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Characterization of Fuzzy, Intuitionistic Fuzzy and Neutrosophic Fuzzy Matrices

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ABSTRACT

In this research endeavour, we will undertake a comprehensive examination of the distinctions existing among various types of matrices, including real matrices, fuzzy matrices, intuitionistic fuzzy matrices, and neutrosophic fuzzy matrices. To enhance our understanding of these matrix types, we will illustrate the concepts with practical numerical examples. Additionally, we will provide a thorough exploration of the limitations and advantages associated with each of these matrix types.

KEYWORDS: Fuzzy Matrices, Intuitionistic Fuzzy Matrices, Neutrosophic Fuzzy Matrices, Interval valued Fuzzy Matrices, Interval valued Intuitionistic Fuzzy Matrices, Interval valued Neutrosophic Fuzzy Matrices.

1.INTRODUCTION

Fuzzy matrices, introduced by Zadeh [1] are an extension of traditional matrices designed to capture and represent the concept of fuzziness and uncertainty. They are a fundamental component of fuzzy set theory, which revolutionized how we handle vague or imprecise data. Fuzzy matrices assign each element a value between 0 and 1, representing the degree of membership of an element in specific set. Fuzzy matrices are instrumental in handling imprecise data. They allow us to represent membership with varying degrees of certainty, making them valuable in fields like artificial intelligence, where uncertainty is prevalent. Fuzzy matrices are widely used in fields such as control systems, decision-making, pattern recognition, and artificial intelligence to handle fuzzy logic and fuzzy reasoning. Thomason [3] has studied Convergence of powers of a fuzzy matrix. Ben-Israel and Greville [4] have studied generalized inverses: theory and applications. Kim and Roush [5] have discussed Generalized fuzzy matrices. Meenakshi [9] has studied Fuzzy Matrix Theory and Applications. Punithavalli and Anandhkumar[12] have studied Interval Valued Secondary k-Kernel Symmetric Fuzzy Matrices.

Intuitionistic fuzzy matrices, introduced by Krassimir Atanassov [2], take the concept of fuzzy matrices a step further. In addition to capturing membership degrees, they also account for non-membership degrees and hesitancy, providing a more comprehensive framework for handling uncertainty. Each element in an intuitionistic fuzzy matrix has three values: μ (membership), ν (non-membership), and λ (hesitancy).Intuitionistic fuzzy matrices are particularly valuable when decision-makers need to express their doubts or conflicts in a more detailed manner. They excel in multi-criteria decision-making and expert systems. Intuitionistic fuzzy matrices find applications in multi-criteria decision-making, expert systems, and situations where decision-makers need to express their uncertainty in a more granular manner. Anandhkumar, Kamalakannan, Chitra, Said [6] have studied Pseudo similarity of neutrosophic fuzzy matrices. Anandhkumar, Kanimozhi, Chithra, Kamalakannan, Said [7] have studied On various

Inverse of neutrosophic fuzzy matrices. . Punithavalli and Anandhkumar [10] have discussed Secondary k-Kernel Symmetric Intuitionistic

Neutrosophic matrices, introduced by Florentin Smarandache, are specifically designed to manage problems involving indeterminacy, inconsistency, and partial truth. They offer a unique approach for dealing with elements that can be true, indeterminate, or false.Each element in a neutrosophic matrix can take one of three values: T (true), I (indeterminate), and F (false), representing the truth, neutrality, or falsity of an element's membership to a set. Neutrosophic matrices are indispensable in situations where information is incomplete or inconsistent. They provide a more versatile tool for decision-making in uncertain environments. Neutrosophic matrices are applied in areas such as decision-making in uncertain environments, information fusion in sensor networks, and situations where information is inconsistent or incomplete. Smarandache [8] has studied Neutrosophic set, a generalization of the intuitionistic fuzzy set. Anandhkumar; Punithavalli; Soupramanien; Said Broumi [11] have discussed Generalized Symmetric Neutrosophic Fuzzy Matrices. Anandhkumar. Kanimozhi,. Chithr, .Kamalakannan,.Said .[13] have studied On various Inverse of Neutrosophic Fuzzy Matrices.

Anandhkumar has taken the initiative to bring forth novel concepts like Range and Column Symmetric Intuitionistic Fuzzy Matrices, Range and Column Symmetric Neutrosophic Fuzzy Matrices, Interval valued Neutrosophic fuzzy matrix, Interval valued Secondary k-kernel symmetric fuzzy matrix, Interval valued Secondary k-Range symmetric Intuitionistic fuzzy matrix. In addition to introducing these new concepts, Anandhkumar has provided concrete numerical examples to elucidate and illustrate the practical applications and implications of these ideas.

1.1 Notations: For Fuzzy Matrix of A ∈ $(F)_n$,

A^T : transpose of A,
R(A) : Row space of A,
C(A) : Column space of A,
N(A) : Null Space of A
A^+ : Moore-Penrose inverse of A ,
$(IF)_n$: Square Fuzzy Matrix.
$F_{[1\times n]}$: The matrix one row n columns.
$F_{[n\times 1]}$: The matrix n rows one column.
IFM: Intuitionistic Fuzzy Matrix
NFM: Neutrosophic Fuzzy Matrix

2. Definitions and Examples

2. Range and Column Symmetric Intuitionistic Fuzzy Matrices

Definition:2.1 Let the intuitionistic Fuzzy Matrix A of order m rows and n columns is in the form of $A=[y_{ij},<a_{ij\alpha},a_{ij\beta}>]$,where $a_{ij\alpha}$ and $a_{ij\beta}$ are called the degree of membership and also the non-membership of y_{ij} in A , it preserving the condition $0 \le a_{ij\alpha}+a_{ij\beta} \le 1.$

Definition 2.2. Suppose a and b are two IFM elements $a=<a_{ij\alpha},a_{ij\beta}>, b=<b_{ij\alpha},b_{ij\beta}>,$ are component wise addition and multiplication are described as,

$a+b=<\text{maximum of } \{a_{ij\alpha},b_{ij\alpha}\}, \text{minimum of } \{a_{ij\beta},b_{ij\beta}\}>$

and $a.b=<\text{minimum of } \{a_{ij\alpha},b_{ij\alpha}\}, \text{ maximum of } \{a_{ij\beta},b_{ij\beta}\}>.$

Definition 2.3. The scalar multiplication of intuitionistic Fuzzy Matrix $A = (< a_{ij\alpha}, a_{ij\beta} >) \in F_{m\times n}$, such that 0≤ a ≤ 1 is denoted by $aA = (<\text{minimum of } \{a, a_{ij\alpha}\}, \text{maximum of } \{(1-a), a_{ij\beta}\} >) \in F_{m\times n}$

Definition: 2.4 Let A be a IFM, if R[A]= R[A^T] then A is called as range symmetric.

Example: 2.1 $A = \begin{bmatrix} <0.2,0.5> & <0,0> & <0.7,0.2> \\ <0,0> & <0,0> & <0,0> \\ <0.7,0.2> & <0,0> & <0.3,0.2> \end{bmatrix}$,

The following matrices are not range symmetric

$$A = \begin{bmatrix} <1,0> & <1,0> & <0,0> \\ <0,0> & <1,0> & <1,0> \\ <0,0> & <0,0> & <1,0> \end{bmatrix}, \quad A^T = \begin{bmatrix} <1,0> & <0,0> & <0,0> \\ <1,0> & <1,0> & <0,0> \\ <0,0> & <1,0> & <1,0> \end{bmatrix},$$

$(<1,0> \quad <1,0> \quad <0,0>) \in R(A)$, $\quad (<1,0> \quad <1,0> \quad <0,0>) \in R(A^T)$

$(<0,0> \quad <1,0> \quad <1,0>) \in R(A)$, $\quad (<0,0> \quad <1,0> \quad <1,0>) \in R(A^T)$

$(<0,0> \quad <0,0> \quad <1,0>) \in R(A)$, $\quad (<0,0> \quad <0,0> \quad <1,0>) \notin R(A^T)$

$R(A) \notin R(A^T)$

Definition : 2.5 Let A ∈ F_n be an Intuitionistic fuzzy matrix , if N(A) = N(A^T) then A is called kernel symmetric IFM where N(A) = {x/xA = <0,0> and x ∈ $F_{1\times n}$},

Example: 2.2 Let us consider IFM

$$A = \begin{bmatrix} <0.4,0.5> & <0,0> & <0.6,0.4> \\ <0,0> & <0,0> & <0,0> \\ <0.4,0.5> & <0,0> & <0.4,0.3> \end{bmatrix},$$

$$N(A) = N(A^T) = \{(<0,0>< a_{ij\alpha}, a_{ij\beta} ><0,0>) / a_{ij\alpha}, a_{ij\beta} \in F\}$$

Definition: 2.6 Let P be a IFM, if C[P]= C[P^T] then P is called as Column symmetric.

Example: 2.3 Let us consider $P = \begin{bmatrix} <0.3,0.5> & <0,0> & <0.7,0.2> \\ <0,0> & <0,0> & <0,0> \\ <0.7,0.2> & <0,0> & <0.3,0.2> \end{bmatrix}$,

The following matrices are not column symmetric

$$P = \begin{bmatrix} <1,0> & <1,0> & <0,0> \\ <0,0> & <1,0> & <1,0> \\ <0,0> & <0,0> & <1,0> \end{bmatrix}, \; P^T = \begin{bmatrix} <1,0> & <0,0> & <0,0> \\ <1,0> & <1,0> & <0,0> \\ <0,0> & <1,0> & <1,0> \end{bmatrix},$$

$(<1,0> \quad <0,0> \quad <0,0>)^T \in C(\mathrm{P})$, $\quad (<1,0> \quad <0,0> \quad <0,0>)^T \notin C(\mathrm{P}^T)$

$(<1,0> \quad <1,0> \quad <0,0>)^T \in C(\mathrm{P})$, $\quad (<1,0> \quad <1,0> \quad <0,0>)^T \in C(\mathrm{P}^T)$

$(<0,0> \quad <1,0> \quad <1,0>)^T \in C(\mathrm{P})$, $\quad (<0,0> \quad <1,0> \quad <1,0>)^T \in C(\mathrm{P}^T)$

Definition 2.7. For Intuitionistic f u z z y matrix A $\in F_n$ is s - symmetric Intuitionistic f u z z y matrix iff $A = VA^TV$

Definition 2.8 For Intuitionistic f u z z y matrix A $\in F_n$ is s-kernel symmetric Intuitionistic fuzzy matrix if $N(A) = N(VA^TV)$.

Definition 2.9 For Intuitionistic f u z z y matrix A $\in F_n$ is s- k-kernel symmetric Intuitionistic fuzzy matrix if $N(A) = N(KVA^TVK)$.

Remark 2.1 We observe that s-k-symmetric Intuitionistic fuzzy matrix is s-k-kernel symmetric Intuitionistic fuzzy matrix because if A is s-k-symmetric then A $= KVA^TVK$, which means that A is s-k-kernel symmetric Intuitionistic fuzzy matrix, then N (A)= N ($KVA^T VK$).

The reverse, however, is not always true. This V is demonstrated in the example that follows.

Example 2.4. Let us consider IFM, $A = \begin{bmatrix} <1,0> & <0.2,0.3> \\ <0.2,0.3> & <0.4,0.3> \end{bmatrix}$,

$$KVA^TVK = \begin{bmatrix} <0.4,0> & <0.2,0> \\ <0.2,0> & <1,0> \end{bmatrix} \neq A$$

Here $A = KA^T K$

Therefore A is symmetric IFM, κ –symmetric IFM, s- κ -kernel symmetric IFM but not s- κ –symmetric IFM.

Example 2.5. Let u consider IFM, $V = \begin{bmatrix} <0,0> & <1,0> \\ <1,0> & <0,0> \end{bmatrix}, A = \begin{bmatrix} <0.5,0.2> & <0.4,0.2> \\ <0.4,0.2> & <0.5,0.2> \end{bmatrix}$

$$KVA^TVK = \begin{bmatrix} <0.5,0.2> & <0.4,0.2> \\ <0.4,0.2> & <0.5,0.2> \end{bmatrix} = A$$

A is symmetric, s-κ-symmetric and hence therefore s- k-kernel symmetric.

Example 2.6 Let us consider IFM

$$K = \begin{bmatrix} <0,0> & <1,0> & <0,0> \\ <1,0> & <0,0> & <0,0> \\ <0,0> & <0,0> & <1,0> \end{bmatrix}, V = \begin{bmatrix} <0,0> & <0,0> & <1,0> \\ <0,0> & <1,0> & <0,0> \\ <1,0> & <0,0> & <0,0> \end{bmatrix}$$

$$K \neq V, K \neq I \text{ and } KV \neq VK$$

$$A = \begin{bmatrix} <0,0> & <0,0> & <1,0> \\ <0.2,0.3> & <1,0> & <0,0> \\ <0.2,0.3> & <0.4,0.5> & <0,0> \end{bmatrix}, KV = \begin{bmatrix} <0,0> & <1,0> & <0,0> \\ <0,0> & <0,0> & <1,0> \\ <1,0> & <0,0> & <0,0> \end{bmatrix}$$

$$VK = \begin{bmatrix} <0,0> & <0,0> & <1,0> \\ <1,0> & <0,0> & <0,0> \\ <0,0> & <1,0> & <0,0> \end{bmatrix}$$

$$KVA^TVK = \begin{bmatrix} <1,0> & <0.4,0> & <0,0> \\ <0,0> & <0,0> & <1,0> \\ <0.2,0> & <1,0> & <0,0> \end{bmatrix} \neq A$$

$$A \neq KVA^TVK$$

Hence A is not s- k-symmetric. But s- k- kernel symmetric.

i.e) $N(A) = N(KVA^TVK) = <0,0>$

3. Range and Column Symmetric Neutrosophic Fuzzy Matrices

Definition : 3.1 Let $P \in F_n$ be a Neutrosophic fuzzy matrix , if $N(P) = N(P^T)$ then P is called kernel symmetric NFM where N(P)= {x/xP = (0,0,0) and $x \in F_{1\times n}$},

Example: 3.1 Let us consider NFM $P = \begin{bmatrix} (0.4,0.5,0.6) & (0,0,0) & (0.6,0.4,0.8) \\ (0,0,0) & (0,0,0) & (0,0,0) \\ (0.4,0.5,0.7) & (0,0,0) & (0.4,0.3,0.6) \end{bmatrix}$,

$$N(P) = N(P^T) = (0,0,0)$$

Definition 3.2 Unit Neutrosophic fuzzy matrix

A square Neutrosophic fuzzy matrix is said to be unit Neutrosophic fuzzy matrix if a_{ii} = (1,1,0) and a_{ij} = (0,1,1) $i \neq j$, for all $i = j$. It is denoted by I.

Example: 3.2 Let us consider NFM, $I = \begin{bmatrix} (1,1,0) & (0,1,1) & (0,1,1) \\ (0,1,1) & (1,1,0) & (0,1,1) \\ (0,1,1) & (0,1,1) & (1,1,0) \end{bmatrix}$

Definition3.3 Symmetric Neutrosophic fuzzy matrix

A square Neutrosophic fuzzy matrix is said to be symmetric Neutrosophic fuzzy matrix if $a_{ij} = a_{j\mathrm{i}}$

Example: 3.3 Let us consider NFM $P = \begin{bmatrix} (0.3,0.5,0.8) & (0,0,0) & (0.5,0.3,0.1) \\ (0,0,0) & (0,0,0) & (0,0,0) \\ <0.5,0.3,0.1> & (0,0,0) & <0.3,0.5,0.7> \end{bmatrix}$,

Definition 3.4. Permutation neutrosophic fuzzy matrix

A Permutation Neutrosophic fuzzy from the same size identity matrix by a permutation of rows. Every row single (1,1,0) with (0,0,1) ‘s everywhere else.

Example:3.4 Let us consider NFPM, $K = \begin{bmatrix} (0,0,1) & (0,0,1) & (1,1,0) \\ (0,0,1) & (1,1,0) & (0,0,1) \\ (1,1,0) & (0,0,1) & (0,0,1) \end{bmatrix}$

Definition: 3.5 Let P be a NFM , if R[P] = R[P^T] then P is called as range symmetric.

Example: 3.5 Let us consider NFM $P = \begin{bmatrix} (0.2,0.5,0.7) & (0,0,0) & (0.6,0.4,0.2) \\ (0,0,0) & (0,0,0) & (0,0,0) \\ <0.6,0.4,0.2> & (0,0,0) & <0.3,0.5,0.7> \end{bmatrix}$,

The following matrices are not range symmetric

$$P = \begin{bmatrix} (1,1,0) & (1,1,0) & (0,0,0) \\ (0,0,0) & (1,1,0) & (1,1,0) \\ (0,0,0) & (0,0,0) & (1,1,0) \end{bmatrix},\ P^T = \begin{bmatrix} (1,1,0) & (0,0,0) & (0,0,0) \\ (1,1,0) & (1,1,0) & (0,0,0) \\ (0,0,0) & (1,1,0) & (1,1,0) \end{bmatrix}$$

$$[(1,1,0)\ (1,1,0)\ (0,0,0)] \in R(\mathrm{P})\ ,\ [(1,1,0)\ (1,1,0)\ (0,0,0)] \in R(\mathrm{P}^T)$$

$$[(0,0,0)\ (1,1,0)\ (1,1,0)] \in R(\mathrm{P})\ ,\ [(0,0,0)\ (1,1,0)\ (1,1,0)] \in R(\mathrm{P}^T)$$

$$[(0,0,0)\ (0,0,0)\ (1,1,0)] \in R(\mathrm{P})\ ,[(0,0,0)\ (0,0,0)\ (1,1,0)] \notin R(\mathrm{P}^T)$$

$$R(\mathrm{P}) \notin R(\mathrm{P}^T)$$

Definition:3.6 Let P be a NFM, if C[P] = C[P^T] then P is called as Column symmetric.

Example:3.6 Let us consider $P = \begin{bmatrix} <0.3,0.5,0.4> & <0,0,1> & <0.7,0.2,0.5> \\ <0,0,1> & <0,0,1> & <0,0,1> \\ <0.7,0.2,0.5> & <0,0,1> & <0.3,0.2,0.4> \end{bmatrix}$,

The following matrices are not column symmetric

$$P = \begin{bmatrix} <1,1,0> & <1,1,0> & <0,0,1> \\ <0,0,1> & <1,1,0> & <1,1,0> \\ <0,0,1> & <0,0,1> & <1,1,0> \end{bmatrix},\ P^T = \begin{bmatrix} <1,1,0> & <0,0,1> & <0,0,1> \\ <1,1,0> & <1,1,0> & <0,0,1> \\ <0,0,1> & <1,1,0> & <1,1,0> \end{bmatrix},$$

$$(<1,1,0> \quad <0,0,1> \quad <0,0,1>)^T \in C(\mathrm{P})\ , \qquad (<1,1,0> \quad <0,0,1> \quad <0,0,1>)^T \notin C(\mathrm{P}^T)$$

$$(<1,1,0> \quad <1,1,0> \quad <0,0,1>)^T \in C(\mathrm{P})\ , \qquad (<1,1,0> \quad <1,1,0> \quad <0,0,1>)^T \in C(\mathrm{P}^T)$$

$$(<0,0,1> \quad <1,1,0> \quad <1,1,0>)^T \in C(\mathrm{P})\ , \qquad (<0,0,1> \quad <1,1,0> \quad <1,1,0>)^T \in C(\mathrm{P}^T)$$

$$C(\mathrm{P}) \notin C(\mathrm{P}^T)$$

Definition 3.7 A neutrosophic matrix Q of order $m \times p$ is defined as Q = $\{x, T_{q(x)}, I_{q(x)}, F_{q(x)}\}$, where T,I,F are called truth – membership, indeterminacy – membership and falsity – membership values of the i, j^{th} element in Q satisfying the condition $0 \le T_{q(x)}, I_{q(x)}, F_{q(x)} \le 3$ for all (i,j).

Definition 3.8 Sum and product of a Neutrosophic Fuzzy Set

Let X be a non - empty set. A neutrosophic fuzzy sets A and B is of the form A = { x,$\mu_{P(x),}$,σ_P(x),ν_P(x): x ∈ X} and B = { x,$\mu_{Q(x),}$,σ_Q(x),ν_Q(x) : x ∈ X} then, the sum, and product of two Neutrosophic fuzzy sets is defined by,

A+B = $\{x, (\mu_P(x) \vee \mu_Q(x), \sigma_P(x) \vee \sigma_Q(x), \nu_P(x) \wedge \nu_Q(x)\}$

AB = $\{x, (\mu_P(x) \wedge \mu_Q(x), 1-\sigma_P(x) \wedge 1-\sigma_Q(x), \nu_P(x) \vee \nu_Q(x)\}$

Example:3.7 Let us consider NFM

$$P=\begin{bmatrix}(0,0,0.5) & (0,0,0) & (0.3,0,0)\\ (0,0,0) & (0,0,0) & (0,0,0)\\ (0.7,0,0) & (0,0,0) & (0.3,0.2,0)\end{bmatrix}, Q=\begin{bmatrix}(1,1,0) & (0,0,1) & (0,0,1)\\ (0,0,1) & (0,0,1) & (1,1,0)\\ (0,0,1) & (1,1,0) & (0,0,1)\end{bmatrix}$$

$$P+Q=\begin{bmatrix}(1,1,0) & (0,0,0) & (0.3,0,0)\\ (0,0,0) & (0,0,0) & (1,1,0)\\ (0.7,0,0) & (1,1,0) & (0.3,0.2,0)\end{bmatrix}, PQ=\begin{bmatrix}(0,1,0.5) & (0.3,1,0) & (0,1,0)\\ (0,1,0) & (0,1,0) & (0,1,0)\\ (0.7,1,0) & (0.3,1,0) & (0,1,0)\end{bmatrix}$$

4.k-KERNEL SYMMETRIC NFM

Definition: 4.1 Let P be a Neutrosophic fuzzy matrix. P belongs to $(NF)_n$ is called k-Kernel symmetric Neutrosophic fuzzy if N(P) = N(KP^TK)

Note:4.1 Let P is k-Symmetric NFM implies it is k-kernel symmetric NFM, for P = $K(P^T)K$ spontaneously implies N(P) = N(KP^T K) . Shows that the converse need not be true.

Example: 4.1 Let us Consider NFM

$$P=\begin{bmatrix}(0,0,0.5) & (0,0,0.4) & (0.3,0.4,0.5)\\ (0.5,0.4,0.6) & (0.1,0.4,0.6) & (0,0,0.4)\\ (0.4,0.5,0.3) & (0.3,0.4,0.5) & (0,0,0.3)\end{bmatrix}, K=\begin{bmatrix}(0,0,1) & (0,0,1) & (1,1,0)\\ (0,0,1) & (1,1,0) & (0,0,1)\\ (1,1,0) & (0,0,1) & (0,0,1)\end{bmatrix}$$

$$KP^TK=\begin{bmatrix}(0,0,0.3) & (0,0,0.4) & (0.3,0.4,0.5)\\ (0.3,0,0.5) & (0.1,0,0.6) & (0,0.4,0.4)\\ (0.4,0,0.3) & (0.5,0,0.6) & (0,0.4,0.5)\end{bmatrix}$$

Therefore, P $\neq$ KP^T K

But, N(P) = N(KP^T K) = $(0,0,0)$

Definition 4.2: A NFM P belongs to F_n is s-symmetric NFM iff P = VP^TV.

Example:4.2 Let us consider NFM $P=\begin{bmatrix}<0.4,0.3,0.2> & <0,0,1> & <0.5,0.4,0.3>\\ <0,0,1> & <0,0,1> & <0,0,1>\\ <0.5,0.4,0.3> & <0,0,1> & <0.3,0.2,0.4>\end{bmatrix}$,

$$V=\begin{bmatrix}<0,0,0> & <0,0,0> & <1,1,0>\\ <0,0,0> & <1,1,0> & <0,0,0>\\ <1,1,0> & <0,0,0> & <0,0,0>\end{bmatrix}$$

Definition 4.4: A NFM P belongs to F_n is s-column symmetric NFM iff C(P) = C(VP^TV).

Example:4.3 Let us consider $P=\begin{bmatrix}<0.7,0.4,0.5> & <0,0,1> & <0.8,0.2,0.1>\\ <0,0,1> & <0,0,1> & <0,0,1>\\ <0.8,0.2,0.1> & <0,0,1> & <0.5,0.7,0.3>\end{bmatrix}$,

$$V=\begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \end{bmatrix}$$

Definition 4.5: A NFM P belongs to F_n is s-k-column symmetric NFM iff C(P) = C(KVP^{T}VK).

Example:4.4 Let us consider NFM $P=\begin{bmatrix} <0.7,0.3,0.4> & <0.5,0.3,0.4> \\ <0.5,0.3,0.4> & <0.7,0.3,0.5> \end{bmatrix}$,

$$\text{K}=\begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix}, V=\begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix},$$

Remark 2.1: We observe that s-k-symmetric NFM is s-k-column symmetric NFM since P = KVP^{T}VK if P is s-k-symmetric NFM. Thus, C(P) = C(KVP^{T}VK) ,indicating that P is an NFM with s-k-column symmetry.

Example 4.5. Let us consider NFM, $V=\begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix}$,

$$P=\begin{bmatrix} <0.7,0.3,0.4> & <0.5,0.3,0.4> \\ <0.5,0.3,0.4> & <0.7,0.3,0.5> \end{bmatrix}, \text{K}=\begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix},$$

$$KVP^TVK=\begin{bmatrix} <0.7,0.3,0.4> & <0.5,0.3,0.4> \\ <0.5,0.3,0.4> & <0.7,0.3,0.5> \end{bmatrix}=P$$

P is symmetric, s-κ-symmetric and hence therefore s- k-column symmetric NFM.

Example 4.6. Let us consider NFM

$$K=\begin{bmatrix} <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \\ <0,0,0> & <0,0,0> & <1,1,0> \end{bmatrix}, V=\begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \end{bmatrix}$$

$K\neq V, K\neq I$ and $KV\neq VK$

$$P=\begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0.5,0.3,0.4> & <1,1,0> & <0,0,0> \\ <0.4,0.2,0.6> & <0.5,0.3,0.4> & <0,0,0> \end{bmatrix}$$

$$KV=\begin{bmatrix} <0,1,0> & <0,1,0> & <0,1,0> \\ <0,1,0> & <0,1,0> & <1,1,0> \\ <1,1,0> & <0,1,0> & <0,1,0> \end{bmatrix}, VK=\begin{bmatrix} <0,1,0> & <0,1,0> & <1,1,0> \\ <1,1,0> & <0,1,0> & <0,1,0> \\ <0,1,0> & <1,1,0> & <0,1,0> \end{bmatrix}$$

$$KVP^TVK = \begin{bmatrix} <0,0,0> & <0,0.2,0> & <0,0,0> \\ <0,0,0> & <0,0,0> & <1,0,0> \\ <0.5,0,0> & <0.4,0,0> & <0,0,0> \end{bmatrix} \neq P$$

$P \neq KVP^TVK$ is not s- κ –symmetric iff not s- κ-column symmetric.

5. Interval valued Secondary k-kernel symmetric fuzzy matrix

Definition 5.1. For a fuzzy matrix $A=[A_L, A_U] \in IVFM_{nn}$ is an Interval valued s - symmetric fuzzy fuzzy matrices iff $A_L = VA_L{}^TV$ and $A_U = VA_U{}^TV$

Definition 5.2 For a fuzzy matrix $A=[A_L, A_U] \in IVFM_{nn}$ is an Interval valued s- ks fuzzy matrix iff N(A) = N(VA^TV)

Definition 5.3. For a fuzzy matrix $A=[A_L, A_U] \in IVFM_{nn}$ is Interval valued s-k- ks fuzzy matrix iff $N(A_L) = N(KVA_L{}^TVK), N(A_U) = N(KVA_U{}^TVK)$

Remark 5.1 If A is interval valued s-k-symmetric, then A_L =$KVA_L{}^TVK$, and A_U =$KVA^T{}_UVK$, indicating that it is interval valued s-k-ks fuzzy matrix, then N(A_L) = N($KVA_L{}^TVK$), N (A_U) = N ($KVA_U{}^TVK$).We note that s-k-symmetric fuzzy matrix is s-k-ks fuzzy matrix.

The opposite isn't always true, though. The example that follows illustrates this V

Example 5.1 Let $K = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, V = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $A=[A_L, A_U] = \begin{bmatrix} [0.2,0.2] & [0.6,0.8] \\ [0.6,0.8] & [0.2,0.2] \end{bmatrix}$,

is an interval valued symmetric, interval valued s – κ symmetric and hence therefore interval valued s – κ kernel symmetric.

Hence , $A_L = \begin{bmatrix} 0.2 & 0.6 \\ 0.6 & 0.2 \end{bmatrix}, A_U = \begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix}$

$$KVA_L{}^TVK = \begin{bmatrix} 0.2 & 0.6 \\ 0.6 & 0.2 \end{bmatrix} = A_L$$

$KVA_L{}^TVK =$ A_L. Similar we can get $KVA_U{}^TVK =$ A_U.

N(A_L)= $N(KVA_L{}^TVK) = \{0\}$

$A=[A_L, A_U]$ is an Interval valued s – κ kernel symmetric.

Example 5.2 . Let us consider Fuzzy Matrices

$$K = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, V = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A=\left[A_L,A_U\right]=\begin{bmatrix}[0,0.1] & [0,0.1] & [0.2,0.2]\\ [0.2,0.3] & [0.3,0.4] & [0.2,0.3]\\ [0.3,0.3] & [0.1,0.1] & [0,0.1]\end{bmatrix}$$ is an Interval valued s – κ kernel symmetric but not an Interval valued s – κ symmetric.

$$A_L=\begin{bmatrix}0 & 0.1 & 0.2\\ 0.2 & 0.3 & 0.2\\ 0.3 & 0.1 & 0\end{bmatrix}, A_U=\begin{bmatrix}0.1 & 0.1 & 0.2\\ 0.3 & 0.4 & 0.3\\ 0.3 & 0.1 & 0.1\end{bmatrix}$$

$$KVA_L^TVK=\begin{bmatrix}0.3 & 0.1 & 0.1\\ 0.2 & 0 & 0.2\\ 0.2 & 0.3 & 0\end{bmatrix}$$

$$A_L\neq KVA_L^TVK$$

Hence A is not s- k-symmetric.

But s- k- kernel symmetric.

i.e) $N(A)=N(KVA^TVK)=\{0\}$

6. Interval valued Secondary k-Range symmetric Intuitionistic fuzzy matrix

Definition:6.1 Interval-valued intuitionistic fuzzy matrix (IVIFM): An interval valued intuitionistic fuzzy matrix (IVIFM) P of order m×n is defined as P = $[x_{ij},< p_{ij\mu},p_{ij\nu} >]_{m\times n}$ where $p_{ij\mu}$ and $p_{ij\nu}$ are both the subsets of [0,1] which are denoted by $p_{ij\mu}=[p_{ij\mu L},p_{ij\mu U}]$ and $p_{ij\nu}=[p_{ij\nu L},p_{ij\nu U}]$ which maintaining the condition $0\le p_{ij\mu U}+p_{ij\nu U}\le 1$, $0\le p_{ij\mu L}+p_{ij\nu L}\le 1$, $0\le p_{\mu L}\le p_{\mu U}\le 1$, $0\le p_{\nu L}\le p_{\nu U}\le 1$ for i = 1,2,⋯,m and j = 1,2,⋯,n.

Example 6.1 Let $P=\begin{bmatrix}<[0.4,0.4],[0.5,0.5]> & <[0.4,0.5],[0.2,0.4]>\\ <[0.4,0.5],[0.2,0.4]> & <[0.4,0.4],[0.3,0.3]>\end{bmatrix}$,

Hence, Lower Limit IFM, $P_L=[P_{\mu L},P_{\nu L}]=\begin{bmatrix}<0.4,0.4> & <0.4,0.5>\\ <0.4,0.5> & <0.4,0.4>\end{bmatrix}$,

Upper Limit IFM $P_U=[P_{\mu U},P_{\nu U}]=\begin{bmatrix}<0.5,0.5> & <0.2,0.4>\\ <0.2,0.4> & <0.3,0.3>\end{bmatrix}$

Remark 6.1 If P is interval valued s-k-symmetric, then $[P_{\mu L},P_{\nu L}]$=KV$[P_{\mu L},P_{\nu L}]^T$VK, and A_U =KVA^T_UVK, indicating that it is interval valued (IV) s-k-RS Intuitionistic fuzzy matrix, then R($[P_{\mu L},P_{\nu L}]$) = R(KV$[P_{\mu L},P_{\nu L}]^T$VK), R($[P_{\mu U},P_{\nu U}]$) = R(KV$[P_{\mu U},P_{\nu U}]^T$VK).We note that s-k-symmetric Intuitionistic fuzzy matrix is s-k-RS Intuitionistic fuzzy matrix.

The opposite isn't always true, though. The example that follows illustrates this V

Example 6.2 Let $K=\begin{bmatrix}<1,0> & <0,1>\\ <0,1> & <1,0>\end{bmatrix}, V=\begin{bmatrix}<0,1> & <1,0>\\ <1,0> & <0,1>\end{bmatrix}$ and $P=<[P_{\mu L},P_{\mu U}],[P_{\nu L},P_{\nu U}]>\in \text{IVIFM}_{nn}$

$$P=\begin{bmatrix}<[0.4,0.4],[0.5,0.5]> & <[0.4,0.5],[0.2,0.4]>\\ <[0.4,0.5],[0.2,0.4]> & <[0.4,0.4],[0.3,0.3]>\end{bmatrix},$$

is an interval valued symmetric, interval valued s – κ symmetric and hence therefore interval valued s – κ range symmetric.

Hence, $P_L = \begin{bmatrix} <0.4,0.4> & <0.4,0.5> \\ <0.4,0.5> & <0.4,0.4> \end{bmatrix}, P_U = \begin{bmatrix} <0.5,0.5> & <0.2,0.4> \\ <0.2,0.4> & <0.3,0.3> \end{bmatrix}$

$$KV = \begin{bmatrix} <1,0> & <0,1> \\ <0,1> & <1,0> \end{bmatrix}\begin{bmatrix} <0,1> & <1,0> \\ <1,0> & <0,1> \end{bmatrix} = \begin{bmatrix} <0,1> & <1,0> \\ <1,0> & <0,1> \end{bmatrix}$$

$$VK = \begin{bmatrix} <0,1> & <1,0> \\ <1,0> & <0,1> \end{bmatrix}\begin{bmatrix} <1,0> & <0,1> \\ <0,1> & <1,0> \end{bmatrix} = \begin{bmatrix} <0,1> & <1,0> \\ <1,0> & <0,1> \end{bmatrix}$$

$$KVP_L{}^T VK = \begin{bmatrix} <0,1> & <1,0> \\ <1,0> & <0,1> \end{bmatrix}\begin{bmatrix} <0.5,0.5> & <0.2,0.4> \\ <0.2,0.4> & <0.3,0.3> \end{bmatrix}\begin{bmatrix} <0,1> & <1,0> \\ <1,0> & <0,1> \end{bmatrix} = P_U$$

$R(P_L) = R(KVP_L{}^T VK)$

$A = [A_L, A_U]$ is an IV s – κ RS.

Example 6.3 . Let us consider IFM

$K = \begin{bmatrix} <1,0> & <0,1> \\ <0,1> & <1,0> \end{bmatrix}, V = \begin{bmatrix} <0,1> & <1,0> \\ <1,0> & <0,1> \end{bmatrix}$ and $P = <[P_{\mu L}, P_{\mu U}],[P_{\nu L}, P_{\nu U}]> \in \text{IVIFM}_{nn}$

$P = \begin{bmatrix} <[0,0.2],[0,1]> & <[0.2,0.5],[0.2,0.4]> \\ <[0.2,0.5],[0.2,0.4]> & <[0.2,0.2],[0.3,0.4]> \end{bmatrix}$, $P_U = \begin{bmatrix} <0,1> & <0.2,0.4> \\ <0.2,0.4> & <0.3,0.4> \end{bmatrix}$,

$$KVP_U{}^T VK \neq P_U$$

Here $P = KP^T{}_U K$

Therefore P is symmetric IFM, κ –symmetric IFM, s- κ -RS IFM but not s- κ –symmetric IFM.

7. Interval valued Neutrosophic fuzzy matrix (IVNFM): An Interval valued Neutrosophic fuzzy matrix P of order m×n is defined as $P = [x_{ij}, < p_{ij\mu}, p_{ij\lambda}\ p_{ij\nu} >]_{m\times n}$ where $p_{ij\mu}$, $p_{ij\lambda}$ and $p_{ij\nu}$ are the subsets of [0,1] which are denoted by $p_{ij\mu} = [p_{ij\mu L}, p_{ij\mu U}]$, $p_{ij\lambda} = [p_{ij\lambda L}, p_{ij\lambda U}]$ and $p_{ij\nu} = [p_{ij\nu L}, p_{ij\nu U}]$ which maintaining the condition $0 \le p_{ij\mu U} + p_{ij\lambda U} + p_{ij\nu U} \le 3$, $0 \le p_{ij\mu L} + p_{ij\lambda L} + p_{ij\nu L} \le 3$, $0 \le p_{\mu L} \le p_{\mu U} \le 1$, $0 \le p_{\lambda L} \le p_{\lambda U} \le 1$, $0 \le p_{\nu L} \le p_{\nu U} \le 1$ for i = 1,2,⋯,m and j = 1,2,⋯,n.

Example 7.1 Consider an Interval valued Neutrosophic Fuzzy Matrix

$$P = \begin{bmatrix} <[0,0],[1,1],[1,1]> & <[0.1,0.3],[0.2,0.4],[0.2,0.5]> \\ <[0.1,0.3],[0.2,0.4],[0.2,0.5]> & <[0,0],[1,1],[1,1]> \end{bmatrix}$$

Lower Limit NFM, $[P_{\mu L}, P_{\lambda L}, P_{\nu L}] = \begin{bmatrix} <0,1,1> & <0.1,0.2,0,2> \\ <0.1,0.2,0.2> & <0,1,1> \end{bmatrix}$

Upper Limit NFM, $[P_{\mu U}, P_{\lambda U}, P_{\nu U}] = \begin{bmatrix} <0,1,1> & <0.3,0.4,0.5> \\ <0.3,0.4,0.5> & <0,1,1> \end{bmatrix}$

and $Q = \begin{bmatrix} <[0,0],[1,1],[1,1]> & <[0.2,0.4],[0.3,0.5],[0.1,0.5]> \\ <[0.2,0.4],[0.3,0.5],[0.1,0.5]> & <[0,0],[1,1],[1,1]> \end{bmatrix}$

Then, $P+Q = \begin{bmatrix} <[0,0],[1,1],[1,1]> & <[0.2,0.4],[0.2,0.4],[0.1,0.5]> \\ <[0.2,0.4],[0.2,0.4],[0.1,0.5]> & <[0,0],[1,1],[1,1]> \end{bmatrix}$

$PQ = \begin{bmatrix} <[0,0],[1,1],[1,1]> & <[0.1,0.3],[0.3,0.5],[0.2,0.5]> \\ <[0.1,0.3],[0.3,0.5],[0.2,0.5]> & <[0,0],[1,1],[1,1]> \end{bmatrix}$

$|P| = <[0,0],[1,1],[1,1]><[0,0],[1,1],[1,1]> + <[0.1,0.3],[0.2,0.4],[0.2,0.5]><[0.1,0.3],[0.2,0.4],[0.2,0.5]>$

$|P| = <[0,0],[1,1],[1,1]> + <[0.1,0.3],[0.2,0.4],[0.2,0.5]> = <[0.1,0.3],[0.2,0.4],[0.2,0.5]>$

Example 7.2 Let us consider interval valued NFM

$$P = \begin{bmatrix} <[0,0],[1,1],[1,1]> & <[0.1,0.3],[0.2,0.4],[0.2,0.5]> \\ <[0.1,0.3],[0.2,0.4],[0.2,0.5]> & <[0,0],[1,1],[1,1]> \end{bmatrix}$$

Lower Limit NFM, $[P_{\mu L}, P_{\lambda L}, P_{\nu L}] = \begin{bmatrix} <0,1,1> & <0.1,0.2,0,2> \\ <0.1,0.2,0.2> & <0,1,1> \end{bmatrix}$,

Upper Limit NFM, $[P_{\mu U}, P_{\lambda U}, P_{\nu U}] = \begin{bmatrix} <0,1,1> & <0.3,0.4,0.5> \\ <0.3,0.4,0.5> & <0,1,1> \end{bmatrix}$

$V = \begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix}$, $K = \begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix}$,

$$KVP_L^T VK = \begin{bmatrix} <0,1,0.2> & <0,0.2,0.2> \\ <0.1,0.2,0.2> & <0,1,0.2> \end{bmatrix}$$

$$KVP_L^T VK \neq P_L$$

Similarly, $KVP_U^T VK \neq P_U$

$$P_L = KP_L K$$

$$KP_L K = \begin{bmatrix} <0,1,0.2> & <0.1,0.2,0.2> \\ <0.1,0.2,0.2> & <0.1,1,0.2> \end{bmatrix} \neq P_L$$

Similarly, $P_U \neq KP_U K$

N (P_L) = $N(KVP_L^T VK) = <0,0,0>$

Therefore P is symmetric NFM, range symmetric NFM, kernel symmetric, but not both κ – symmetric and s- κ - symmetric NFM.

Example 7.3. Let us consider Interval valued NFM,

$$P=\begin{bmatrix} <[0.7,0.2],[0.3,0.4],[0.4,0.6]> & <[0.5,0.4],[0.3,0.3],[0.4,0.2]> \\ <[0.5,0.4],[0.3,0.3],[0.4,0.2]> & <[0.7,0.2],[0.3,0.4],[0.4,0.6]> \end{bmatrix} V$$

$$=\begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix},\ K=\begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix},$$

Lower Limit NFM, $P_L=\begin{bmatrix} <0.7,0.3,0.4> & <0.5,0.3,0.4> \\ <0.5,0.3,0.4> & <0.7,0.3,0.4> \end{bmatrix},$

Upper Limit NFM, $P_U=\begin{bmatrix} <0.2,0.4,0.6> & <0.4,0.3,0.2> \\ <0.4,0.3,0.2> & <0.2,0.4,0.6> \end{bmatrix}$

$$KVP_L{}^TVK=\begin{bmatrix} <0.7,0.3,0.4> & <0.5,0.3,0.4> \\ <0.5,0.3,0.4> & <0.7,0.3,0.4> \end{bmatrix}=P_L$$

P is symmetric, RS, s-κ-symmetric and hence s- k- kernel symmetric.

Example 7.4 Let us consider Interval valued NFM

Lower limit NFM, $[P_{\mu L},P_{\lambda L},P_{\nu L}]=\begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0.5,0.3,0.4> & <1,1,0> & <0,0,0> \\ <0.4,0.2,0.6> & <0.5,0.3,0.4> & <0,0,0> \end{bmatrix}$

$$K=\begin{bmatrix} <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \\ <0,0,0> & <0,0,0> & <1,1,0> \end{bmatrix},V=\begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \end{bmatrix}$$

$$KV=\begin{bmatrix} <0,1,0> & <0,1,0> & <0,1,0> \\ <0,1,0> & <0,1,0> & <1,1,0> \\ <1,1,0> & <0,1,0> & <0,1,0> \end{bmatrix},\ VK=\begin{bmatrix} <0,1,0> & <0,1,0> & <1,1,0> \\ <1,1,0> & <0,1,0> & <0,1,0> \\ <0,1,0> & <1,1,0> & <0,1,0> \end{bmatrix}$$

$$KVP_L{}^TVK=\begin{bmatrix} <0,0,0> & <0,0.2,0> & <0,0,0> \\ <0,0,0> & <0,0,0> & <1,0,0> \\ <0.5,0,0> & <0.4,0,0> & <0,0,0> \end{bmatrix}\neq P_L$$

$$P_L\neq KVP_L{}^TVK$$

Hence P is not s- k-symmetric and not RS. But s- k- kernel symmetric.

i.e) $N(P_L)=N(KVP_L{}^TVK)=<0,0,0>$

8. Theoretical Approaches

8.1Difference between classical matrices and fuzzy matrices

Classical matrices and fuzzy matrices are mathematical concepts that differ in their representation and handling of uncertainty:

Classical Matrices: Deterministic Values: Classical matrices contain precise, deterministic values. Each element of a classical matrix is a real number or an element from a specific set (e.g., integers) with no ambiguity or uncertainty.

Exact Arithmetic: Operations on classical matrices are based on exact arithmetic, such as addition and multiplication of real numbers or integers.

8.2Fuzzy Matrices: Fuzzy Values: Fuzzy matrices involve values that represent uncertainty or vagueness. Instead of exact numbers, elements in a fuzzy matrix are often membership degrees in a fuzzy set ranging from 0 to 1. These degrees indicate the extent to which an element belongs to a particular set.

Fuzzy Arithmetic: Operations on fuzzy matrices are typically based on fuzzy arithmetic, which involves rules for combining fuzzy values, taking into account uncertainty.

In summary, the key difference lies in the nature of values within the matrices. Classical matrices deal with precise, crisp values, while fuzzy matrices incorporate fuzzy, uncertain values that can represent vagueness or ambiguity in real-world data and decision-making scenarios. Fuzzy matrices are particularly useful in fields like fuzzy logic, decision-making, and expert systems where uncertainty needs to be considered.

Intuitionistic fuzzy matrices and neutrosophic matrices are both extensions of classical matrices designed to handle uncertainty, but they differ in their representation and approach. Here are the differences, advantages, and limitations of each:

8.3 Intuitionistic Fuzzy Matrices: Representation: Intuitionistic fuzzy matrices use three parameters for each element: membership degree (μ), non-membership degree (ν), and hesitation degree (λ).

These parameters represent the degree of membership, degree of non-membership, and degree of uncertainty or hesitation associated with each element.

Advantages: Comprehensive Representation: Intuitionistic fuzzy matrices provide a more comprehensive and nuanced representation of uncertainty compared to traditional fuzzy matrices, allowing for a more detailed modeling of complex decision-making scenarios.

Handling Hesitation: The inclusion of hesitation degrees allows decision-makers to explicitly express their uncertainty or reluctance about certain decisions.

Useful in Complex Problems: They are particularly useful in complex decision-making problems where uncertainty is high and different degrees of membership and non-membership need to be considered.

Limitations: Increased Complexity: The inclusion of three parameters per element makes computations more complex compared to traditional fuzzy matrices.

Data Collection Challenges: Determining the three parameters (μ, ν, and λ) for each element can be challenging in practice, especially when dealing with large datasets.

8.4 Neutrosophic Fuzzy Matrices: Representation: Neutrosophic matrices use three components for each element: truth degree (T), indeterminacy degree (I), and falsity degree (F).

These components represent the truth, indeterminacy, and falsity aspects of each element.

Advantages: Handling Contradictions: Neutrosophic matrices are capable of handling contradictions and inconsistencies in data, making them suitable for situations where conflicting information is present.

Flexibility: They offer flexibility in representing uncertainty, allowing for more complex and diverse uncertainty scenarios.

Useful in Conflicting Data: Neutrosophic matrices are particularly useful when dealing with data sources that provide conflicting or uncertain information.

Limitations: Complex Arithmetic: Performing arithmetic operations on neutrosophic matrices can be more complex compared to traditional or intuitionistic fuzzy matrices.

Lack of Standardization: Neutrosophic logic is less standardized and widely accepted compared to fuzzy logic, which may limit its practical application in some fields.
Data Interpretation: Interpreting the three components (T, I, F) can be subjective and context-dependent, which may lead to challenges in real-world applications.
In summary, the choice between intuitionistic fuzzy matrices and neutrosophic matrices depends on the specific problem and the nature of uncertainty and contradictions present in the data. Intuitionistic fuzzy matrices offer a more structured and nuanced representation of uncertainty, while neutrosophic matrices excel in handling conflicting information and are more flexible in uncertain scenarios. However, both approaches come with computational complexities and challenges in data interpretation.

Conclusion:8.5 In summary, these three types of matrices each offer a distinct approach to handling uncertainty and ambiguity in data. Fuzzy matrices focus on membership degrees, intuitionistic fuzzy matrices offer a more detailed representation with membership, non-membership, and hesitancy values, and neutrosophic matrices are tailored for dealing with truth, indeterminacy, and falsity in membership information. The choice of which matrix to use depends on the specific context and the nature of uncertainty involved, as outlined by their respective. We present equivalent characterizations of an Interval valued Fuzzy matrices, Intuitionistic Fuzzy matrices and Neutrosophic Fuzzy matrices explain and numerical examples are given. Also we discussed range symmetric, kernel symmetric, column symmetric of these three types of Matrices including numerical results.

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