu_B(\varphi(g_1), q), \nu_B(\varphi(g_2), q))\\ &= C(\varphi^{-1}(\nu_B)(g_1, q), \varphi^{-1}(\nu_B)(g_2, q))\end{aligned}$$

so

$$\varphi^{-1}(\nu_B)(g_1g_2, q) \leq C(\varphi^{-1}(\nu_B)(g_1, q), \varphi^{-1}(\nu_B)(g_2, q)). \ (b)$$

Moreover

$$\varphi^{-1}(\mu_B)(g_1^{-1}, q) = \mu_B(\varphi(g_1^{-1}), q) = \mu_B(\varphi^{-1}(g_1), q) \geq \mu_B(\varphi(g_1), q) = \varphi^{-1}(\mu_B)(g_1, q)$$

and

$$\varphi^{-1}(\mu_B)(g_1^{-1}, q) \geq \varphi^{-1}(\mu_B)(g_1, q). \ (c)$$

Finally

$$\varphi^{-1}(\nu_B)(g_1^{-1}, q) = \nu_B(\varphi(g_1^{-1}), q) = \nu_B(\varphi^{-1}(g_1), q) \leq \nu_B(\varphi(g_1), q) = \varphi^{-1}(\nu_B)(g_1, q)$$

and

$$\varphi^{-1}(\nu_B)(g_1^{-1}, q) \leq \varphi^{-1}(\nu_B)(g_1, q). \ (d)$$

Thus from (a) to (d) we give that $\varphi^{-1}(B) \in QIFSN(G)$. □

Proposition 3.14. *Let $A = (\mu_A, \nu_A) \in NQIFSN(G)$ and H be a group. Suppose that $\varphi : G \to H$ be an epimorphism. Then $\varphi(A) \in NQIFSN(H)$.*

Proof. By Proposition 3.11 we have $\varphi(A) \in QIFSN(H)$. Let $x, y \in H$ and $q \in Q$. Since φ is a surjection, $\varphi(u) = x$ for some $u \in G$. Then

$$
\begin{aligned}
\varphi(\mu_A)(xyx^{-1},q) &= \sup\{\mu_A(w,q) \mid w \in G, \varphi(w) = xyx^{-1}\}\\
&= \sup\{\mu_A(u^{-1}wu,q) \mid w \in G, \varphi(u^{-1}wu) = y\}\\
&= \sup\{\mu_A(u^{-1}wu,q) \mid w \in G, \varphi(u^{-1})\varphi(w)\varphi(u) = y\}\\
&= \sup\{\mu_A(u^{-1}wu,q) \mid w \in G, \varphi^{-1}(u)\varphi(w)\varphi(u) = y\}\\
&= \sup\{\mu_A(w,q) \mid w \in G, \varphi(w) = y\}\\
&= \varphi(\mu_A)(y,q).
\end{aligned}
$$

Thus

$$\varphi(\mu_A)(xyx^{-1},q) = \varphi(\mu_A)(y,q). \quad (a)$$

Also

$$
\begin{aligned}
\varphi(\nu_A)(xyx^{-1},q) &= \inf\{\nu_A(w,q) \mid w \in G, \varphi(w) = xyx^{-1}\}\\
&= \inf\{\nu_A(u^{-1}wu,q) \mid w \in G, \varphi(u^{-1}wu) = y\}\\
&= \inf\{\nu_A(u^{-1}wu,q) \mid w \in G, \varphi(u^{-1})\varphi(w)\varphi(u) = y\}\\
&= \inf\{\nu_A(u^{-1}wu,q) \mid w \in G, \varphi^{-1}(u)\varphi(w)\varphi(u) = y\}\\
&= \inf\{\nu_A(w,q) \mid w \in G, \varphi(w) = y\}\\
&= \varphi(\nu_A)(y,q).
\end{aligned}
$$

Thus

$$\varphi(\nu_A)(xyx^{-1},q) = \varphi(\nu_A)(y,q). \quad (b)$$

Therefore from (a) and (b) we get that

$$\varphi(A)(xyx^{-1},q) = (\varphi(\mu_A)(xyx^{-1},q), \varphi(\nu_A)(xyx^{-1},q))$$

$$= (\varphi(\mu_A)(y,q), \varphi(\nu_A)(y,q)) = \varphi(A)(y,q)$$

and then $\varphi(A) \in NQIFSN(H)$. □

Proposition 3.15. *Let $B = (\mu_B, \nu_B) \in NQIFSN(H)$ and $\varphi : G \to H$ is a group homomorphism.Then $\varphi^{-1}(B) \in NQIFSN(G)$.*

Proof. As Proposition 3.12 we obtain that $\varphi^{-1}(B) \in QBFSN(G)$. Now for any $x, y \in G$ and $q \in Q$ we obtain

$$
\begin{aligned}
\varphi^{-1}(\mu_B)(xyx^{-1},q) &= \mu_B(\varphi(xyx^{-1}),q)\\
&= \mu_B(\varphi(x)\varphi(y)\varphi(x^{-1}),q)\\
&= \mu_B(\varphi(x)\varphi(y)\varphi^{-1}(x),q)\\
&= \mu_B(\varphi(y),q)\\
&= \varphi^{-1}(\mu_B)(y,q).
\end{aligned}
$$

Then

$$\varphi^{-1}(\mu_B)(xyx^{-1}, q) = \varphi^{-1}(\mu_B)(y, q). \quad (a)$$

Also

$$\begin{aligned}
\varphi^{-1}(\nu_B)(xyx^{-1}, q) &= \nu_B(\varphi(xyx^{-1}), q) \\
&= \nu_B(\varphi(x)\varphi(y)\varphi(x^{-1}), q) \\
&= \nu_B(\varphi(x)\varphi(y)\varphi^{-1}(x), q) \\
&= \nu_B(\varphi(y), q) \\
&= \varphi^{-1}(\nu_B)(y, q).
\end{aligned}$$

Then

$$\varphi^{-1}(\nu_B)(xyx^{-1}, q) = \varphi^{-1}(\nu_B)(y, q). \quad (b)$$

Therefore (a) and (b) give us that

$$\begin{aligned}
\varphi^{-1}(B)(xyx^{-1}, q) &= (\varphi^{-1}(\mu_B)(xyx^{-1}, q), \varphi^{-1}(\nu_B)(xyx^{-1}, q)) \\
&= (\varphi^{-1}(\mu_B)(y, q), \varphi^{-1}(\nu_B)(y, q)) \\
&= \varphi^{-1}(B)(y, q)
\end{aligned}$$

and then $\varphi^{-1}(B) \in NQIFSN(G)$. □

4 Open problem

In this paper, we defined the concepts of intersection, normality, direct product, homomorphism and anti homomorphism of $QIFSN(G)$ and investigated their properties of them. Now one can study and obtain $QIFSN(M)$ of R-modules M as we did $QIFSN(G)$ for groups and this can be an open problem.

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